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COVER

This is an illustration of Sandra Keith's poem, *And this is the Tale that Zeno Told,* where the classic tale of Achilles and the turtle illustrate the concept of convergence of an infinite series.

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Table of Contents

From the Editor

The essays and poetry in this issue are a rich intellectual feast. The journal and the movement are growing and are part of the climate of math reform. In the AMS Notices (1/95) editor Hugo Rossi pleads for more understandable, even if less technical, mathematical exposition, while letter writer John Wermer of Brown University complains of unintelligible colloquium talks, with one notable exception.

I think that the high level of exposition and the range of topics in our journal encourage math reform. The world wide network of readers and authors contribute to and define humanistic mathematics and influence doing, learning and teaching mathematics.

MESA,the UCLA Graduate School of Education, and the humanisitic math network are joining in a three year effort to enhance the perception of mathematics among middle school teachers and their students, and to participate in the mathematics education reform efforts.

Please note the enlarged list of associate editors. It is expected that the circulation will grow and our journal will eventually be published three times a year. I want to welcome the additional associate editors who will solicit essays, referee contributions, do some proof reading, and help determine the direction of the journal.

Thanks to the Exxon Education Foundation for their continuing support.

It is with bittersweet emotions that I celebrate the graduation in June and moving on of Michelle Ivey who has been the production manager of the journal as well as the MAA Notes volume *Essays in Humanistic Mathematics*. Michelle is a skilled technician and a conscientious manager, and a valuable partner in this endeavor. Her hand is evident in the handsome appearance and the editorial smoothness of the journal.

From the Production Manager

During the past three years I have had the unique experience of working side by side with Professor White on this journal and to publish a book entitled Essays in Humanistic Mathematics, published by the Mathematics Association of America. As a chemistry major, mathematics was no stranger to me, but *humanistic* mathematics?? Working on proofreading and layout of these essays really made me think about the ideas and ideals the authors were trying to express. I became more and more involved in a concept that was originally completely foreign to me; humanistic mathematics is teaching mathematics as a human creation, and therefore subject to the prejudices and whims of the people involved. This is far removed from the conventional approaches to mathematics where everything is in its final, "perfect" form, with no sense of the evolution of the concepts now taken for granted.

It has been a wonderful experience, and I have really enjoyed working with the articles and the authors. I applaud the work of the journal and the authors who contribute to it, and I hope that the journal will continue to be the wonderful forum for expression of ideas. I will miss being so closely involved in the production, but I am looking forward to receiving it at whatever graduate school I attend next year.

Our colleague, Stavros Busenberg succumbed to Lou Gehrig's disease last year at the age of 51. There was an outpouring of shock and sadness from around the world. Friends and colleagues came from great distances to participate in his memorial services.

The following year 200 colleagues, collaborators and friends joined in a four day international conference at Harvey Mudd College on mathematics, mathematical biology and other topics of interest to Stavros. It was a tribute to the energy and scope of his contributions and to the great esteem and affection with which he was regarded.

In memory of Stavros Busenberg

Mario Martelli
Harvey Mudd College
Claremont, CA

He was my best friend in the USA. With him I felt at home, like in Florence. We spoke Italian to each other, we talked about Italian literature, which he knew quite well, and every Thursday we went together to the Italian table at the Oldenborg center. We spoke Italian when we were doing research or we were talking about difficulties we found in teaching certain topics or strategies we had devised for achieving clarity and effectiveness in our lectures. We were frequently the first to arrive in the morning at the department and the last to leave in the evening. Many times our day together did not end with the words "Ciao a domani (see you tomorrow)", said before the short ride back home, because we called each other after dinner, if a new idea or a new question had crossed our mind. He always told me: "Mario, we will do great things together". He was right, except for the time.

We have done great things together. One was the organization of the 1990 International Meeting to honor a common friend, Ken Cooke, in his 65th birthday. I still remember when on the final day of the meeting, after the last talk, Stavros took the stand and said: "See you all at the turn of the century for another great meeting in Claremont". At that time I would have never imagined that the next International Meeting in Claremont would come only four years after the one in 1990 and would be in memory of Stavros. We worked side by side for the success of the meeting in honor of Ken, and we devoted several months and a lot of energy to edit two beautiful volumes of proceedings. Many participants have repeatedly told us that they have great memories of the meeting and many colleagues have expressed their admiration for the volumes. Springer-Verlag printed a poster to publicize one of them. Stavros and I also obtained some remarkable results. He talked about them in Europe, in China, in New Zealand. Back in Claremont from these trips, he would always tell me : "Mario, people really like the results. Everybody says that they are really beautiful. We must collect them in a book". We started planning it, we wrote the scheme and a few pages. We could not finish it, because Stavros left too soon.

What was amazing for me was the energy and versatility Stavros had. One day we would work together. The next day he would work with his students. The day after he had a visitor and they were working on another problem. Then came his work with Kenneth Cooke. Stavros' knowledge of mathematics was by far deeper than one could have expected in a first rate professional. He had an open mind for new ideas, new methods, and modern technology. TeX and LaTeX came and Stavros learned them immediately. Electronic mail came and Stavros was among the first to use it. Chaos theory was still in its infancy and Stavros already knew almost everything about it. I don't know how he could find the time to learn and master so many different things.

He has left in my life an empty space that will never be filled. Sometimes when I look back at my notes I find a page or two written by him while we were working together. I stop and go back with my memory to those beautiful days and it seems almost impossible that he is not with us anymore. Holding that piece of paper I almost hear his voice, I see his gestures, I look at him writing. I see Stavros and myself going to the photocopy

machine to make an extra copy for him or for me.

Our families were very close. We all enjoyed each others' company immensely. The toy railway Stavros built for his two sons, George and John, and all the equipment that came with it, became the toy railway of my son Teddy. It is still under the porch of my house and my youngest daughter Lisa plays with it. I still remember the many times Stavros brought me apricots or figs from his backyard. He knew that I like these fruits and he never forgot to pick some for me. We never came around to make some wine together, That was one of the projects we promised each other to do "when we had time". Unfortunately that time has never come.

An Italian poet has written:

We are
like leaves
on trees
in the Fall.

How true that is! Every leaf is going to fall, but surely Stavros, for some destiny which is hard to understand, fell too soon. He accomplished in his brief life what would have taken two lives for many others to complete. And during his brief but intense illness he gave a beautiful example of strength and dignity. My dearest friend, it has been a great privilege to have known you, and a great honor to have worked with you. I will keep your memory with me forever.

A tribute to Stavros Busenberg 1942 to 1993

Graeme Wake
Massey University
New Zealand

It is a pleasure to have this opportunity of writing a tribute to Professor Stavros Busenberg, late of Harvey Mudd College in Claremont. My contact with him began in 1988 when he indicated his desire to come to New Zealand as the first of three Fulbright Visiting Programs to assist in the establishment of "Mathematics-in-Industry Programs" within New Zealand. The record shows that his contribution was great and provided an example to us all in terms of achievement, professionalism, and tenacity. We endeavored to set in place during his three month stay, June through August 1989, a program of industrially funded Ph.D. projects. This is continuing in two or three places in New Zealand thanks in no small way to his input.

Stavros, Bonnie and family made their first visit to the Southern Hemisphere "the other side," or "down under" with a genuine sense of adventure towards exploring new parts previously unseen. They coped with the wet winter weather and our smallness with good humor and became our close friends and mentors as a result. It was with a deep feeling of regret and honor that I made the visit here to speak at the conference in his memory in the summer of 1994.

His contributions to our country was more than just in the professional mathematics sphere. He graced the finest aspects of the Fulbright Program with skill and was the best possible ambassador for his profession and country that could be imagined. He acquired a number of lasting contacts in New Zealand which led to ongoing publications. (I am pleased to be able to state that my Busenberg index equals unity!)

What made the work of Stavros so striking was his underlying gifts of good communication skills, clarity of thought, and sensitivity towards people and places. The hallmark of so many good mathematicians is their lack of ability to communicate with the non-mathematical community. Perhaps it was his training as an industrial mathematician, or his Southern European origin, or a combination of the two, that made him a strong exception to this rule. Stavros could communicate with anyone at any level on almost any topic. Our profession needs this skill and we're much poorer for having lost one of the leading figures in the industrial and biological mathematics scene who showed us how to do it. Of course, Stavros was an individualist with strong views but his keenness to get things done was

always tempered by his warm generous nature. He believed deeply in the Clinic experience at Harvey Mudd College and the Claremont Graduate School and was determined to see its needs influence the curriculum at Claremont.

Stavros was a great traveler and was known in many parts. We seem to be the only place in the Southern Hemisphere to be touched by his skills at first hand. This gives me a unique way to make this tribute with affection and gratitude to one of our most dedicated, capable, and courageous members. His loss at the peak of his career is felt by all of us "down under."

In Memoriam

Courtney Coleman
Harvey Mudd College
Claremont, CA

Stavros was like a ball of fire—brilliant, warming, illuminating, and awesome. Wherever he spoke, seminar room, classroom, lecture hall, he filled the space with his presence. How did he do it? The trace of an accent from his childhood Greek gave a touch of the exotic to his speech. But what shone through the accent and the showmanship of writing on the blackboard with either hand, or both at once, was the searching intellect, the love of learning, the astonishing breadth of knowledge in mathematics and physics and engineering and biology and literature and language. A few years ago he learned to speak Italian because he was to spend a sabbatical semester in Italy. But for him learning Italian meant more than picking up a Berlitz book or working his way through a mathematics vocabulary. To Stavros, Italian meant going to the master, to Dante, and that is just what he did. A few years ago he started to apply mathematical techniques to some epidemic models - so he learned biology and epidemiology. Many years ago, after he had earned degrees in mechanical engineering, he fell in love with epsilons and deltas (his Greek heritage?) and got his doctorate in mathematics. To him the world of the intellect had no boundaries. Marvelous things were out there waiting to be discovered and understood and explained and used. Applied mathematics, pure mathematics, science, engineering were all open to him and he refused to draw lines between them. His title was Professor of Mathematics, but the last two words in that title were too confining, too defining. He was truly a professor in the renaissance sense. Stavros' life will be for those of us who were privileged to work with him a shining example of humanism in the academy.

A Mentor and Friend Remembered

Melissa Aczon
Harvey Mudd College, '93

I first met and got to know Professor Busenberg during my sophomore year in college. He taught the very first differential equations class I took. As a teacher, he was one of the best I ever had. From the very first day, he amazed me with the energy and enthusiasm he brought to the class. He always motivated the equations and theorems, making the subject alive.

In the last semester that he was with us, I had another class with him, and I sometimes had the chance to assist him in another one of his classes. Although I could tell he was often in pain, he was still as incredibly enthusiastic and entertaining in the classroom as ever!

Much as he was a great teacher in the classroom, he was even more incredible as a research advisor. I was very fortunate to have had the opportunity to work on research with him for almost two years.

He introduced me to the beauty and joy of

mathematical research and opened up so many new and exciting areas for me to explore. He always worked hard to help me, and as a result, he also demanded and expected a lot. I am very grateful for that because I have been able to accomplish more than I ever imagined.
Although he was demanding, he was so in quite a cheerful way that never discouraged me. I never found it hard to get excited over the work I was doing with him because his enthusiasm and energy were contagious!

I did have my moments of frustration with my research—when I was "stuck" and did not know where to go next. After research sessions with him, my questions were answered, and I felt confident I could do anything once again.

Although he often pushed me hard, he also understood my difficulties, not only with my research project but with my other school work as well. He always asked me how my other classes and activities were going and listened to me while I talked about them. When I needed extra time to work on my other classes, he gave me time off from research.

He became not only my mentor, but my friend also. He readily offered advice when I needed it, whether it was for research, classes, summer work opportunities, or graduate school. He was one of the few professors I knew who truly cared about his students. He certainly took very good care of me.

Poetry by Lee Goldstein

Mathematizing

Mathematics begins
Upon a denominative
Foundation,
Where the forleading nonverbality
Is in place,
And when a verbal undifferencing
Is eliminative,
Then the symbolic shift
Does take.

—1993

Mentalism

Spiral of the belike,
Hoping of a like to be at the spiral center
Where the avoidant dislike is the reality principle
And is typically assumed to be from outside,
But where a dislike might, too, be about the center;
And there is the reality within,
And of that spiral wish,
When the dislike, he or she might convert it,
At least., as the spiral, into the neither like nor dislike

—1983

Psychosis

Nooscopic unrelatedness
Can drive the human intelligence of an incognizable numinosity
Thenceforeward, to the equations of the sphere,
Whilst this exceeding positive transposition
Can also beget, through the unconscious, an incipient eidetic
That splits the personal
And abets an insurgence of psychical energies
Unto the hallucinatory,
That is seeming or chaotic.

—1987

nooscopic: pertaining to the examination of the mind

Noesis

Emication of thought is not love,
Because it has no exteriority;
Yet whatever is muted willfully
Has a countenance;
Nay, the autoptic [   ] relativistic range of things
Can be a beauteous species,
If thought can be transmuted,
Even as in a mirror,
By law, or homologically into strings

emication: a sparking; a springing forth
autopic: relating to or based on personal observation

The lacona in the fifth line arises out of Wittgenstein's private language argument. I am informed that there is a connection between superstring theory and homology theory.

—1987

A New Start For College Mathematics; Or Mathematics' Greatest Hits

Harald M. Ness
University of Wisconsin Center
Fond du Lac, Wisconsin

After a great deal of success with their mathematics program for liberal arts students, *For All Practical Purposes*, COMAP has now embarked on a very ambitious project, with financial support from the National Science Foundation, to develop a new one year introductory mathematics course for majors in mathematics and mathematics intense fields. A draft of the two volume text, *Principles and Practice of Mathematics (Math 101-102)*, has been written, and COMAP has an agreement with Springer-Verlag for publication. The project is under the direction of the very capable and indefatigable Walter Meyer. The project can take pride in a very impressive list of authors and advisors: Andrew Gleason, Joe Malkevitch, David Moore, Henry Pollak, Alan Tucker, Joe Gallian, Frank Giardano, and Mike Olinick, to name a few.

I had the pleasure and privilege of attending a summer workshop on the project held at the West Point Military Academy where some of the authors presented topics and background material and interacted with the participants. In studying the materials and attending the workshop, I became aware that there is much in this program and its philosophy that is consistent with the philosophies expressed at the sundry meetings of the Humanistic Mathematics Network, and in articles and essays in the Network Journal, as well as the book published by MAA. I would like to relay my impressions of this important and innovative program.

They exit with no knowledge of the diversity of modern mathematics or the variety of applications and little motivation to continue in mathematics and mathematics related fields.

There are certain basic premises upon which the development of this program has been based. One major premise is that the prevailing narrow linear path through algebra, trigonometry, three semesters of calculus, and differential equations that has comprised precollege and the first two years of college mathematics for so long is no longer applicable, is counterproductive as far as motivating good students to continue in mathematics and mathematics related fields, and is inconsistent with the current culture.

The current curriculum dates back to when the only significant uses for mathematics were in engineering and physics and gives the impression that in mathematics, time has stood still since the eighteenth century. Mathematics has exploded in many different directions; the current curriculum, through the sophomore college year, does not give the students the flavor of this diversity. It gives them no knowledge of the large variety of modern applications of mathematics.

Rather than the metaphor of mathematics education as a pipeline, Sheldon Gordon, in his position paper on the project, refers to the current preparation for calculus-calculus curriculum viewed by the students as a tunnel, "... a long, dark, subterranean tunnel that they slowly crawl along, constantly scraping their knees, as they pass through poorly seen chambers of algebra, geometry, trigonometry, precalculus, calculus I,II, and III, and differential equations." If they survive the rats waiting to devour them along the way (my metaphor), they exit the tunnel with no knowledge of the diversity of modern mathematics or the variety of applications and, for most, little motivation to continue in mathematics and mathematics related fields. I'm sure that after exiting the tunnel, the shock of the bright light of further mathematics and other math related courses is considerable.

Many people, including C.P. Snow, Morris Kline, Jacob Bronowski, and others have been concerned with the isolation of mathematics from the rest of the culture. We in mathematics, to a large extent,

are responsible for this sad state of affairs. Our aloofness is reflected in our curriculum as described above, the way we teach mathematics, the way we relate to people outside of mathematics, and the way we write. A course in tune with contemporary culture showing the variety of aspects, including operations research, statistics, computer programming, business management, and most scientific and engineering professions would bring us back into the mainstream of the culture. Even after fourteen years of mathematics, our students have had no exposure to the contributions of mathematics to the technology of CAT scans, image processing, compact discs, fax machines computers, and the list goes on. The 1990 report, *Mathematics Outside of Mathematics Departments* by Sol Garfunkel and Gail Young shows that more students are enrolled in advanced mathematics courses taught outside of mathematics departments than are enrolled in courses taught within mathematics departments. This is further evidence that we are not providing the mathematics needed in the rest of the culture, but rather are isolating ourselves by continuing to teach from a very narrow viewpoint.

Our aloofness is reflected in our curriculum as described above, the way we teach mathematics, the way we relate to people outside of mathematics, and the way we write.

Another basic premise of the project is that students should become, early on, familiar with the diversity of modern mathematics. Hence, rather than the very intense depth of a small number of phenomena of the current curriculum, this program stresses breadth. The introductory college courses in other disciplines are of a survey nature, elucidating the various branches of the disciplines. It is believed that doing a similar thing in the introductory mathematics course would whet the appetite of students for further study of mathematics and mathematics related fields.

Immediate applicability of the mathematics to contemporary cultural needs is another premise upon which this project rests. To me, there is a clear departure in this program from the focus on the development of mathematics for mathematics sake to a focus on the culture and the place of mathematics in this culture as well as the contributions of mathematics to the rest of the culture and vice versa. Cultural stress (Ray Wilder's term) is responsible for the development of most of mathematics. This course clarifies that role.

Principles and Practice of Mathematics (the name may change) attempts to weave together a study of change which includes a prelude to the fundamental methods of calculus in a discrete setting, geometry with emphasis on vectors laying a foundation for linear algebra and calculus, linear algebra which is fundamental and permeates mathematics and its applications, discrete topics of graph theory and algorithms with an emphasis towards computer science, algebraic (and geometric) structure, and probability. Although an excellent job of considerable weaving takes place—certainly far more than current curricula—it is not as much as we idealists would prefer.

Is this, then, the ideal introductory college mathematics curriculum? Of course not; there is and cannot be one. Curriculum should be a dynamic endeavor, not a static structure. However, this is a start, and I believe an excellent start, on developing a more meaningful curriculum. I encourage all to peruse this fine initial attempt; a cursory glance will not do it justice.

I have advocated, for years, a core curriculum for the first two years of college mathematics. Both Sheldon Gordon and Joe Malkevitch, in their position papers, spoke to the desirability of removing the artificial barriers that separate various "branches" of mathematics. These barriers constitute a deterrent to effective and efficient mathematics education. These so called branches of mathematics are all intertwined. They sustain each other. They are one. Why can't we teach mathematics that way?

There is a great deal of resistance to change, especially in the college mathematics community. Remember the old cigarette commercial, "I'd rather fight than switch"? On a take off from that, Sol Garfunkel, in a recent editorial in *Consortium*, stated, "I'd rather work than fight". I'm with Sol.

Missing Dimensions of Mathematics Instruction

Walter Meyer
Adelphi University

A few years ago, I began work leading a project, funded by the National Science Foundation, to create a new gateway course to the mathematics major which would be an alternative to Calculus. It is not my purpose here to argue the pros and cons of this idea (interested parties should contact COMAP[1] for sample materials or more extensive descriptions). The original grant had an objective which was consistent with, but not explicitly centered on a humanistic approach. During the work of the grant, I formulated some further ideas for our authors to think about and circulated it as a position paper[2]. In that paper I explicitly addressed questions of a humanistic nature. As the ideas in that paper were not an explicit part of the original proposal and were not mentioned while recruiting authors, I did not feel justified in insisting that those ideas be addressed in writing our curriculum materials. Nonetheless, one member of the Humanistic Mathematics Network, Professor Harald Ness (see the companion article in this issue), feels that our curriculum materials do show a humanistic touch. It is impossible to publish our entire 1-year course here, but readers of this Journal might find my position paper of interest. The present article is a slightly edited version of that original position paper.

By restricting the scope of mathematics instruction to mathematics as an isolated endeavor, we have walled mathematics off from our culture to a degree that its future is problematical.

In writing the proposal for this project, my thoughts were mainly on how to express the breadth of mathematics itself. But there are other dimensions along which we can seek breadth in our instruction, dimensions which are little stressed at the present time. These missing dimensions could be decisive in making mathematics attractive to a larger number of students by making mathematics seem less isolated and more tied to thoughts and experiences that our students find familiar and congenial.

These dimensions include:

1. mathematics as an element of culture, evolving as civilization does;
2. mathematics as a social experience as well as a solitary one;
3. the hands-on approach to learning mathematics;
4. the connection of mathematics with major themes in our quest to understand the world about us;
5. the roots of mathematics in student interests and experience; and
6. mathematics as a story.

Other dimensions could perhaps be added. And, of course, the ones presented here are not independent. (For example, the value of applications occurs in a number of these dimensions.)

It is hard enough to teach the basic technicalities of mathematics. Why complicate things with all these other dimensions? Because, by restricting the scope of mathematics instruction to mathematics as an isolated endeavor, we have walled mathematics off from our culture to a degree that its future is problematical. Mathematics instruction mostly ignores these dimensions and is normally presented in a purely rational, emotionally flat and self-contained way, often detached from the interests and experience of ordinary people. The result is an image problem for mathematics and mathematicians. What is worse, many mathematicians acquiesce approvingly in the idiosyncratic image created, as illustrated by the following quote:

> An eloquent mathematician must, from the nature of things, ever remain as rare a phenomenon as a talking fish.... He has to turn his eye ever inwards, to see everything in its driest light, to train and inure himself to a habit

of internal and impersonal reflection and elaboration of abstract thought, which makes it most difficult for him to touch or enlarge upon any of those themes which appeal to the emotional nature of his fellow men [Sylvester 1991].

I believe that such a "dry" attitude toward mathematics is not inevitable. Imagine what it would have been like to have been a mathematics student of Galileo! Or to have been present at the banquet when Pythagoras (allegedly) sacrificed an ox in honor of the discovery of his famous theorem. Or to have been on deck when the follower of Pythagoras was thrown overboard because of his discovery that the square root of 2 was irrational. Of course, these stories about Pythagoras may be myths, but the people who constructed them surely cared deeply about mathematics because they regarded it as something more than mental gymnastics.

Throughout its history, mathematics was often regarded as deeply meaningful. Consequently, it often aroused passion. We have the example of Gauss being afraid to publish his thoughts on non-Euclidean geometry for fear of ridicule; and Cantor reaping a harvest of ridicule and opposition for his ideas about infinity, perhaps because of their mystical-religious connotations. In our own time, I have heard heated arguments develop over the value (or lack of it) of chaos, fractals, and catastrophe theory. But today's mathematics instruction in our classrooms is mostly about craftsmanship. Our teaching style makes it hard for students to see that we find mathematics deeply meaningful and feel strongly about it.

Let me elaborate on some of these thoughts about the missing dimensions of instruction in mathematics. I leave as an unsolved problem how items on this list can influence the design of a broad one-year introduction to college mathematics.

1. Mathematics as an Element of Culture

Mathematics has played a central role in our culture, but we mostly ignore this in our teaching. It is also true (but less well understood) that culture influences how mathematics evolves. A course that reveals this interplay can help dispel the notion that mathematics is a static body of revealed knowledge. Consider, as an example, the fact that the relatively modern study of connectivity (in topology and especially in graph theory) parallels the fact that there is, in the modern world, an unprecedented degree to which human beings and their institutions and towns are connected. This changes human consciousness. A traveler returning from China today does not make news, but when Marco Polo did so, this caused a sensation that reverberates in the history books. The telephone company advertising jingle, "We're all connected" could be a subtitle for large chunks of modern mathematics.

An important aspect of this is that, as culture evolves, so does mathematics. In a time when much of the world's geography has been explored, and space exploration is restricted to astronauts, mathematics offers fertile ground for exploring the unknown. A broad approach to mathematics instruction should include much modern mathematics, including some which displays the symbiosis between mathematics and culture.

2. Social Meaning of Mathematics

One of the many revolutions that challenge us is the development of computer software capable of doing manipulations that were once the hallmark of a capable and promising student. It has been remarked that for a modest sum, one can buy a pocket computer which could get a high grade on a traditional Calculus final. Our initial response is to change what we teach—more understanding and less rote. This is an excellent response but it is not a complete solution. Artificial Intelligence programs that can solve word problems have already been devised, and computers are being taught to prove theorems as well. The program SRE+VE, developed by L. Hines and W. Bledsoe, can prove that the sum of two continuous functions is continuous [*Notices* 1991]. Perhaps in the long run computers will be able to get high grades on anything that most of our students can be taught to

In a time when much of the world's geography has been explored, and space exploration is restricted to astronauts, mathematics offers fertile ground for exploring the unknown.

do. If we depend upon mechanical competence to distinguish us from machines, we will, at best, be forever one step ahead of the Artificial Intelligence researchers. We need instead to insist that an essential component of mathematics is that it is done by human beings, as a social activity, to which meaning is attached and about which people feel deeply. Although we need to take advantage of computers, we cannot define mathematics to be just that which cannot be done by machines.

One way in which one might emphasize the social meaning of mathematics, is to have mathematics be learned as a social activity instead of a solitary one, perhaps using small-group learning methods, which are currently under investigation. Is it possible to structure a text so that it can equally well be used for traditional instruction and for small-group instruction as well?

3. A Hands on Approach

Mathematics is already a hands on subject since we are always manipulating symbols to do calculations or construct proofs. But we can supplement this with "real" objects for experiment: puzzles, soap films, physical devices giving rise to chaos, computer software, etc. Thurston, Doyle and Conway [1991] have devised a "Geometry and the Imagination" course at Princeton exploiting the possibilities inherent in paper, tape and scissors. In England, Celia Hoyles has been the mathematical host of a prime-time TV program on mathematical puzzles [Howson and Kahane 1990].

In our pencil and paper work with symbols, we need to make sure that students understand the symbols they are manipulating. We are often preoccupied with "covering" the material. We should be mindful of the double meaning of "cover" and concentrate instead on revealing the mathematics. (I am indebted to Bill Thurston, Peter Doyle and John Conway for this neat linguistic distinction.)

4. Stress Major Themes and Imagination as well as Craftsmanship

Einstein has remarked [1950] that the hallmark of science and mathematics, curiosity to understand the world around us, is inherent in children and is sometimes extinguished as they grow up. But we can try to reawaken this childlike curiosity. One way to be successful at this is to link our instruction to major themes of perennial interest, such as the following three.

A. One such major theme is our desire to predict phenomena in the world around us. Today we can predict the arrivals of comets and solar eclipses with precision far beyond that needed for most practical purposes, but we can't tell whether it will rain tomorrow. This extra-ordinary unevenness of scientific development could be a useful "hook" to enlist student interest in dynamical systems. Devaney has written a recent book [1990] that presents some of these ideas using no more than high school mathematics. The idea of sensitive dependence on initial conditions can be communicated without lots of mathematical machinery. Examples where the initial conditions are not very crucial can be shown for comparison. This is an obvious example where computational assistance, especially computer graphics, can be valuable.

B. Another major theme in mathematics that often evokes wonder in ordinary people is the concept of infinity. Infinity is inherently mysterious (and psychologically difficult) and has philosophical and religious resonances for many students. None of this is exploited in Calculus where the treatment is quick and matter-of-fact. If we give students a variety of experiences with infinity we may be able to make it more understandable while still exploiting its potential for intriguing students. We could dwell on geometric series, contrasting it with the harmonic series. We could describe the philosophical problems Zeno raised. We could show some fractal pictures. Although our goal is to build understanding and not create confusion, we might even display some of Cantor's paradoxical discoveries.

Today we can predict the arrivals of comets and solar eclipses with precision far beyond that needed for most practical purposes, but we can't tell whether it will rain tomorrow.

C. A third major theme concerns one of the great intellectual watersheds of the twentieth century: the

discovery that the physical world doesn't conform too well to common sense. When dealing with the very small or the very large (quantum theory or relativity), only mathematics is capable of dealing with phenomena that appear odd to common sense. A small corner of this story can be approached through consideration of non-Euclidean geometry. Prenowitz and Jordan [1989] begin their geometry book with a short, informal, high school level introduction to non-Euclidean geometry.

5. Root Our Instruction in Student Interests and Experience

In beginning courses, we need to stress what students actually care about. The usual arrangement between professors and students is for students to learn to care about what the professor says they should care about. When market demand propels students into courses, they eagerly absorb professorial values. But market demand for mathematics is invisible to many of today's students. Consequently, we need to show them how mathematics is relevant to things they care about.

Partly this means applications, since seeing the usefulness of mathematics gives a hint of market demand (even though students would prefer to see "mathematician wanted" ads in the newspapers). The desirability of an applied flavor for some of our courses has been widely accepted by our community and needs little further elaboration here. However, I would like to urge that the applications come first as motivations for beginning students. Calculus has always been an applications-oriented course, but some calculus books have begun with a review of the formal properties of the real numbers. (A recent book begins with a discussion of an automobile odometer and the relation of its readings and the number of miles driven, and this is a great improvement.)

The usual arrangement between professors and students is for students to learn to care about what the professor says they should care about.

Probability is a wonderful subject through which we can make contact with the vital interests of ordinary people. Human beings have very poor intuition about matters of chance. There are many reasons for this, not the least of which are emotional biases that prevent us from feeling in our hearts what calculation tells us to be true. Do we not all know someone who is afraid of flying but has no fear of the much greater danger of driving the same mileage in an automobile? Students instinctively realize that they live in a world of probabilistic hazards and opportunities and that proper management of these risks can bring great dividends. Students who do not enter careers in mathematics also realize that if improving their risk management requires remembering theorems and algorithms throughout their lives, they will have to do without the benefit. Might it be possible to use the theorems and algorithms to sharpen intuition; something which might remain even if the technical basis gets rusty?

6. Mathematics as a Story

Can we make a mathematics book which is a "page-turner"— one where students are wondering what happens next? The characteristic of a page-turner is that it is a good story. Many aspects of mathematics can be presented as good stories.

Mathematicians have many of the same passions as poets and politicians and so mathematics partakes of the timeless human dramas of intellectual curiosity and pursuit of beauty, of ambition, rivalry and egotism, even the drive for power and wealth. There are historical aspects to the story too: traditions we followed, and later cast off in the realization that they were blinders. Here and now there are axes being ground and philosophical apple carts being upset. New ideas come along and the old guard beats them back. But some ideas survive anyhow and change things. These are the stories that need to be told.

Here are three great stories from our history, all of which are still evolving. Can we find a way to tell some of these stories at the freshman level?

A. Mathematics and the Social Sciences

The great triumph of Newtonian physics in the seventeenth and eighteenth centuries led intellectuals to wonder whether the domain of human affairs could not be made predictable and understandable through mathematics. Condorcet,

one of the towering figures of the French Enlightenment wrote:

> This advance of the physical sciences ... could not be observed without enlightened men seeking to follow it up in the other sciences; at each step it held out to them the model to be followed

In a eulogy for a friend and mentor, Turgot, he also writes of views he absorbed from this friend:

> A great man whose teaching and example, and above all whose friendship I shall always mourn, was convinced that the truths of the moral and political sciences are susceptible of the same certainty as those forming the system of the physical sciences, even those branches of the these sciences which, like astronomy, seem to approach mathematical certainty .... It was for him that I undertook this work subjecting to calculation questions relating to the common utility. [Baker 1975, p. 197]

Condorcet believed that probability theory would be a mechanism for rational design of laws and the consequent perfection of society. His optimism has not borne fruit. But the story is not over. There continues to be slow but steady application of mathematics to social science. Will mathematics ever be as effective in this area as in the physical sciences? And if not, can we give a mathematical account of where the limitation lies (an "impossibility theorem")? This is one of the major unfinished stories of mathematics.

B. Geometry and the Nature of Space

Modern mathematicians have an understanding of the word "geometry" that would be surprising, perhaps even appalling to Euclid. Gauss, one of the greatest of all mathematicians, was afraid to publish his thoughts on non-Euclidean geometry because they seemed so radical. Similarly, the idea of n-dimensional space is a modern revolution. We have mentioned that non-Euclidean geometry can be approached with a high school mathematics background. Likewise the powerful ideas of dimensionality have been made "popular" by many talented expositors such as Abbot, Banchoff and Rucker. How about revealing some of this to our students?

C. Functions

The concept of function that we teach today is a relative newcomer to mathematics. For the longest time, function meant polynomial with relatively low degree or one of the trigonometric functions. Eventually exponential and logarithmic functions arrived. Still later, we have functions obtained from clever limiting processes or defined as solutions to differential equations. Today we often contemplate, especially in discrete mathematics and computer science, functions that do not deal with numbers (e.g., boolean functions). This increasing generality is part of mathematics' increasing power. Could we tell some of this story?

This increasing generality in the nature of functions we can deal with parallels the increasing generality of geometric structures we deal with. Euclid's geometry is all about rectilinear figures and a few (very few) curvilinear ones—chiefly the conic sections. Oddly, most of the shapes we see about us in the world of nature are neither rectilinear nor conic. But today, we can deal with fractal representations of clouds, mountains, human faces. How is it possible that the geometry of Euclid, which describes so little of our natural world, was so successful? There is a story here. And will our new-found ability to deal with clouds, mountains and faces, be a great turning point in the story or just a little twist in a minor subplot? The promise and uncertainty in this development has many mathematicians on the edge of their seats with anticipation. Shouldn't students know about this current and hot story?

Can we write mathematics in the style described here and make it "work"? The view of mathematical writing described here is only partly new. Great expositors have been stressing these missing dimensions for years, especially in books for the layman. Of course, these books for the layman leave out the precision, rigor, and skill building that we have stressed in our textbooks. I think our students are not so different in motivation from laymen and so it would be useful to combine the two writing styles into one grand synthesis. High-quality books for the layman have been influential in attracting many of today's mathematicians to the field, and, for me, that is good evidence that including the missing dimensions does work.

[1] Available now for field-testing from COMAP, 57 Bedford, Lexington MA 02173, or r.slade @comap.com. To be published by Springer-Verlag.

[2] Originally published in Math 101-102: A New Start For college Mathematics, ed. by Walter Meyer, COMAP, 1992.

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To Stephen W. Hawking

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(originally in Bulgarian)

The space-time collapse,
in the space-time fist,
rises in waves and spins.
Where is Now? When is Here?

Rock at the edge
of human want, chance, and mystery,
what are our faults today?
Where is the eternal Fire?

The Question looks for an Answer.
The Answer asks a new Plan.
Universe of dreams, great
is your temple of stars.

Space-time of the spirit
of desire and eclipse,
what are your strivings?
When will come the Spring?

The Study of Mathematics and Growth in the Spirit

Rosemary Schmalz, SP

A cursory look at the history of mathematics attests to the fact that there is a longstanding relationship between mathematics and religion. From Pythagoras to the turn of the century, there have always been mathematicians with an intense interest in religious questions. Some mathematicians, such as Leibnitz and Pascal, actually wrote religious treatises. Likewise, there are writings in many of the world religions on the mystical meaning of numbers in scripture and dogma. Furthermore, in earlier times, theologians as well as mathematicians considered some mathematical entities as God-given and decried any question of them. Even in the last century, Cantor's work on the transfinite was criticized by theologians as well as mathematicians.

In our century, David Eugene Smith, in a chapter of the 1931 NCTM yearbook entitled "Mathematics and Religion," proposed to demonstrate "the influence of elementary mathematics upon the religious instincts of youth."[1] He states that the study of mathematics affords students contact with the infinite. It also allows them to consider the nature of time and space, introducing them to the possibility of dimensions which they cannot experience through the senses. Such contacts lure them to give up "the childish boast that we believe only what we see... we have come to see how full of awe we are in the presence of the awful Infinite."[2] He further states that the study of mathematics lets students see that rigorous questioning of the basis of a discipline (e.g., Euclidean geometry) leads to the demise only of nonessentials and results in a more firm understanding of the essentials. Thus, students may surmise from this realization that "modern religious thinking... has nothing to fear from honest study; if its nonessentials go, the essentials will stand more firmly."[3] Smith concludes that studying mathematics allows a student to grasp the Eternal with what he calls the unseen tendrils of his mind without the external authority of book, priest, or teacher. Such an experience gives one freedom from some of the imposed certainties of childhood and broadens one's concept of reality, thus facilitating the student's search for the higher ideals of humanity.

I am convinced with Smith that studying mathematics can influence the religious instincts, though I would prefer the word "spiritual" to "religious." However, I propose an influence beyond what Smith proposes. His points are related to the *content* of mathematics. I propose that the *doing* of mathematics can influence the human spirit, that *the wholehearted study of mathematics develops those disciplines which facilitate spiritual growth.*

Such a conviction has its roots in my own experience and has been affirmed by my having stumbled upon, a number of years ago, quotations from mathematicians *about mathematics* that closely parallel the writings of spiritual masters about the spiritual life. This led me to search out more such quotations in the writings of mathematicians, a sample of which appear below. Please note that I am NOT proposing that mathematicians are inherently good persons or become good persons through doing mathematics or that doing mathematics leads a person to faith in deity or to the embrace of a particular belief system. I propose simply that the study of mathematics develops certain disciplines and that these disciplines are the same as those essential for progress along a spiritual path. I substantiate this claim by demonstrating the similarities in the writings of mathematicians and spiritual masters.

The faculty of the intuition

The basis of the similarities lies in the fact that much of spiritual experience and mathematical discovery are rooted in the faculty of intuition. Popularized now as "right-brained knowing," this faculty along with reason (the activity of the left brain) constitute the two faculties for knowing. A dictionary of psychological terms defines intuition as "direct and apparently immediate knowledge..."[4] Mathematician Alan Turing defines it as that faculty which "consists in making spontaneous judgments which are not the result of conscious trains of

reasoning."[5] Psychologist Jerome Bruner states that "Intuition implies the act of grasping the meaning or significance or structure of a problem without explicit reliance on the analytic apparatus of one's craft."[6] A current spiritual writer, Thomas A. Kane, defines the intuition as "that faculty of the mind that apprehends truth as *immediate* knowing without deduction or reasoning."[7] These definitions, from quite different sources, each suggest that intuition is that faculty of the mind for which comprehension is spontaneous and immediate as opposed to rational and linear, and very often, though not always, sudden.

The experience of intuition

It is clear from the writings of mathematicians and spiritual writers that the faculty of the intuition is of primary importance in each area. In the area of spirituality, it is generally accepted that the Divine cannot be known through reason alone. Jewish theologian Abraham Heschel writes:

> The awareness of God... does not enter the mind by way of syllogisms nor can the certainty of faith be presented on a silver platter of speculation. Logical plausibility does not create faith nor does logical implausibility refute it...[8]

Similarly, mathematician Jacques Hadamard claims that there is hardly any completely logical discovery. "Some intervention of intuition issuing from the unconscious is necessary at least to initiate the logical work."[9]

Spiritual literature is replete with descriptions of intuitive insight. For example, Ignatius Loyola, founder of the Jesuits, writing of himself in the third person, gives this account:

> As he sat there, the eyes of his understanding began to open. Without having any vision he understood—knew—many matters both spiritual and pertaining to the Faith and the realm of letters and that with such clearness that they seemed utterly new to him...[10]

Thomas Merton describes one of his intuitive breakthroughs in these words:

> In Louisville, at the corner of Fourth and Walnut, in the center of the shopping district, I was suddenly overwhelmed by the realization that I loved all those people, that they were mine and I theirs, that we could not be alien to one another even though we were total strangers. It was like waking from a dream of separateness... This sense of liberation from an illusory difference was such a relief and such a joy to me that I almost laughed out loud. A member of the human race! To think that such a commonplace realization should suddenly seem like news that one holds the winning ticket in a cosmic sweepstake.[11]

Mathematicians describe similar experiences. The most famous descriptions of such experiences in mathematics are those of Henri Poincare. He relates that he had spent fifteen days on a problem about Fuchsian functions without success. Then one evening having drunk black coffee at a late hour, he could not sleep. He claims that throughout the night, ideas collided and some interlocked. The next morning he knew the solution to his problem. Some time later while on vacation where he had temporarily put aside his mathematical work, he had the following experience:

> When I put my foot on the step [of the bus] the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian function were identical to those of non-Euclidean geometry. I did not verify the data... but I felt a perfect certainty.[12]

On his return, he took up work on something seemingly unrelated but without much success. He relates:

> Disgusted with my failure, I went to spend a few days at the seaside, and thought of something else. One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformations of indeterminate ternary quadratic forms were identical with those of non-Euclidean geometry.[13]

Philip J. Davis of Brown University relates that he recently watched a film on the hypercube produced by his colleagues Banchoff and Strauss and though impressed, he was disappointed because he gained no *feeling* for the hypercube. However, after a session at a computer graphics console which allowed him through controls to manipulate its image on the screen, he relates, "Suddenly I could *feel* it. The hypercube had leaped into palpable reality..."[14]

Paul Halmos relates that when taking a complex function course from Pierce Ketchum during his first year as a graduate student:

> I had absolutely no idea of what was going on. I didn't know what epsilons were, and when he said take the unit circle, and some other guy in class said "open or closed", I thought that silly guy was hair-splitting, and what was he fussing about... Then one afternoon something happened. I remember standing at the blackboard in Room 213 of the mathematics building talking to Warren Ambrose and suddenly I understood epsilons. I understood what limits were, and all of the stuff that people had been drilling into me became clear... All of that stuff that previously had not made any sense became obvious... That afternoon I became a mathematician.[15]

This is not only an expression of insight but also a description of "conversion," which will be discussed below.

The disciplines undergirding intuition

Whether in mathematics or the spiritual life, the masters concur that a strict discipline is necessary for the intuition to work. As Bruner says, "Discovery, like surprise, favors the well-prepared mind."[16] This discipline of preparation consists of similar factors whether described by mathematicians or spiritual writers. I will discuss four particular similarities: an initial conversion and sustained dedication, a willingness to devote oneself to "useless" activity, the recognition of the value and necessity of sustained attentiveness, and detachment from preconceptions and results.

CONVERSION AND DEDICATION

Lives of holy persons in all religions are filled with conversion stories. Some of the more famous of these are the stories of the apostle Paul, Francis of Assisi and Siddhartha Gautama, the Buddha. Conversion usually includes a sense of call, a sense of finding meaning in the call, followed by a reordering of one's life in keeping with the call.

There seems to be in the lives of many mathematicians a conversion process. Sometimes there is an early time when they showed no interest and aptitude for mathematics, but whether this is the case or not, there is frequently some instant when interest is born of such great magnitude that the study of mathematics becomes a lifelong commitment. The words and phrases they use to describe their experiences have all the aspects of religious conversion. There are allusions to various aspects of call, to finding meaning in the call, to changing one's lifestyle to meet the call.

Sonya Kovalevskaya writes in her autobiography:

> Besides arithmetic, Malevich taught me elementary geometry and algebra. Not until I grew somewhat more familiar with this latter field did I begin to feel an attraction to mathematics so intense that I started to neglect my other studies.[17]

Bertrand Russell reports in his autobiography:

> At the age of eleven, I began Euclid with my brother as my tutor. This was one of the great events of my life, as dazzling as first love. I had not imagined that there was anything so delicious in the world. After I learned the fifth proposition, my brother told me that it was generally considered difficult, but I had no difficulty whatever. This was the first time it had dawned on me that I might have some intelligence. From that moment until Whitehead and I finished *Principia Mathematica*, when I was thirty-eight, mathematics was my chief interest, and my chief source of happiness.[18]

Georg Cantor, on receiving permission from his father to pursue a career in mathematics, wrote to him the following:

> I hope that you will still be proud of me one day, dear Father, for my soul, my entire being lives in my calling; whatever one wants and is able to do, whatever it is toward which an unknown secret voice calls him, *that he will carry through to success*.[19]

Not only their initial commitment but their continued dedication is described in similar superlatives. Angus Taylor refers to his career in mathematics as "a love affair."[20] Stanislaus Ulam reports in his autobiography that early in his career:

> I gave my own little talk (at the Congress in Zurich in 1932) feeling only moderately nervous. The reason for this comparative lack of nervousness, I think, in retrospect, was due to my attitude, compounded of a certain drunkenness with mathematics[21] and a constant preoccupation with it.[22]

Thus, mathematicians leave us with no doubt that they consider doing mathematics a call, the

response to which demands dedication and commitment that gives meaning to their lives.

USELESSNESS

The presence of a contemplative/monastic tradition in most world religions attests to a common conviction that there is value in a life committed to the pursuit of the absolute. Persons committed to such a life are usually not involved in direct service to humanity and critics may see such lives as "useless;" yet, there remains a conviction within the religious traditions that such a life is extremely worthwhile. Raimundo Panikkar, a contemporary theologian with roots in Christianity and Hinduism, quotes the *Bhavagad-Gita* as saying that contemplation is that activity that maintains the world in cohesion.[23] Ancient Chinese philosopher Chuang-tzu tells us, "Every man knows how useful it is to be useful. No one seems to know how useful it is to be useless."[24] Medieval mystic Meister Eckhart admonishes, "You should do all your works out of this innermost ground with why."[25]

Mathematicians speak often of the uselessness of their work. They do not apologize that their work has no immediate application. G. H. Hardy makes this observation:

> If useful knowledge is, as we agreed provisionally to say, knowledge which is likely, now or in the comparatively near future, to contribute to the material comfort of mankind, so that mere intellectual satisfaction is irrelevant, then the great bulk of higher mathematics is useless. Modern geometry and algebra, the theory of numbers, the theory of aggregates and function, relativity, quantum mechanics—no one of them stands the test much better than another, and there is no real mathematician whose life can be justified on this ground. If this be the test, then Abel, Riemann, and Poincare wasted their lives; their contribution to human comfort was negligible, and the world would have been as happy a place without them.[26]

John von Neumann remarks:

> But still a large part of mathematics which became useful developed with absolutely no desire to be useful, and in a situation where nobody could possibly know in what area it would become useful; and there were no general indications that it ever would be so. By and large it is uniformly true in mathematics that there is a time lapse between a mathematical discovery and the moment when it is useful and that this lapse of time can be anything from 30 to 100 years, in some cases even more; and that the whole system seems to function without any direction, without any reference to usefulness, and without any desire to do things which are useful.[27]

Davis and Hersh, in trying to describe what mathematicians do, write:

> The ideal mathematician's work is intelligible only to a small group of specialists, numbering a few dozen or at most a few hundred. This group has existed only for a few decades and there is every possibility that it may become extinct in another few decades. However, the mathematician regards his work as part of the very structure of the world, containing truths that are valid forever, from the beginning of time, even in the most remote corner of the universe.[28]

This last quotation summarizes well the sense of all mathematicians that their work is of timeless value even though it is not immediately useful. It closely compares to contemporary theologian and psychologist Henri Nouwen's description of a contemplative monastery as "the center of the world."[29]

ATTENTION

In most religious traditions, some practice that I will call "prayer" is considered essential. For this paper, let us define prayer (as opposed to "saying prayers") as some form of regular, dedicated practice which includes an element of yearning for or awareness of the unlimited which human life touches. Methods of prayer most directly related to contemplative traditions usually consists in some form of disciplined attention, whether chanting, focusing on a question or an image, or repeating a mantra.

Evelyn Underhill, a well known authority on mysticism, describes contemplative prayer thus:

> The true asceticism is a gymnastic... of the mind. It involves training in the art of recollection, the concentration of thought, will, and love upon the eternal realities which we commonly ignore. The embryo contemplative ... must acquire and keep a special state of

> inward poise, an attitude of attention, which is best described as "the state of prayer," that same condition which George Fox called "keeping in the Universal Spirit." If we do not attend to reality, we are not likely to perceive it... It means hard work, mental and moral discipline of the sternest kind.[30]

Claudio Naranjo has this to say:

> The trait that all meditation has in common... is *dwelling upon* something. While in most of one's daily life the mind flits from one subject or thought to another... meditation practices generally involve an effort... to set our attention upon a single object, sensation, utterance, issue, mental state, or activity... As you may gather from this statement, the importance of dwelling upon something is not so much in the *something* but in the *dwelling upon*. It is this concentrated attitude that is being cultivated, and with it, attention itself.[31]

Mathematicians all know the daily work of "dwelling upon" inherent in doing mathematics. They know that, at times, they take leaps in understanding and produce results, but their work is mostly to look, to ponder, to concentrate—in short, to follow the directives of the spiritual masters to become contemplative, attentive.

Ulam describes Stefan Banach and his students at the coffee house near the university thus:

> There would be brief spurts of conversation, a few lines would be written on the table, occasional laughter would come from some of the participants, followed by long periods of silence during which we drank coffee, and stared vacantly at each other. The cafe clients at neighboring tables must have been puzzled by these strange doings. It is such persistence and habit of concentration which somehow becomes the most important prerequisite for doing genuinely creative mathematical work.[32]

He describes his own work thus:

> From him (Stanislaw Mazur) I learned much about the attitudes and psychology of research. Sometimes we would sit for hours in a coffee house. He would write just one symbol like y = f(x) on a piece of paper, or on the marble table top. We would both stare at it as various thoughts were suggested and discussed. These symbols in front of us were like a crystal ball to help us focus our concentration. Years later in America, my friend Everett and I often had similar sessions, but instead of a coffee house they were held in an office with a blackboard.[33]

Kovalevskaya alludes to the discipline of attentive "looking" in her autobiography:

> I understand your surprise at my being able to busy myself simultaneously with literature and mathematics. Many who have never had an opportunity of knowing any more about mathematics confound it with arithmetic, and consider it an arid science. In reality, however, it is a science which requires a great amount of imagination, and one of the leading mathematicians of our century states the case quite correctly when he says that it is impossible to be a mathematician without being a poet in soul. Only, of course, in order to comprehend the accuracy of this definition, one must renounce the ancient prejudice that a poet must invent something which does not exist, that imagination and invention are identical. It seems to me that the poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing.[34]

How closely this echoes the directives of St. Teresa of Avila, "I do not require you to form great and serious considerations in your thinking. I require you only to look."[35] All great mathematicians are skilled in this art of looking, of being attentive.

DETACHMENT

When writing on the fruits of prayer, most spiritual writers are quick to caution against expecting "results." They encourage fidelity to the work of prayer, (all of them agreeing that it is indeed work), pointing out that true enlightenment is a transformation deep within, a gradual, at times imperceptible change. The Bhagavad-Gita tells us:

> But these actions
>   Abandoning attachment and fruits
> Must be performed, O Partha
>   This is my definite and highest doctrine.[36]

Dominican theologian Richard Woods gives a similar warning:

> Thus meditation and contemplation must be approached for what they are, not for what they can do for us. Otherwise, paradoxically, they

> won't do much of anything, because they can't.[37]

Alan Watts says this of Zen practice:

> The practice of Zen is not the true practice so long as it has an end in view, and when it has no end in view, it is awakening...[38]

Compare this to von Neumann's remark:

> Successes were largely due to forgetting completely about what one ultimately wanted or whether one wanted anything ultimately.[39]

However, detachment does not mean the absence of energetic commitment; it must not be equated with indifference. One certainly must work toward the goal with great desire whether it is enlightenment of the spiritual or mathematical nature, but when one works with detachment, he/she works in a relaxed way, without *anxiety* for results.

James Borst writes:

> One often meets the idea of "concentration" in connection with this [contemplative] prayer. Concentration yes... but not as the result of a mighty and tense effort; only as a gentle letting go of things, a relaxing of our nervous grip on people and situations and the release from worry and anxiety.[40]

Constance Reid gives us this picture of David Hilbert at work in his garden that fits the above admonition perfectly. She tells us that a large blackboard was attached to the neighbor's wall. He interrupted his work by bicycling or walking around the flowerbeds or doing some pruning. She quotes a description of him by Courant, who observed him at work from an upper window, as "a fantastic balance between intense concentration and complete relaxation".[41]

Working anxiously, without relaxation, causes one to focus on the self rather than on the work at hand. In both mathematics and the spiritual life, it can lead to form of meditation, inaccuracies and retardation of real progress. John Main, writing on the mantra, warns:

> We must not self consciously ask ourselves "How far have I got?"... If we try to force the pace or to keep a constant self-conscious eye on our progress, we are, if there is such a word, non-meditating because we are concentrating on ourselves, putting ourselves first, thinking about ourselves.[42]

Norbert Wiener echoes this sentiment in writing about a colleague:

> He [is] a hard worker, and he makes the greatest demands on the sincerity and industry of those about him, demands which are only exceeded by those he makes upon himself. According to my way of looking at things, he involves himself too deeply in the expected outcome of a particular piece of research, so that if in fact it comes out differently, he will be disproportionately worried and spend excessive effort at trying to salvage that which has already proved itself unsalvageable.[43]

A prime example of the effect of lack of detachment in mathematics history is the work of Girolami Saccheri. Saccheri set out to prove definitively that Euclid's fifth postulate was deducible from the first four. His work led him to discover many of the now classical theorems in non-Euclidean geometry. However, he found his results so puzzling and so repugnant that he called the results contradictory and claimed to have succeeded in proving that Euclid's fifth postulate was indeed a theorem. Mathematics historian Howard Eves notes, "...Saccheri lamely forced into his development an unconvincing contradiction involving hazy notions about infinite elements. Had he not seemed so eager to exhibit a contradiction here, but rather had admitted his inability to find one, Saccheri would today unquestionably be credited with the discovery of non-Euclidean geometry."[44] Because he was working for a preconceived result and was not able to be detached from it, he was not able to see the meaning of his results.

Thus, in both mathematics and spiritual pursuits, one must never allow anxiety about results or preconceived judgments to distort the focus of one's work. In both areas, this impedes progress and increases the possibility of error.

Conclusion

These quotations give us some basis to conclude that great mathematicians, in doing mathematics, are following the same disciplines necessary for spiritual development. I propose that we may also conclude that the study of mathematics develops these disciplines also in the "less-than-great," ourselves and our students. As I stated earlier, this

is not to say that the study of mathematics brings one to a faith commitment. Neither am I equating the gift of spiritual enlightenment with the gift of mathematical discovery. However, I propose that we can further conclude that if one who is committed to doing mathematics is already a person of faith or becomes a person of faith in the future, such a person has already developed those disciplines necessary for growth in the spiritual life. Through the disciplines of attention with detachment and dedication to what does not seem immediately useful, such a person is developing the inner freedom necessary to allow the human intuition to reach its full potential. Philosopher and activist, Simone Weil, writes:

> If we concentrate our attention on trying to solve a problem in geometry, and if at the end of an hour we are no nearer to doing so than at the beginning, we have nevertheless been making progress each minute of that hour in another more mysterious dimension. Without our knowing or feeling it, this apparently barren effort has brought more light into the soul. The result will one day be discovered in prayer.[45]

Her words affirm my belief that the doing of mathematics is by its very nature enriching to the human soul.

Perhaps the same could be said of other disciplines or of study in general. I can only speak for the discipline of mathematics and hope that persons of other disciplines may wish to reflect on such similarities in their own areas. I simply offer evidence from the great mathematicians that no matter what are the conscious goals of persons doing mathematics, such activity, when undertaken with commitment provides the opportunity for development of that faculty of the soul with which the human touches the divine.

1. David Eugene Smith, "Mathematics and Religion," in W.D. Reeve, editor, *Mathematics and Modern Life,* (New York: Bureau of Publications, Columbia University, 1931), p. 54.

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3. *Ibid.*, p. 56.

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Poetry by Jonathan Post

PI : A Mnemonic Poem

[The length of each word, in letters, is a digit in the decimal expansion of pi=3.14159265358979323846 2643383279502884197169399375105820974944592307816406286208998628034825342117067982148 08651328230 + "O!" is zero, as well as circularity and the groaned response to pun]

NOW I know a value
    (certainly do!)
Number whose use
    gives circular functions
Purpose: numerical, too.

"Pi" its cognomen
    when verbal
    or symbol;
Uses for all,
    interest for he
    science intrigues.
Think O! to assemble
    decimals' form
    I memorized stanzas

I summed endlessly
    the character mnemonics
And finally cried I
    "O! That's harmonic!"

Pi, O! marvelous decimal
    that eternally will grow
    while computers go
    and, O! compute
Numerals I deduce flee
    0! surely
    to infinity!

Poetry, my O! pathetic
    technique retentive
conceals circle
    of geometry: O!
The fine pleasure it shall
    now give.

So I a hundred
    O! verily hundred
Multiplex decimals
    of a Math Produced
    (O!)
Fabulous number know.
A few in addition
    to end. O!

ca.1965

Pythagorean Nightmare

I was a child the first time I had this dream
so there was no horror in the windy beach, the ocean,
the salt smell in the lightning, and the storm's voice
was my father's voice, dark, but to be trusted

The Hurricane cried "Know me: I am in the zero."
and the crashing surf called "I am in the wave."
The tumbling sand-grain hovered by my eye,
a perfect crystal—"I am in geometry."

The lightning laughed, and showed me a magic life
which was my life; a line of rocks which ran
out into the foam, breakwater in the spray
and each rock was a year, each rock further from the shore

I stand on the fifth rock with a chain of jewels in my hand;
these are my counting numbers, these bright ones, I can take them
anywhere, for long days, former friends, or railroad cars—
I run them sparkling through and through my fingers

Another rock, the shore is misty, Algebra—
where numbers put on masks, the candles lit,
the presents wrapped, the cake is cut, the horns
silent for a moment, 'til the singing starts

Jump, jump, here I pursue the nightingale
through tangled woods, through rifts and booming caverns
whose shadows hold infinity; soft predicate boots
laced with quantifiers, bookmark feather in my hand

One of the last rocks—birdsong fills the night
of moonlight on strange machines building a golden cage
and finding themselves within; I too press against the bars:
can I fit between them? They are the Undecidable.

I hold my breath at the edge of the final boulder—
dull fuzzy shapes assemble in the mist—
dry books, some girl, a door—I gesture them away
and thunder speaks the message I'd forgotten.

Nightmare, Nightmare' How could I be fooled?
This dream again—the windy beach, the ocean—
moaning in sweat I grope for the light and glasses—
touch them—shiver—and am barefoot on the granite

The Hurricane cries "Know me: I am in the zero."
and salt tears at my beard, sleet trembles on my thighs,
everything else was false! This beach is my only home
and I have been leaping, dazzled, away from shore...

0600-0845
30 mar 75

e : Mnemonic to the Base of Natural Logarithms

$e \sim 2.718281828459045...$

It: natural,
I: personal,
so exponent
I appraise.
It: enabling
logs' table—
logarithm,
0 base,
amaze!

0330-0345
2050-2100
11 July 83

Some Notes on How Students Perceive Mathematics

Joan Countryman
The Lincoln School

Constance Reid's charming biography of E.T. Bell[1] has a chapter entitled, "The Human Side of Mathematics." She begins with the sentence, "It has been said, possibly by Bell himself, that the human side of mathematics is mathematicians." One thing I have learned in years of work with high school math students is that few students see anything human (or humane) about mathematics. This may be due, at least in part, to the fact that they do not see any mathematicians in mathematics. What they do see, and I quote here from an essay by Alfinio Flores, "The Shadows of Mathematics," published in the *Arithmetic Teacher* of April, 1993, "is rote learning, meaningless procedures, unrelated topics, and memorizing formulas. For them, learning mathematics is developing skills in symbolic manipulation of numbers and formulas, little understanding, and no fun."

Ruth Parker, in *Mathematical Power* [2], a lively discussion of teaching math in elementary classrooms, reminds us of the contrasts between the mathematics that is done in school and the mathematics that mathematicians do. (See Table 1)

Twenty five million children study mathematics in school in the United States; most of their time is devoted to computation, practicing tasks that hand calculators can do faster and more efficiently. If you ask those children to tell you what it means to do mathematics they will tell you add, subtract, multiply, divide." Unfortunately, as they get older, many of those children will say that they hate mathematics.

You will hear this in the words of my students, as they describe their experience of mathematics. Here is the voice of K..., a student in an honors course:

> My earliest math memory is associated with failure. [In a new school] I didn't know how to multiply and the rest of the class was doing multiplication problems. I struggled to memorize the multiplication tables but I could not understand the concept of multiplication ... The next math memory that seems relevant is one during my seventh grade year. We began to learn algebraic equations and I couldn't understand what the "x" meant in an equation like $5x + 3 = 18$. The concept of what "x" stood for escaped me. I felt useless and stupid when I couldn't even imagine how to solve or what to solve for in these equations.

Table 1

School Math	Real Math
Neat and concise	Messy
Speed, getting answers quickly	Persistence and flexibility
Right answers	No answer book
Arithmetic and manipulation of symbols	Diverse domains, including geometry, patterns,functions, logic, data analysis, etc
Calculators after basic skills	Tools available to examine and represent ideas
Math done alone	Math used to make sense of information; collaborative work[3]

K... had become a student who wanted the teacher to "just tell me how to do it;" but C... was more confident about his own ability:

> In elementary school I was a very good math student and always found myself working on math level tests much higher than those of my classmates. My teachers pushed me very hard, and I often got private lessons with the teacher while the rest of the class was working on other problems. I remember being proud of being the only third grader in the school to know long division.

Nevertheless even C... saw long division as central; it represented a mathematics that was complicated and difficult.

These themes arise frequently in the comments students make about themselves as learners. Here is a calculus student, Molly, who was skeptical about her ability to do mathematics, an activity which she equated with getting many right answers.

> "I think that I am not a mathematical person. I enjoyed the class, but I really don't think I learned anything except how to flunk with dignity . . . I think that this is a hard course. I tried but my best just didn't make it. I am glad that I stuck with it, though. I really don't care for integrals, limits, etc."

Hilary, a student with a more positive image of herself as a learner, said, "Math is fun when you get it right. Otherwise it's frustrating."

> "I think that some math is fun. So far I like algebra. I never learned how to do math really well, but what I do learn I remember, or at least I try to remember. When I was in sixth grade I learned the most math."

> "I'm not bad at math once I get the hang of what I'm doing, but I usually rush and make dumb mistakes. Math to me should be just +, -, x, ÷.

Although my intention here is to describe, and not to prescribe, I do want to suggest a direction that might help disabuse students of the notion that math is arithmetic and arithmetic is only boring and/or difficult. In a high school course[4] that was rich and thick, *mathematical,* if you will, I found students commenting in their journals about a discipline that seemed much closer to the math that real human beings do.

> I still really love the whole idea of closed and recursive equations. It seems that if you can talk about the way a function's input and output are related in such a general way, then you really know a lot about the numbers you're dealing with. The relationships between the numbers transcend what the values of the numbers may be.

Listen to the voice of a high school mathematician, a student in the same course, describing his work in this longer entry:

> It seemed to me when I was reading the section about building an irrigation system that the whole problem was misdirected. After several pages of calculations and explanations, I thought that the problem of even water distribution could have been easily solved by substituting a long "flat" water spout (sort of like the ones in fancy bathtubs) in place of several round ones. Of course I may not have thought of an alternative system except for the complexity of the calculations, which means that the lengthy explanation was good because it got me to think: "Isn't there a simpler way?" This is sort of like the tape recorder problem where after several pages of calculations and formulas the authors declare that something is wrong and that we need to do the problem all over again. I think that this brings us to a fundamental question we need to consider in the construction of mathematical models: is our answer the solution to our problem? Often, in the course of adding, subtracting, and otherwise mathemetizing, we lose sight of the ultimate goal, and if we answer the wrong question, or if our answer is complicated, then what good is that answer?

When the mathematics that we teach is real our students will perceive its humanity. I share with Ted Sizer, whose Coalition of Essential Schools is currently receiving national attention, the conviction that in high school we need to give all students practice in thinking hard about problems that matter. What if the goal for every math course were that at the end of the course students would want to take another? It is a challenging vision.

Notes:

[1] Constance Reid, *The Search for E.T.* Bell.Washington, DC: MAA, 1993

[2] Ruth Parker, *Mathematical Power*. Portsmouth, NH: Heinemann Books, 1993

[3] from *Mathematical Power*.

[4] Contemporary Precalculus with Applications. North Carolina School for Science and Mathematics,. Janson, publisher.

Poetry by Monte J. Zerger

Adams State College
Alamosa, CO

Mistress of mine, time and
Again you have wooed me with your
Theorems and proofs,
Held me captive with your abstract beauty, and
Enchanted me with your dance.
Mistress of mine, time and again I have been
Awed by the
Transcendent melodies you weave and the
Infinite tapestries you spin from only a sparse
Collection of symbols and
Signs. Mistress of mine, it has been a long and glorious romance.

Match Mates

All day in this game I equate
So I find it perfectly great
    that anagrammatics
    transforms "mathematics"
Into these three words, "I match mates"

MAA Convention

They're nearly obsessed
        with obscure quests
    and thoughts that merely bore the rest

They track rare game
        on abstract planes,
    are quick to chase anything arcane

Now what could I
        possibly find
here amidst these quirky minds?

Amidst these beards
        and equally weirds
    who've gathered here
        in this House of Mirrors?

Möbius
(an eternal limerick)

I journeyed once to Deja Vu
Where nothing, not one thing, was new
    no this side or that
    no future, no past
Just one, long, continuous loop

Student Lament

I found trigonometry neat
As easy as π to complete
    I knew all the angles
    But now, here I dangle
Since Newton and Leibniz hung me

Writing Assignments in an Abstract Algebra Course

Krystina K. Leganza
Department of Mathematical Sciences
Ball State University
Muncie, Indiana

Including writing assignments in my algebra courses came about due to my college's (at the time Saint Mary's College) and my own belief that students should "write across the curriculum" and in particular in their majors' courses. Many papers have discussed the importance and benefits of writing in math courses. In this paper I will discuss three specific writing assignments that I have designed and used and some observations about each.

The first time I taught Abstract Algebra I used the sample writing assignment described by Anne Brown in Chapter 29 of *Using Writing to Teach Mathematics*. My subsequent assignments were modeled after this one. However, I have found it is not necessary to give the students a detailed outline of the paper.

When designing my own writing projects, I have three main goals: the assignment can be broken down into parts; the assignment relates to the mathematics covered in the course but also introduces new topics; and the assignment is not too mathematically challenging so that the students can concentrate on the writing and not get bogged down in the mathematics.

They have a chance to rewrite and correct the mathematics before the final paper is turned in. This has allowed the students to concentrate on their writing when they work on the final paper.

The exercises in the assignments are standard. Although students are already expected to write proofs in complete sentences, these assignments go beyond normal homework exercises. The students are required to include all relevant definitions, some examples, lemmas, and finally the proof of a theorem. The papers are usually five to ten pages long.

Breaking the Assignment Down into Parts

There are several benefits to breaking the assignment down into parts. I noticed in my earlier writing assignments in which I did not do this that the students were afraid to sit down and start writing. They did not think that the words "write" and "math" belonged in the same sentence. They viewed the assignment as some enormous project hanging over their heads and tended to procrastinate. Thus, one or two nights before the project was due, they were panicking and frantically trying to write a math paper. By breaking the assignment down into smaller pieces, they could work on one portion at a time. The students have commented on course evaluations that they appreciate the project being structured in this manner.

Another positive aspect of breaking the assignment down into parts is that the students get feed back along the way. They have a chance to rewrite and correct the mathematics before the final paper is turned in. This has allowed the students to concentrate on their writing when they work on the final paper. I no longer require a draft of the final paper since these earlier exercises are drafts. However, the students are free to turn in drafts that I will make comments on, but not grade.

The first part of the assignment is to write a short paper introducing the necessary terms, giving examples, and explaining notation. The middle parts of the paper tend to introduce new topics tangential to the course and involve some proofs that will be used in the last part of the assignment. This last assignment is the goal of the entire paper and usually involves proving a theorem.

The parts of the assignment are designed so that writing the final paper only requires putting the components together with transitions and an introduction and conclusion. I have found that I have to repeatedly remind students of the importance of transitional sentences and paragraphs. They think that I am being funny when I say that the final paper should flow like a novel.

DISTRIBUTING THE ASSIGNMENT TO THE STUDENTS

Copies of three of my assignments as they were given to the students can be found at the end of this paper. When I make the assignment, I give them all of the parts as well as a written explanation of how the paper will be graded. At this time I also give them all due dates. These due dates tend to be one week apart with a couple of weeks between the last part and the final paper.

GRADING

As mentioned above I give the students some written guidelines on how the paper will be graded when it is assigned. (A copy of this is also attached.) These are based on the departmental guidelines for evaluating writing used by the Mathematics Department at Saint Mary's College. I grade each part of the assignment separately and include these scores in the student's homework grade. The student also receives a grade for the final paper.

COMMENTS ABOUT THE ASSIGNMENTS

The first two assignments were used in a sophomore level Algebraic Structures course. The third assignment was used in a junior/senior level second semester Abstract Algebra course.

One problem I encountered with the first assignment listed was that I gave the names of the new concepts defined in part (2). For some reason the students spent time worrying about the terms "center" and "centralizer" and lost sight of the fact that they were just trying to prove a set was actually a subgroup. Originally, I included the new terms so that they would be familiar with them and also so that they could look them up in reference books. I do not think that they used resources other than their own text even though they were told that they could. In fact, the proof that the center of a group is a subgroup of the group is in the text book we used in a chapter that we did not cover. After they turned in their papers, I pointed this out to them. No one had even noticed this.

Notice in the second assignment I decided not to give the names of the new concepts. This seemed to work out better from the students' point of view. When these ideas came up naturally in later class lectures, I introduced the formal terminology. Parts (lb) and (lc) were included to reinforce the concepts of cyclic and abelian groups. However, I might exclude these from the assignment in the future because neither is required to prove the theorem listed in part (4) and the students seemed distracted by these parts.

The third assignment was probably the most successful. Even the students who had done writing assignments for me in other courses mentioned that this was their favorite. I liked this particular assignment because it tied together so many topics: ring, commutative ring, unity, ideal, and field. Obviously, the mathematics is not difficult, but it requires a clear understanding of the topics and some thinking beyond the course work. Writing the paper helped the students review for the test over this material. One of the test questions was the converse of the theorem proven in the paper. When we went over the test in class, I pointed out that with the paper and that test question we now had an if and only if statement. Linking the test and the paper made the paper a more natural part of the course from the students' perspective.

Although it can be difficult at times to design writing assignments that meet all of my goals, it can be done with some careful planning before the semester begins. I have found these projects to be very worthwhile and will continue to incorporate them into my courses.

CONCLUSION

Although it can be difficult at times to design writing assignments that meet all of my goals, it

can be done with some careful planning before the semester begins. I have found these projects to be very worthwhile and will continue to incorporate them into my courses.

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APPENDIX A

MATHS 311
AUTUMN 1991
WRITING ASSIGNMENT

You may not work together on this assignment. However, you may see me for help.

1. Write and type a 1-2 page paper explaining the concepts of group, abelian group, subgroup, etc. Include some examples. Explain notation.

DUE: Thursday, Sept. 26

2.a) Let G be a group. Define the <u>center of G</u> to be
$Z(G) = \{ a \in G \mid ax = xa \text{ for all } x \in G\}$.
Prove that Z(G) is an abelian subgroup of G.

b) Let G be a group and $a \in G$, a fixed element. Define the <u>centralizer of a in G</u> to be
$C(a) = \{ g \in G \mid ga = ag\}$.
Prove that C(a) is a subgroup of G.

DUE: Thursday, 0ct. 10

3. Prove the following theorem.

<u>THEOREM:</u> $Z(G) = \cup_{a\in G} C(a)$. (This means the intersection of all subgroups of the form C(a).)

DUE: Thursday, Nov. 7

APPENDIX B

MATHS 311
SPRING 1992
WRTTING ASSIGNMENT

You may not work together on this assignment. However, you may see me for help.

1. Write and type a 1-2 page paper explaining the concepts of group, abelian group, cyclic group, subgroup, etc. Include some examples. Explain notation.

DUE: FRIDAY, FEBRUARY 7

2. Let G be a group and H a subgroup of G. For any x in G, define $x^{-1}Hx = \{ x^{-1}hx \mid h \in H \}$.
a) Prove: $x^{-1}Hx$ is a subgroup of G
b) Prove: If H is cyclic, then $x^{-1}Hx$ is cyclic.
c) Prove: If H is abelian, then $x^{-1}Hx$ is abelian.

DUE: FRIDAY, FEBRUARY 14

3. Let G be a group and let H be a subgroup of G. Define
$N(H) = \{x \in G \mid x^{-1}Hx = H\}$.
Prove that N(H) is a subgroup of G.

DUE: FRIDAY, FEBRUARY 21

4. Let H be a subgroup of G and x an element of G.

Prove $N(x^{-1}Hx) = x^{-1}N(H)x$.

DUE: FRIDAY, FEBRUARY 28

The final paper is to put all of the above parts together into one logically connected body.

DUE: FRIDAY, MARCH 20

APPENDIX C

MATH 354
SPRING 1990
WRITING ASSIGNMENT

1. Write a 1-2 page paper explaining the concepts of ring, commutative ring, unity, units, ideal, field, etc. Include some examples. Explain notation.

2. For any element a in a ring R, define $\langle a \rangle$ to be the smallest ideal of R that contains a. If R is a commutative ring with unity, show that $\langle a \rangle = aR = \{ar \mid r \in R)$.

3. Let R be a commutative ring with more than one element. Prove that if for every non-zero element a of R, we have aR = R, then R is a field.

PAPER: Prove the following theorem.

Theorem. Let R be a commutative ring with unity. Suppose the only ideals of R are {0} and R. Show that R is a field.

DUE DATES:	EXERCISE 1	Feb. 4
	EXERCISE 2	Feb. 11
	EXERCISE 3	Feb. 18
	DRAFT (TYPED)	Mar. 8
	FINAL PAPER	Apr. 8

APPENDIX D

Your grade on the paper will be based on the following:

<u>Knowledge of math</u>
- insight
- clear, concise, complete
- examples
- notation, symbols, variables
- accurate

<u>Originality</u>
- independent thought
- audience

<u>Organization</u>
- orderly, logical manner
- smooth transitions
- intro and conclusion

<u>Vocabulary</u>
- math vocabulary
- word choice

<u>Sources</u>
- references in text
- bibliography

<u>Grammar</u>

And This is the Tale That Xeno Told

Sandra Z. Keith
St. Cloud State University

Remember great Achilles, most handsome, lithe and tall...
You sing of your Olympians—why he'd outrun them all!
Achilles favored turtle soup, but to obtain that feast,
He had, you understand, to stoop and catch the beast.
Imagine! Great Achilles—was challenged by a turtle!
Said Turtle: "you can't catch me!", but Achilles merely chortled;
"I walk ten times as fast as you—I'll catch you near or far!"

He said he'd give the beast a start; he'd yield a thousand yards.
He was not man but demi-god, and had to save his face,
For all the gods had gathered there to watch that famous race.

Great Zeus himself! And Phoebus, too—the driver of the Sun;
Sly Hermes, grim Haefestus, all the Muses, every one!
I can't describe them all, you know, the clever and the mighty,
But goddesses were there as well—Diana, Aphrodite!

Now to assure for fairness, so there'd be no dirty pool,
Lord Zeus in his great wisdom laid down this rigid rule:
Achilles mustn't run or jump or reach ahead with spear
No matter how annoyed he is, and never-mind how near.
There'd be no victory turtle feast and no festivities
Until he comes upon the point where haughty Turtle is.
(A straight line, too, is de rigeur, a changing path no-go;
The turtle yet was ignorant of present times LOGO.)

Achilles with an easy stride, through a thousand yards did fare;
But when he got to Turtle's start...the turtle wasn't there!
The turtle now, you understand, was a hundred yards ahead,
So to complete this hundred yards, Achilles promptly sped.
Alas now, magic goal, ten yards! Although by far no "league",
Achilles with a blistered foot, experienced some fatigue.

What! One yard more? He got there too, but only to discover
There still was something like four inches more he had to cover!
"The race is won!" Achilles thought, "This race is such a cinch!"
And yet to his surprise he found there yet was three-eighths inch!
(Achilles hated fractions: in school he somehow missed them;
He hoped that all his troubles would be solved by the metric system.)

I'm getting bored, and so are you. I shall condense the rest.
As time went on, Olympic gods as well lost interest.
They went quite far away, in fact, to their respective stars:
Zeus to the planet Jupiter, and Ares—off to Mars.
Hermes, of course, to Mercury and Aphrodite to Venus.
(In this way though forgotten, they still aspire to please us.)
And the Muses—ah the Muses! Imagine if you're smart—
How all those ancient Muses would react to modern art!
But sadder yet to think upon—unless they all deceive us...
Is how that cruel Gravity messed up the poor god Phoebus!

Still, if you look in microscopes, I'm sure that you will find
It's zero millimeters that Achilles lags behind.
Achilles often sulks you know, but now he's getting mad.
The viewpoint of the turtle, though, is: things are not so bad.
If he will only persevere, and persevere he will!
(He has some hopes as to the rumors 'bout Achilles' heel.)

And while the situation here appears to becoming static,
It forms a classic stumbling block of modern mathematics.
And if you think, dear student, that all this isn't serious...
Indeed it is. In fact, it's called "convergence of the series".
When does the turtle meet his doom? Consider, and decide:
When he's gone a thousand over nine. Take pencil and divide!

What is Mathematics?
An Answer to our Liberal Arts Dilemma

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I was introduced to the Humanistic Mathematics Network about 18 months ago by Harald Ness, Associate Editor of the Journal, and it was the professional equivalent of finding a cache of jewels or the proverbial pot of gold. Just think, there is a whole network of people who recognize mathematics as more than (greater than?) a narrow, skill-oriented, utilitarian discipline or a collection of abstractions wedded to logic! !

I realize that the ferment in Mathematics education has provided a renewed interest in the role of mathematics in the core curriculum. The American Mathematical Monthly published the results of the Committee on Undergraduate Programs in Mathematics (CUPM) panel on "appreciation" courses in 1983. The National Endowment for the Humanities outlined a year-long mathematics course in their publication: *50 Hours, a Core Curriculum for College Students* in 1989 and in the same year the *Report on the NSF Disciplinary Workshop on Undergraduate Education* featured a mathematics course under the heading "Mathematical Literacy". Each of these documents outlined courses which reflected the spirit of the Humanistic Mathematics Network.

"A survey in breadth rather than depth of a variety of mathematical topics. While emphasis is on the spirit, concepts and structure of modern mathematics, manipulative skills and techniques are also developed."

Many of the recommendations made by the CUPM Panel are realized in a course which has been a staple in the basic studies program of my university for over twenty years. An outline of this course and a brief discussion of its underlying philosophy may be of benefit to network members who are considering their liberal arts offering(s).

The three-credit semester course is entitled *Activities in Mathematics* and carries the following catalog description: "A survey in breadth rather than depth of a variety of mathematical topics. While emphasis is on the spirit, concepts and structure of modern mathematics, manipulative skills and techniques are also developed." The textbook is *Mathematics-A Human Endeavor*, 2nd edition, by Harold Jacobs.

The course is structured around an answer to the question, "What is Mathematics?" The students recognize on the first day of class that this answer is *not* a definition of mathematics but rather a description that focuses on attributes of mathematics. The four facets of this description are:

- Mathematics as a study of patterns
- Mathematics as an organized body of knowledge
- Mathematics as an art form
- Mathematics as a tool

The mathematical content associated with each part of this description is examined below. Detail on two topics within each category will provide evidence of the depth of treatment of specific topics.

MATHEMATICS AS A STUDY OF PATTERNS

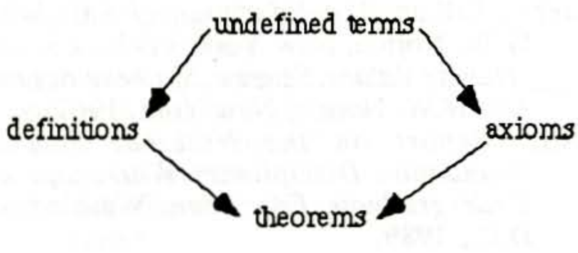
Topics covered in illustrating pattern searches include: arithmetic and geometric sequences, number bases, patterns on pool tables, infinite sets, the Konigsberg bridge problem and networks, "n" dimensions, functions, conic sections, Fibonacci sequences and the golden ratio. Inductive reasoning and the role of counter examples are also discussed. Contemporary applications of each topic, when available, show the pervasiveness of mathematics in today's world.

The development of "n" dimensions and base two indicate two typical class presentations. The "Prisoner Model" is the pattern used to initiate the work with various dimensions. Point (person) P is trapped between two points on a line, within a closed curve in the plane, and finally within a closed three dimensional "container" in space. In each instance, escape is possible by stepping out into the next dimension and returning outside the constraint. The next model looks at the patterns related to the triangle, the tetrahedron and their 4th, 5th, and 6th dimensional analogs. This search results in Pascal's Triangle. The final pattern is the typical ordered "n-tuple" representation of points in the various dimensions. (This is picked up later in the course with the equations for hyperspheres and hyperparaboloids.) Applications include hypercube architecture in computers, translation of signals from satellites in "n" dimensional spaces, medical illustrations and the dimensions of "superstring" theory.

Base two is taken as a special case within a general treatment of positional numeration systems. It is the variety of applications of the binary system which are featured. Students are introduced to digital radio, product coding, filmless photography, and compact discs in addition to the obvious use of base two in the computer.

MATHEMATICS AS AN ORGANIZED BODY OF KNOWLEDGE.

The recognition of mathematics as an organized body of knowledge features this simple model illustrating the deductive system



Truth tables are developed as a vehicle to introduce tautologies and the three basic inference patterns. A definition for proof and a discussion of methods of proof provides a setting for work related to inference patterns, equivalent statements and indirect reasoning. Axioms are studied in depth, culminating in an introduction to non-Euclidean geometry and the "loss of certainty."

The nature of the if-then statement is singled out for particular emphasis. Illustrations using advertisements, strategies in tic-tac-toe and poker, and expert systems help students see the many roles of implication. They are also forced to confront their errors in the use of the converse and/or the inverse.

The investigations of axioms and the "loss of certainty" in mathematics are among the high points of the course. The student is led from Euclid to Spinoza to Newton to Jefferson to contemporary illustrations of axioms. The notion of absolute truth in mathematics is then refuted through examining the contributions of Bolyai, Lobachevski and Riemann. Comments related to the revolution in algebra and the cultural impact in the arts help to show the influence of non-Euclidian geometry in the history of mathematics.

MATHEMATICS AS AN ART FORM

Mathematics as an art form is the only part of the description of mathematics that has just one class period devoted to its development. This is because the student needs to have a stronger background in the content of mathematics before the more creative and beautiful aspects of the discipline show through. Comparisons are made between mathematics and music, literature and sculpture as a way of illustrating this creativity within mathematics. In addition, words such as rhythm, symmetry, design and unity which mesh with the student's understanding of the arts are used to support the linkage between mathematics and other arts.

MATHEMATICS AS A TOOL

By viewing mathematics as a tool, one opens up a broad range of possibilities. Because the textbook has excellent chapters in the following areas, the course relies on counting techniques, probability and statistics to illustrate the "unreasonable effectiveness of mathematics." Most of the standard introductory topics in each of these areas are uncovered with such classic examples as the birthday problem, probability in the courts, and predictions from a sampling tray, are also included.

Students have many questions as they work the exercises associated with these chapters, which reduces the class time available for additional applications. However, one topic that does

provide for expansion is the fundamental counting principle. Four examples that provide an opportunity to illustrate this counting principle are: the 256 ways of topping a hamburger used by an area fast food restaurant, a daily newspaper that used the "impressive three word phrase" exercise in its business pages, a local professional bowler who introduced his book on spares by stating there are 1023 different possible spares and finally the matching house key problem that was experienced by the course instructor.

Personal experience on the part of students is used to see the strengths and weaknesses of the measures of central tendency. An overhead transparency containing eight different situations (e.g., charity contributions, cost of living, test scores) provides a setting for a discussion of these measures. They are then linked together (and linked to probability) through the introduction of the normal curve.

words such as rhythm, symmetry, design and unity which mesh with the student's understanding of the arts are used to support the linkage between mathematics and other arts.

Generally, textbook assignments and reading are the responsibility of the student. Class time is used to respond to students' questions, point out connections and expand upon this description of mathematics through additional, non-textbook, examples.

Another feature of the course is the writing/project component. The students are required to *react* (three to five pages) to one of two books written by Lillian Lieber. One of these is *The Education of T.C. Mits* and the other is *Human Values, Science, Mathematics and Art*. Two of these student papers are presented following this article. If the student has a particular interest and would like to complete a project rather than the paper, this can be done after consultation with the instructor. Examples of projects include designs related to arithmetic sequences, music composed and played using number patterns, a welded brass golden rectangle showing the smaller golden rectangles determined by cutting off squares, a switch and light box showing a logic pattern and a "pyramid power" experiment.

Student reaction to this course has been remarkable. Positive feedback from students has reflected a change in attitude toward mathematics and a lessening of anxiety about studying mathematics. One student responded: "Why haven't I had an experience like this in my previous study of mathematics?" It has been particularly heartening to recognize the transformation of students who are also parents in terms of the importance of mathematics in their children's future.

REFERENCES

1. Cheney, V. Lynn, *50 Hours. a Core Curriculum for College Students*, National Endowment for the Humanities, Washington, D.C., 1989.
2. CUPM, "Mathematics Appreciation Courses: The Report of a CUPM Panel", *American Mathematical Monthly*. 90 (1983) 44.
3. Jacobs, Harold, *Mathematics, A Human Endeavor*, W.H. Freeman, San Francisco, 1982.
4. Lieber, Lillian, *The Education of T.C. Mits*, W.W. Norton, New York, 1942.
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Human Values and Science, Art & Mathematics

Cindy Cichy

This is not intended to be
written as free verse.
By writing each phrase on a separate line
I am facilitating rapid reading,
because we all know
how busy
math professors are these days.
And because
Lillian R. Lieber does
in her book:
Human Values and
Science, Art and Mathematics

The first concept Lieber touches on
is the concept of
CAPABILITIES and LIMITATIONS
of the human mind.
Which then lead into the old saying about
"Life,Liberty, and the Pursuit of Happiness.

Life is talked about in the sense
that it means a lot more than
shear existence.
therefore life is not just to exist .
Happiness is talked about as
a feeling, an emotion,
not a dollar sign
as some people think.
and the the pursuit of happiness;
it's not a job or money;
it is also a feeling and emotion;
it's enjoying life, the happiness
you have made for yourself.
But we are all so busy
and in such a hurry
that we forget
to smell the flowers
and pet the animals.

But by now you're asking
what does this have
to do with this book?
Well frankly a lot.

Science, Art and Mathematics
are all man-made;
just like our world.
By
one man or many men.
Just, like the flowers and animals
which were made by one man,
God.
And now with His help
and the help of Science, Art, and Mathematics
we, the common man
can create them.

Mathematics is a lot more than
counting the change
in your pocket,
or the day of the week.
It is a group
of man-made
assumptions
that are proven, tested, and postulated.
With math we are able
to produce jobs,
build ships,
and solve the national hunger problem.
It is these assumptions
or self-evident truths
that have given us freedom,
and freedom is
Good.
It is the combination
of Freedom
and Control
that makes Math and Science
both Creative and Conservative.
This combination can and does
serve as a model for
thinking.
About anything
whether it be
Democracy or
Censorship
or Whatever!

Okay—okay
enough of Lieber's perception
and on to Cindy Cichy's

This book has taught me
that these three concepts—
Science, Art and Math
surround us wherever
we go.
There is no getting away
from them.
They're in our homes, our cars,
our books, our schools,
our pasts, presents, and futures,
Everywhere

And they need to be taken
a lot more seriously
then we treat them.
Coming into this course
I said—
"I hate Math"
and I said the same thing
when I took
my science and art classes.
But now
my eyes have been opened.
I am no longer ignorant
of the facts.
I always asked why?
Why do I need
these science classes,
one art class, and
one mathematics class.

And now I know why.
It is to teach us where
we have evolved from

In a world as complex
as ours
we can not tolerate
being ignorant anymore.
Our world is falling apart
and all we can do
is push it away.
But no more.
The system has become
one stronger
because I believe in it.
And if we realize
that we can't have
the latter without
the former
a lot more people will
believe, too.

One last quote:
Einstein said in his telegram
to the people:
support our efforts
to bring realization
to America
that mankind's destiny
is being decided:
TODAY
NOW
THIS MOMENT.

Reaction to *Human Values, Science, and Mathematics* by Lillian R. and Hugh Gray Lieber

Jeanette Nelson

They are RIGHT!
The more you KNOW, the more
there IS to know.
And then you really know
LESS than before because
there is SO MUCH more
to know about. (page 9)

I know VERY LITTLE about
mathematics. After reading
this book and almost
finishing the "Activities in Mathematics"
course, I find I know
SO much LESS about math
than I
ever IMAGINED. And life is
no longer blissful in that
part of my brain
that numbers have INVADED.

This book reiterated in
greater depth
QUITE a bit of what the instructor
covered in the course. I found it
interesting that his and Lieber's
LOGIC CIRCLES and
ELECTRIC SWITCHES were identical!

My math knowledge limited
comprehension
of most of the math in
the BOOK. But that doesn't
bother me a lot.

My feeling
after finishing
reading
the book reinforces a
view of MINE that there
is more than one way
to SKIN A CAT!

There are many different
ways to reach the
same CONCLUSIONS as
evidenced
by the COORDINATE SYSTEM
on page 103, Chapter IV.

Scientists and mathematicians
are more PATIENT and
TOLERANT with their PEERS
than the general population is
with one another.

Often, we the general PUBLIC,
get stuck on one and only one
way of thinking
about a problem and CAN'T
FIND an answer. By
looking at a situation in
as many DIFFERENT WAYS
as possible, and through
EYES not connected
to the problem,
NEW IDEAS for successful
solutions often are reached.

No ONE of the solutions needs
be BETTER than the others—it is
just a DIFFERENT WAY to get
from point A to point B
without getting STYMIED in the
middle, as in the case
by the lake. (Page 102)

I thought maybe the AUTHOR
was attempting to demonstrate
through mathematical
THEOREMS and POSTULATES
and FORMULAS what it is like
to live your life HAPPILY.

There are CERTAIN RULES which
must be followed
in life to live HAPPILY and to
get from the beginning
to the end.

It has been said
that LIFE is like a TRIP.
It is the journey that
COUNTS because once you get
to your DESTINATION, the
trip is OVER. So I was
thinking about this
as I read LIEBER'S book.

I was thinking how little I
UNDERSTAND of what she is
explaining. And it is so
DAMN FRUSTRATING when you
have thought all
of your life that 2 and 2
are 4, and all of a sudden here's
a NEW WAY to look at that
with BOOLEAN ALGEBRA. (Page 49)
But I'll accept that
and try something new to me.

By the way, I'm putting
all these PAGE NUMBERS and
parentheses in because
that was
the ONE ASPECT of the book
I did not LIKE along
with CAPITALIZED WORDS
which tend to distract the
EYE (and MIND) from
reading along smoothly.
But then I re-read so
many pages to get the DRIFT
it didn't matter all that much.

I'm not sure that I got
much more feel
for MATHEMATICS from the
book, but maybe now I
can understand better
why my husband and daughter
DEMAND such precision
and LOGIC in their most
casual conversations.

They are the SCIENTIST
and MATHEMATICIAN. I am not and
it doesn't bother me. I think
it bothers them that it
doesn't bother me. But
that doesn't make ANY of us
BETTER or WORSE than the OTHER.

We and everyone else in the
WORLD have
"'EQUAL RIGHT AND EQUAL SUCCESS,'
even though they all use
DIFFERENT COORDINATE SYSTEMS,
that is,
even though
each one approaches the problem
in his own way!" (page 105)

That is the beauty of our
DEMOCRATIC SOCIETY. We
are FREE to THINK ANY WAY
we choose
as long as it is LAWFUL
and does not HARM ANYONE.

I see this book written
in 1961 as a PLEA FOR PEACE
and CONTROL
of NUCLEAR POWER as well as
for personal freedom.

So there you have it!
My IMPRESSIONS
of what this book was
ALL ABOUT...not so
much mathematics
as how to LIVE LIFE.

Letter Division

Paul J. Tobias

```
          THE
KEYTO | SUCCESS
        OYXUT
         UHUTS
         KEYTO
         HOOCHS
         HKYSCE
          TOKHK
```

Test your Math-Logic

Each letter stands for a digit from 0-1-2-3-4-5-6-7-8-9 the same digit for the same letter throughout the problem

(answers on the next page)

```
         THE
MATH | MYSTERY
       ATEHA
        AAYYR
        AZHHT
         YMMHY
         YXTSE
          YMSH
```

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          MATH
IMAG | INATION
       IMAG
        XYMI
        HAGX
        MOHTO
        MTHAY
         AYION
         MXOGG
          MHIT
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look for more letter division in future issues

special thanks to:
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Answers to Letter Division

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34926 7500477 214

MATH MYSTERY THE
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IMAG INATION MATH
3124 3925379 1256
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