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Gary Stogsdill
Prescott College

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Being Reasonable: Using Brainteasers to Develop Reasoning Ability in Humanistic Mathematics Courses

Gary Stogsdill

Prescott College, Prescott, AZ, USA
gary.stogsdill@prescott.edu

Synopsis

Developing reasoning ability is often cited as one of the principal justifications of a mathematics requirement for liberal arts undergraduates. Humanistic math courses have become recognized as a paradigm for liberal arts mathematics, but such courses may not provide the opportunity to develop reasoning ability. The author describes his procedure for using brainteasers to promote reasoning in a humanistic math course for liberal arts undergraduates.

Developing reasoning ability is often cited as one of the principal justifications of a mathematics requirement for liberal arts undergraduates. Sporn [4] surveyed educators and philosophers from antiquity through the 20$^{\text{th}}$ century regarding the purpose of a liberal arts education and the role of mathematics therein. Prominent among the consensual views is that "mathematics is an excellent subject, perhaps the best, to use to train the student in analytical thinking, logic, and reason" [4, page 118].

Yet those of us who teach college mathematics know that liberal arts undergraduates often struggle to succeed or find value in traditional skills courses such as College Algebra. Further, Sporn [4] and George [1] found that liberal arts undergraduates tend not to be well served by traditional skills courses. Both researchers describe a century-long movement to create more meaningful math-course options, often referred to generally as liberal arts mathematics.

One of the more interesting developments within this movement is humanistic mathematics, which aims to provide liberal arts undergraduates

with an informed appreciation of significant math-related ideas and developments that have helped to define the world we live in and what it means to be human. Humanistic math courses can be taught much like a course in art appreciation or music appreciation. Such courses in "mathematics appreciation" have been singled out by George [1] for their "relative success ... as a paradigm for liberal arts mathematics" [1, page 141]. But how can students also develop reasoning ability in such courses? Brainteasers can fill this role and nicely complement any humanistic math course.

I stumbled onto the practice of using brainteasers in college math courses when I began teaching traditional skills courses at Prescott College in 1990. My students were all liberal arts undergraduates needing to meet their math requirement, and most of them entered my classes with about as much enthusiasm as one brings to a root canal procedure. Further, the prevalence of math anxiety among these students had the effect of dampening whatever reasoning ability was already present.

One way I dealt with this situation was to create a math therapy exercise for the purpose of alleviating math anxiety [5]. Another way was to try starting every class with a couple of brainteasers that would hopefully be perceived as fun and not like math. To my encouragement, the math zombies in class began to perk up and become engaged, sometimes displaying a surprising degree of creative thinking during brainteaser time. All students enjoyed processing the brainteasers, which caused the overall class energy to become more positive and more open to learning. Because of these results I have continued to use brainteasers in each of my math courses and have arrived at a procedure that I believe is optimal for developing reasoning ability.

Since creating this course in 2004, I have taught *Mathematical Explorations*, a humanistic math course that meets the mathematics requirement for liberal arts undergraduates at Prescott College. Brainteasers are an integral part of this online course for limited-residency students. Following is my procedure with rationale.

First and foremost, I emphasize to students that the purpose of the brainteasers is not to get a right answer but primarily to engage their minds in a reasoning process. My intention with this emphasis is to alleviate the anxiety-inducing pressure of traditional skills courses that focus entirely on getting the right answer. Students in *Mathematical Explorations* are graded mainly on their willingness to put time and energy into the reasoning process and to endure the discomfort of not knowing an immediate way to proceed

with a problem. This mirrors real-life problem solving where we often do not know how to proceed initially and where sustained creative thinking may be required.

Second, I encourage students to have fun and be playful. The all-too-common notion that higher learning, especially math, must be grim and stressful in order to be rigorous is simply misguided. Only a little time spent with young children reveals that natural learning is inherently playful and enjoyable. It is surely no accident that one of the behavioral criteria for distinguishing higher forms of life is their ability to engage in play. By encouraging students to have fun and play, they can more easily relax into the kind of creative thinking that is vital to solving real problems and that allows the gift of sudden insight.

Third, I use humor in teaching with brainteasers. Where possible, I try to make the actual wording of each brainteaser funny or absurd. I also try to use occasional and appropriate humor when interacting with students about their responses to the brainteasers. Appropriate humor has been shown to increase student interest and motivation, decrease anxiety, and improve learning and retention [2, 6].

Finally, I emphasize brainteaser etiquette, which mainly consists of students waiting until everyone has had plenty of time to contemplate the brainteasers before offering answers. With my current online course, *Mathematical Explorations*, students begin contemplating each weeks set of three brainteasers on Monday, with responses being allowed to the Brainteaser Forum on Wednesday. Students are not permitted to post just answers; they must also describe their entire reasoning process. On Thursday, I post to the Brainteaser Forum a series of hints, as needed for each group of responses, to aid the reasoning process. Students have through the weekend to post new responses or further thoughts, and by Monday I post not only the official answers but also a detailed explanation of the reasoning process or processes that can be effective in solving each brainteaser. Meanwhile, students are allowed to respond to each others posts by offering encouragement and ideas in a collaborative spirit.

The level of brainteaser difficulty progresses from not too challenging to fairly difficult over the course of the semester. Here is an example of a brainteaser from the beginning or not too challenging stage, although it may not be as easy as it seems at first glance:

> Now for a tragedy (with a happy ending): A poor snail has fallen to the bottom of a 10-foot-deep dry well. This is a courageous snail, and every day it is able to crawl 3 feet up the side of the well. At night, however, our poor snail slips back down 2 feet. How long will it take our hero to reach the top of the well and experience freedom?

Here is an example of a brainteaser from the middle stage of the course:

> You find yourself in a brainteaser nightmare (sorry) surrounded by 3 boxes labeled "Apples," "Oranges," and "Apples and Oranges". You are told that, while these are the correct categories, each box is labeled incorrectly and that your life will only be spared if you can label each box correctly after selecting just one fruit from one box. How can you do this? You cannot see into the boxes, nor can you use the sense of smell. (I sure hope you get this one right!)

And here is an example of a brainteaser from toward the end of the course in the fairly difficult stage:

> You come to see me in my office to express how much you're enjoying this course. Right away you start enjoying the course less because you notice I'm wearing a tag that reads: THIS STATEMENT IS FALSE. Then, of course, I point to the tag and ask you, "Is it true or false?" Please explain your answer. (Running out of my office is not an answer.)

The brainteaser immediately above has no simple answer and illustrates my practice of occasionally choosing brainteasers that allow me to discuss at length in my explanations some of the fascinating math-related ideas that enrich our lives in interdisciplinary ways. In this case I discuss the history and nature of paradox, Gödel's Incompleteness Theorem, other singular ideas of Gödel, and the limits of rational knowing.

Students respond exceptionally well to this process of using brainteasers to develop reasoning ability. Although the time commitment and possible points for this process comprise only one-quarter of the total course, this is often what students choose to comment on most extensively in their narrative

self-evaluations completed at the end of the course. Following are representative student comments from recent offerings of *Mathematical Explorations*.

> "*Every week I waited eagerly to post my answers to the three offered brainteasers. The excitement of discovering an answer, posting it, waiting, feeling successful, or the frustration of not quite getting it, fulfilled the need for a back and forth dialogue-between students and students and [students and] teacher. The weekly brain workout kept me sharp and always looking for the hidden mathematical answer waiting behind most things.*"
>
> "*The weekly brainteasers opened my mind to creativity and unique ways to problem solve. I can easily take this practice into my daily life to find solutions from a different perspective.*"
>
> "*The brainteasers I pondered every week helped me to understand a gentler approach to problem solving. I became more relaxed and less performance oriented, and [I] focused on a process that was unique to me where I would download the brainteasers, read them, and then ponder the questions. I began to notice my classmates processes and fine tuned my strategies to unravel the puzzles. I enjoyed the interaction with the other students and appreciated the guidance from [the instructor]. . . The calming effect that these processes have had on my anxious mind helped me gain confidence when I began to see that I have a natural logical and analytical style*"
>
> "*With each weekly assignment [of brainteasers] I have gained an understanding to the underlying math concepts, which allows more pieces of the puzzle to fall into place. I have become more confident in my ability to apply math concepts and skills to solve real-world problems. . . I have learned to be a better quantitative thinker and decision maker.*"
>
> "*What I appreciated most about his course was its focus on logic and the application of mathematical thinking in daily life [through the brainteasers]. While most of us know by now that we are not going to become PhD mathematicians, being able to think logically and systematically is an important skill in any profession. I appreciated the opportunity to practice this type of thinking and I believe it will serve me well in the future.*"

> "*The brainteasers we worked on each week stumped me more often than not, but because we were encouraged to approach them creatively I could let go of being overly concerned about getting them right on my first attempt. Along the way, I found an enjoyment of the process... I felt better about embracing uncertainty than in any math class Ive taken before, and this inspiration created a new determination in my heart to continue exploring mathematics!*"
>
> "*Although intimidating at first, I loved the weekly brainteaser exercises and found it refreshing to mull over possibilities first inside my own head and then collaboratively in the class forum. It was so interesting to reflect upon the different ways my classmates and I came to answer the brainteasers. It was also helpful for me to explore other reasoning styles so that I could revisit my own and learn new approaches for the future.*"
>
> "*The brainteasers were enjoyable because they continually challenged me to cultivate new perspectives and think outside the box. As a result, I found myself approaching every-day life situations in fresh, positive ways.*"

These representative student comments[1] invite four observations. First, students describe an improvement in their ability to problem solve in everyday life. This is significant because 90 years of research into outcomes of traditional math skills courses has found repeatedly that "there is little or no relationship between [traditional] math problem solving and real life problem solving" ([3], as cited in [4, page 172]). These student comments may suggest that using brainteasers in a humanistic math course accomplishes something that traditional skills courses cannot: the transfer of problem solving ability from the classroom into everyday life.

Second, students refer to the lessening of anxiety, or as one student put it "the calming effect," that this brainteaser process allows. Alleviating the typical stress level that math problem solving induces can nurture creative thinking and insight. Third, students describe the benefit of working collaboratively and how this improves their own approach to problem solving.

[1] I acknowledge here that I completed Prescott College's IRB process regarding quoting student comments, and that I have obtained written permission from all students to include these quotes in this document.

Finally, while students mention the substantial challenge presented by the brainteasers, they also refer to enjoying this challenge, gaining confidence, and wanting to explore mathematics further.

Brainteasers make a valuable addition to any humanistic math course, and they are easy to find and use. Numerous books and other resources are available that offer a variety of brainteasers at a variety of levels. It can be an enjoyable and personally enriching process for instructors to search out just the right brainteasers for their courses. I am also happy to share with anyone who is interested my own series of brainteasers used in *Mathematical Explorations*.

Humanistic math courses can be a rewarding experience for students and teachers alike. Developing reasoning ability need not be sacrificed in humanistic mathematics because brainteasers offer a fun and effective way to promote reasoning for liberal arts undergraduates.

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