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Mathematics as an Aesthetic Discipline

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INTRODUCTION

This brief paper offers a defense of the study of mathematics. It is intended for those people who are convinced either that mathematics is not worth studying or that mathematics is "just not for them."

This paper is especially intended for those humanists and the literati who thrive in the world of art, music, and literature, but who think that mathematics is a mechanical, cold, unimaginative discipline, suitable only for unartistic, uncreative "computer-types." This paper will suggest that these humanists have confused mathematics with the discipline that went by that name in their schooling. In short, this paper will suggest that the literati who think that the study of mathematics needs defending are completely unaware of the meaning of the word "mathematics." And thus they are really demanding a defense of something else; namely, the memorization of formulae and equations and the mechanical manipulation of numbers that was forced upon them in school. They will find no such defense here.

The reader is alerted to a caveat: this paper is not intended for those who find the entire academic enterprise in need of defending. Those who demand a defense of the study of music, poetry, philosophy, biology, chemistry, and mathematics are advised to look elsewhere. They will not find it here.

THE COMMON DEFENSE

Usually, the study of mathematics is defended almost exclusively along the lines of its effectiveness as an instrument. Legions of so called "mathematics" teachers attempt to sell mathematics to their students as nothing more than a manipulative and a practical tool.

Of course mathematics is useful and practical as a utensil, but only to the professional scientist and engineer. Almost everyone else will use no more "mathematics" in their everyday life than the simplest of grammar school arithmetic: balancing a check book, counting change. One needs little more. The notion that anyone other than a scientist will ever use even the most elementary trigonometry or algebra is laughable. Imagine the absurdity of being in a car or on a plane when suddenly the need arises to solve a quadratic equation or to graph a trigonometric function. But this is precisely the scenario that the traditional defense has coerced us into accepting as realistic. Clearly this is absurd. And so is our complicity.

Of course, students realize this. They become apathetic or openly hostile towards this "mathematics." And who can blame them? Why should anyone care about mathematics if its only value is its practicality, a practicality relegated either to the simplest of childish arithmetic or to the arcanely out of reach, complex world of the professional scientist's mathematics? If this is mathematics, then something is wrong with the student who likes mathematics!

"My early teachers chanted the notion of practical value like a litany. It was repeated at each level, in each course, from grade one through high school. They meant to justify mathematics on the basis of its utility in the conduct of one's daily life.

There is nothing wrong with this except they went too far and claimed too much. Mathematics is useful in this sense. But, with this narrow connotation of 'value,' a little goes a long way. Counting change, measuring carpet, or balancing one's checkbook requires only the slimmest knowledge of mathematics. From early on, I wondered why such pedestrian activity required so much schooling.

The true value of mathematics lies outside commonplace activity." J. P. King [Ki].

THE NEW DEFENSE

The new defense of the study of mathematics does not rely on the utility of mathematics. The cornerstone of

this new defense is the beauty of mathematics, a notion singularly alien to the general public.

AESTHETICS

We study mathematics for the same reasons we study poetry or music or painting or literature: for aesthetic reasons. Simply put, we study mathematics because it is one of the loveliest disciplines known to man.

"A mathematician, like a painter or a poet, is a maker of patterns.... The mathematician's patterns, like the painters or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics." G. H. Hardy [Ha].

"...this character of beauty and elegance [in mathematics is] capable of developing in us a sort of aesthetic emotion." Henri Poincare [Po].

"There is, first of all the motivating force for mathematics which is beauty." J. P. King [Ki].

The fashioners of this sublime beauty, artists indeed, must possess a rare creativity and an imagination of the highest order.

"The moving power of mathematical invention is not reasoning but imagination." Augustus de Morgan [HA].

"There is an astonishing imagination even in the science of mathematics... We repeat, there is far more imagination in the head of Archimedes than in that of Homer." Voltaire [HA].

"The essence of mathematics is its freedom." Georg Cantor [HA].

"The science of pure mathematics, in its modern developments, may claim to be the most original creation of the human spirit." A. N. Whitehead [Wh].

One of the most compelling aesthetic features of mathematics is its refined austerity. Its unadorned gracefulness is unique among the arts. In fact, part of the very essence of mathematics is its precision. People are referring to this quality when they suggest that mathematics teaches "clear thinking." Mathematics' precision does not lie in any claims of universal truth. But rather this precision, and hence power, lie in the acknowledgment of exactly the points at which mathematics consciously and deliberately abandons claims of universal truth. Mathematics is the only discipline that I am aware of that does this. And this precision and austerity allow for an elegant economy, an economy that comes from the elimination of the cluttering mire of imprecision.

"Strange as it may sound, the power of mathematics rests on its evasion of all unnecessary thought and on its wonderful saving of mental operations." Ernest Mach [Be].

"Mathematics is precise or it is nothing." J. P. King [Ki].

"Mathematical knowledge adds vigour to the mind, frees it from prejudice, credulity, and superstition." John Arbuthnot [Mo].

"One cannot escape the feeling that these mathematical formulae have an independent existence and an intelligence of their own, that they are wiser than we are, wiser even than their discoverers, that we get more out of them than we originally put into them." Heinrich Hertz [HA].

"Calculus is the most powerful weapon of thought yet devised by the wit of man." N.W.B. Smith [Mo].

The mathematician, however, is not merely an ascetic, cold and austere. He or she is an expressive artist involved in the richly human struggle to create and to discover.

"...a mathematician experiences in his work the same expression as an artist; his pleasure is as great and of the same nature." Henri Poincare [Be].

"I have heard myself accused of being an opponent, an enemy of mathematics, which no one can value more highly than I, for it accomplishes the very thing whose achievement has been denied me." Goethe [Be].

"A mathematician who is not also something of a poet will never be a complete mathematician." Karl Weierstrass [Mo].

"Other qualities of a far more subtle sort, chief among which in both cases is imagination, go to the making of a good artist or a good mathematician." Maxime Bocher [Mo].

Sadly, most people, including the otherwise sensitive and culturally sophisticated, are completely unaware of the intrinsic aesthetic features of mathematics.

"The useful combinations are precisely the most beautiful, I mean those best able to charm. This charm is the special sensibility that all mathematicians know but of which the profane are so ignorant as often to be tempted to smile." Henri Poincare [Po].

"Nothing lives further from the intellectual experience of members of the educated public than the notion that mathematics can have aesthetic value." J. P. King [Ki].

The common defense is not, however, supplanted by the new defense, but rather it is subsumed by it. This subsumption takes the unexpected form of an appreciation for the utility of mathematics. By this I mean that to most students of mathematics, the utility of mathematics should be presented in something like the same fashion as music is presented to students of music history, namely as a marvel to be appreciated, not an instrument to be operated. Those students interested in actually creating music (i.e., in becoming musicians or composers) are advised to study performance or composition. Similarly, those students interested in actually harnessing the utilitarian powers of mathematics (i.e., in becoming engineers, scientists, and mathematicians) are advised to study engineering and applied mathematics. But for the vast majority of mathematics students, a simple, honest appreciation of the remarkable utility of mathematics should be seen as the ultimate "real-world" goal. In short, the sense of agency developed in most students regarding the utility of mathematics should be of an appreciative nature, not an instrumental nature. And since "appreciation" is an aesthetic term, not a scientific term, for most students, the traditional defense of the study of mathematics as a tool is subsumed by the aesthetic perspective of the new defense.

"There is no branch of mathematics, however abstract, which may not someday be applied to phenomena of the real world." Nicolai Lobachevsky [HA].

"The mathematician, carried along on his flood of symbols, dealing apparently with purely formal truths, may still reach results of endless importance for our description of the physical universe." Karl Pearson [Be]

"Algebra is the intellectual instrument which has been created for rendering clear the quantitative aspects of the world." K. N. Whitehead [HA].

"Mathematics is the queen of the sciences." Carl Fredrich Gauss [Be].

"It is mathematics that offers the exact mathematical sciences a certain measure of security which, without mathematics, they could not obtain." Albert Einstein [Be]

"A book on the new physics, if not purely descriptive of experimental work, must be essentially mathematical." P. A. M. Dirac [Di].

"The great book of nature can be read only by those who know the language in which it was written. And this language is mathematics." Galileo [Be].

GREAT THINGS

The study of great things, including the study of great ideas, needs no defense. And many of the greatest of human thoughts have taken the form of mathematics.

"...not the mere fact of living is to be desired but the art of living in the contemplation of great things." Bertrand Russell [Ru].

"This therefore is Mathematics, she reminds you of the invisible forms of the soul; she gives life to her own discoveries; she awakens the mind and purifies the intellect; she brings light to our intrinsic ideas; she abolishes oblivion and ignorance which are ours by birth." Proculus Diadochus [HA].

"Mathematics is the only good metaphysics." Lord Kelvin [Be].

"To create a healthy philosophy you should renounce metaphysics but be a good mathematician." Bertrand Russell [Be].

"Number rules the universe." Pythagoras [Be].

"God ever geometrizes." Plato [Be].

"The Great Architect of the Universe now begins to appear as a pure mathematician." J. H. Jeans [Je].

Mathematics is created by human beings. It was not carved on tablets and handed down by a god. The most brilliant members of our species have exerted, and continue to exert, the most noble effort to give us this mathematics.

When school children study analytic geometry, they should be made aware that this seemingly trivial and esoteric subject exists to us only because of the heroic efforts of a succession of brilliant minds, culminating in the work of Descartes. Its depth, originality, and profundity are lost on students. It has been carefully polished and refined so exquisitely, presented so elegantly and simply, that students myopically receive it as a trifle.

"Though the idea behind it all is childishly simple, yet the method of analytic geometry is so powerful that very ordinary boys of seventeen can use it to prove results which would have baffled the greatest of the Greek geometers--Euclid, Archimedes, and Apollonius. The man, Descartes, who finally crystallized this great method had a particularly full and interesting life." E. T. Bell [Be].

"(Analytic geometry), far more than any of his metaphysical speculations [which include, "Cogito ergo sum."] immortalized the name of Descartes and constitutes the greatest single step ever made in the progress of the exact sciences." John Stuart Mill [Be].

When calculus students give a sleepy, disinterested yawn during the discussion of the fundamental theorem of calculus, they should be told that the most outstanding human minds struggled for over two millennia to find this seductively simple formula. Until Newton and Liebnitz finally uncovered it for us, no human eyes had ever gazed upon it, although the greatest intellects had searched for it.

Today, we present this masterpiece to teenage students in a ten-minute lecture. And students receive it in the same spirit that it's presented: as just another boring, god-given, inhuman formula to memorize. Clearly this is unacceptable. Students must learn that mathematics is the most human of endeavors. Flesh and blood representatives of their own species engaged in a centuries long creative struggle to uncover and to erect this magnificent edifice. And the struggle goes on today. On the very campuses where mathematics is presented and received as an inhuman discipline, cold and dead, new mathematics is created. As sure as the tides.

Students deserve the truth: Mathematics is vibrant and dynamic, an incredibly rich and human discipline, a liberal art, and a humanity in the purest sense.

"...the mathematics of a mathematician is profoundly personal." Seymore A. Papert [Pa].

"Although mathematics itself is 2,500 years old, more has been created in the last fifty years than in all the previous ages combined..." Jerry King [Ki].

"In mathematics alone each generation builds a new story to the old structure." Hermann Hankel [Kl].

"(Arithmetic) is one of the oldest branches, perhaps the very oldest branch, of human knowledge; and yet some of its most abstruse secrets lie close to its tritest truths." H. J. S. Smith [Be].

Educated men and women, from the dilettante to the cognoscente, must be at least modestly literate in all fields of intellectual inquiry. Imagine the literate who is not acquainted with the theories of evolution, relativity or quantum mechanics. Imagine the sophisticate who is unfamiliar with the works of Shakespeare, Picasso, or Mahler. But most so-called educated people know nothing of mathematics.

"One's intellectual and aesthetic life cannot be complete unless it includes an appreciation of the power and the beauty of mathematics. Simply put, aesthetic and intellectual fulfillment requires that you know about mathematics." J. P. King [Ki].

"What is there about mathematics that compels so many men and women to work at it with the fervor of dedicated artists and yet keeps it simultaneously outside the experience of the rest of intellectual society?" J. P. King [Ki].

"Outside of the closed circle of professional mathematicians, almost nothing is known of the true nature of mathematics or of mathematics research." J. P. King [Ki].

Most people, at least most 20th century Americans, are interested in the lives of public figures. Even the lives of some intellectuals are of interest to the average citizen: Einstein is a pop icon. Amadeus, a movie about Mozart, was a popular success. There was a recent movie about the physicist Stephen Hawking shown in American popular movie houses. The rank and file recognize references to artists and thinkers as diverse as Heisenberg, Schroedinger, Beethoven, Picasso, Stravinsky, Monet, Plato, Aristotle, Freud, Jung, Camus, and Sartre. But almost no one knows

even the names of the most important mathematicians. Who but the mathematician has heard of Gauss, Galois, Cantor? They are thinkers of the first rank. But unlike their counterparts in every other discipline, their names are completely unfamiliar. Clearly, if the masses were aware of the humanness of the mathematics enterprise, natural human curiosity would demand that mathematicians be included in the class of thinkers worth knowing.

"Those who have never known a professional mathematician may be rather surprised on meeting some, for mathematicians as a class are probably less familiar to the general reader than any other group of brain workers. The mathematician is a much rarer character in fiction than his cousin the scientist." E. T. Bell [Be].

The human essence includes an amazingly robust sense of wonder. If students realize that they have been banned access to a tremendously rich body of knowledge (mathematics), this natural wonder, if properly cultivated, will transform the "banned" into the "tempting." And students will demand to know of it. Bertrand Russell perfectly captured this refined sense of wonder in his autobiography. It is a fitting epigram for this paper.

"There was a footpath leading across fields to New Southgate, and I used to go there alone to watch the sunset and contemplate suicide. I did not, however, commit suicide, because I wished to know more of mathematics." Bertrand Russell [Ru].

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Mathematizing

Lee Goldstein

Mathematics begins
Upon a denominative
Foundation,
Where the anteceding nonverbality
Is in place,
And when a verbal undifferencing
Is eliminative,
Then the symbolic shift
Does take.