Revised Problem Statement:
An Algebraic Approach to Voting Theory

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Voting theory has been explored mathematically for the past three and a half centuries. A significant amount of the focus has been on conflicts between positional and pairwise voting schemes for three or more candidates. Significant developments have been made recently by mathematician Donald Saari, of UC Irvine, using geometric approaches to analyze these conflicts. I intend to utilize his ideas and his new approaches, and expand upon these developments from an algebraic perspective. More specifically, I will use tools from representation theory to elicit some of the natural behaviors of voting profiles. I will also make generalizations to similar results for partially ranked data.

The resulting thesis will contain three major contributions. The first will be a short history of progress made in the field of voting theory. I will explore the evolution of concepts and methods of voting, as well as methods of analyzing voting, as they led into mathematical analysis.

Secondly, it will provide a translation of many of the concepts within voting theory into representation theoretical terms. Saari has already begun the work of putting voting into framework of vector spaces. However, there are many more tools within abstract algebra which will prove to be very useful. It will allow us to easily decompose voting spaces directly in terms of the tally maps we perform on them. Also, there are concepts from representation theory which provide very elegant ways of generalizing to partially ranked data.

Thirdly, this thesis will contain a reworking of voting theory concepts, in an attempt to recover and extend previously developed results. More significantly though, I intend to use tools from representation theory to decompose profile spaces associated with partially ranked. The goal will be to develop analogs to theory previously stated for fully ranked data. Very specifically, I hope to study the partial ranking of $n$ alternatives, where the voters are asked to fully rank their favorite $k$ alternatives. This is a natural first case for partial ranking since it is a simplification of a full ranking. Hopefully, any work toward analyzing this case will lead to techniques for analyzing other partial rankings.
References


various kinds of paradoxes that arise from different methods of tallying votes.

[10] Donald G. Saari. Explaining all three-alternative voting outcomes. *J. Econom. Theory*, 87(2):313–355, 1999. This article mostly concentrates on decomposing the voting space in the case of 3 candidates. This is a good introductory article to Saari’s geometric methods in that it stays in the context of an example to help simplify the ideas.


[13] Donald G. Saari. *Decisions and Elections: Explaining the Unexpected*. Cambridge University Press, 2001. This book spends extra time building intuition for why our expectations might be as they are. It was written in a similar tone to his article “Symmetry, Voting, and Social Choice”, in that it is geared for the educated layman. It does, however, also cover Saari’s geometric methods.
