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I will consider mathematics education only at the university level, where the habits and values of the university mathematician are decisive. I start with a couple of little "anecdotes".

When members of the mathematics department at University X get together to talk about course Y in their Undergraduate Committee, their agenda does not include quality of teaching or student difficulties. The only point concerning course Y is the syllabus.

Practically all mathematics courses descend from an ancient tradition. Even modern courses like linear algebra have now been going on for decades. So they all, including course Y, already have a well-established syllabus, as can be seen from the table of contents of any respectable textbook on the subject. For calculus and pre-calculus courses, that syllabus is dictated (so it is believed) by the needs of the succeeding courses. Intermediate algebra must prepare the student for college algebra, college algebra must prepare the student for calculus 1, calculus 1 for calculus 2, calculus 2 for calculus 3, and so on. There comes a moan, "It's too much material to cover in a semester. Half of them always flunk!" And the familiar answer: "There's nothing we could take out without messing them up for the next semester." Other reforms besides tinkering with the syllabus are neither proposed nor considered.

What about constraints imposed by other departments? A Mathematical Emissary walks over to the Engineering School to talk to a committee of engineering professors about the syllabus for some course in engineering mathematics. To every possible topic, the engineers cry, "Yes! Very good! They should know that too!" The M.E. secretly suspects that not all her engineering colleagues around the table "know that too." Never mind, they want their students to know it. When the M.E. gets back to the math department, her colleagues quickly decide that the engineering profs are "out of their gourds," and cut the swollen syllabus back to traditional size.

Such troubles with syllabi are semi-trivial. Much more serious are the troubles that come from self-defeating teaching styles, and from the teacher's false conception of the nature of his subject.

Nearly all U.S. university professors, including math professors, have been shaped by a shared trauma: their graduate training. They have survived an intensive apprenticeship as aspiring Ph.D.'s, struggling for years to win their advisor/supervisor's approval.

They would learn that treating mathematics students as if they were human beings ("humanistic mathematics education") is the way to avoid mathematics avoidance.

For many, this ordeal has permanently imprinted upon their thinking their advisor's way of thinking and teaching. (Sometimes, it is true, the imprint is reversed. After a "stormy" advisership, the student may seek to teach and think in a way opposite to the advisor's.)

In research, this tendency is well known. An experienced reader recognizes the writing, not only of Professor X, but also of X's students. It is not surprising that somethinng similar happens in teaching style. This tendency is mentioned less often because teaching, unlike publication of research, is a private performance. Not totally private, of course, since it is done in the presence of students. But so far as the math professor's colleagues and fellow mathematicians are concerned, it is definitely private. If mathematicians A, B, and C are asked about the teaching of mathematician D, generally none of them will have any knowledge of it, except what they could conclude from hearing D talk at a research seminar or at a meeting of the American Mathematical Society. D's performance in the classroom will be unknown to any of the three, unless some stray student once commented about it. What the university mathematician does in

the classroom is virtually unknown to colleagues, even in her department. And it's strongly influenced by her experience as a graduate student.

What then is the character of graduate mathematics teaching, which indirectly determines the character of all university math teaching, graduate and undergraduate? The purpose of graduate mathematics teaching is to produce new mathematicians. If enough students from University A get Ph.D.'s, publish papers, and become recognized mathematicians, then University A's graduate mathematics program is a success. If some students in the program fail to follow the lectures, fail to do the exercises, fail to complete the program, that is of little consequence.

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An influential graduate mathematics professor is of necessity deeply embedded in his research. In his teaching he uses the same language, assumptions, viewpoints as in conversation with his colleagues. These may well be unfamiliar to the graduate student. The student who succeeds has to overcome the disorientation of the lecture room and somehow leap into the gestalt of research level talk.

There is a connection between teaching style and writing style. One vicious trait of the periodical research literature in mathematics is the exclusion of motivation or heuristics. An author is not usually permitted to tell the reader about the false leads and blind alleys which led ultimately to success. Neither is she encouraged to tell why the problem in question is interesting or useful.

In the classroom, the graduate professor is not constrained by journal editorial policy. Nevertheless, his lectures are usually as barren of heuristics or motivation as are his research articles. From a certain so-called "rigorous" point of view, all that is necessary in mathematics is to state the theorems accurately and prove them correctly (rigorously). Where they come from and what they are good for are not considered to be part of the mathematics. Indeed, the graduate professor himself may not have much of a clue where his subject came from, or what it's good for. He may well have been educated in the same abstract, dogmatic style he now perpetuates.

The university mathematicians who are educated this way tend naturally to teach this way. They start as teaching assistants while still attending graduate classes, so the influence from graduate class to precalculus teaching is immediate and direct. They usually are given no training in teaching or lecturing, no obervation or criticism by more experienced teachers. Instead, they are just handed a textbook, a classroom number, and a meeting time.

Later, as assistant professors, they persist in the habits acquired as teaching assistants. After all, nobody ever told them to do differently. Their main concern now is the struggle for tenure, which means—not teaching, but publication. True, there are student evaluations. But students don't usually explain very well what they like or don't like. Anyhow, their evaluations don't matter much, unless they are catastrophic.

Despite all this, some teaching assistants are good natural teachers. And some who aren't naturally good teachers learn after a while to listen to their students, and achieve communication with them. This is a personal matter. Nothing in the university system requires it or rewards it.

Given teachers indoctrinated in this manner, it is no surprise what happens in the undergraduate classroom. As in the graduate class, the lecture method is supreme. Interruptions are not desired. Students are there to take notes, not to engage in dialogue. The important thing is for the professor to give a correct statement of the facts ("theorems"). Failure to mention an exception or a condition is considered "dishonest." If he can possibly do it, he should prove everything he says. ("Prove" means "prove rigorously," leaving nothing out.) This ideal is seriously struggled for in upper-division and graduate courses. In calculus and pre-calculus, everyone admits that it's impossible. That is part of the reason why math faculty dislike teaching these courses. Someone has to teach them, of course. It is done by teaching assistants, part-timers from local high schools, and a few full faculty members forced to take a turn at this sub-mathematical chore.

Most university mathematicians are "pure" (not "applied"). They are ill at ease teaching concepts from biology or physics. If the textbook contains such applications, such teachers prefer to quickly pass over them. Calculus students don't hear the names of Copernicus or Galileo or Kepler. No one expects them to understand the part that calculus played in the scientific revolution that created the modern world. Calculus is just something you do with formulas ("functions") and graphs.

I hasten to say that not all graduate mathematics courses are unmotivated piles of dogma hurled at the heads of hapless graduate students. Some great mathematics researchers are natural teachers. Some are eager to explain the heuristic behind their work. The inspiration from such professors can be carried forward in the teaching of their students, just as the smug dogmatism of other professors can be carried forward in the teaching of their students.

No one will be surprised to hear that the inspiring teachers and graduate classes are not the majority. To be appointed to a graduate faculty of mathematics it is not necessary that one's teaching be brilliant, or even passable. Whether a graduate professor of mathematics does or doesn't take pains with his teaching, his colleagues won't be delighted and won't be upset. What he does with his classes is his business, not theirs.

So the most important disciplinary constraints on the reform of mathematics teaching are the teaching style mathematicians learn in graduate school, and the institutional values which neglect teaching quality in deciding tenure and promotion.

Every newspaper-reader knows that recently a lot of talking, meeting-going, and some Federal money are being spent to reform mathematics education. Should we expect this activity to bring significant results? I would like to think so. But it is my impression that the reform being promoted is curriculum reform, especially increasing the use of computers. The problems of teaching style and of a mistaken idea of the nature of the subject are hardly mentioned. Why so? To answer that question would take a whole separate paper. But so long as it is so, what changes do come will be little more than reshuffling what we already have, not creating anything essentially new or different. (Readers who are curious about what I mean by "mistaken idea of the nature of the subject" can look up references 1, 3, or 4.)

How can we change this lamentable situation, where bad teaching of mathematics is propagated down from one generation to the next?

I can imagine two different ways. One is <u>segregation</u>. Impose a sharp separation between research and teaching in mathematics.

Future teachers of undergraduates would receive a training appropriate for future teachers. That means that along with creative problem-solving and correct calculating, a central place would be reserved for communicating. Both in writing and in speech, both in speaking and in listening. Not only answering questions, but understanding questions. And also knowing mathematics, not only in itself, but also in relation to history and philosophy, to the human sciences as well as the natural sciences. This solution is possible only in theory. The distribution of power in the academic-governmental world makes such a reform inconceivable.

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Giving up on solution #1, we turn to solution #2: change the thinking of the big-wigs of American math. The top math professors in the top grad schools, the top research managers in the top industrial labs, the top math bureaucrats in the U. S. Office of Education and the 50 State Departments of Education, the top editors of math texts in the top math text publishing companies, the math ed professors in the top Colleges of Education, the top officers and staff of AMS, MAA, and SIAM.

All these people would learn to care how math is taught, not just what math is taught. They would learn that independent work by students is essential in all mathematics classes, K through 20. They would learn that realistic, credible applications of mathematics are indispensable, from K thnrough 20. They would learn

that treating mathematics students as if they were human beings ("humanistic mathematics education") is the way to avoid mathematics avoidance.

That is my "solution."

But what kind of solution do you call that? It's like the solution to the cat problem in Aesop's fable. To be safe from her claws, the mice need only hang a bell round Kitty's neck. But where will they find a mouse willing to bell the cat?

How can we achieve a revolution in the established views on mathematics education? If at all, only by organized effort, by education, lobbying and agitation, continued over years. That is how change is sometimes achieved in the U.S. cultural-political system. Who will do that organizing, educating, lobbying and agitating? Only those who care enough about it to spend the time and effort.

Presently one small organization is engaged in this work (the Humanistic Mathematics Network c/o Prof. Alvin White, Mathematics Department, Harvey Mudd College, Claremont, California, 91711). If enough people care, more organizations will appear. More people will join in their activity. In time, something may happen.

If, on the other hand, we aren't able to turn around mathematical education in the large, at least we should be able to change it in the small. In our own schools, our own departments, our own courses, we can teach students (not merely "teach the material"). We can make sure we understand where the mathematics came from and where it is going, and share this inside information with our students. We can insist on interaction in the classroom, not tolerating a passive audience that merely copies formulas from the blackboard. To change the old saying slightly, we can light a candle or two, even while we curse the dark.

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