Research Proposal: Rupture in Thin Fluid Films

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1 Introduction

For a thin fluid film in one dimension whose height is given by $h(x,t)$, we can model the film by the thin film equation

$$h_t + (h^n h_{xxx})_x = 0,$$

(1)

where the physical values for $n$ are 1, 2, and 3. We can study the behavior of this equation for various boundary conditions using numerical methods and functional analysis of the energies of $h$. For instance, we require that the mass $\int h dx$ of the film be conserved. We may also find it useful to consider the arclength of the film and other higher-order energies that have less in the way of physical interpretation.

If $h$ experiences a singularity, we say that the film has ruptured.

2 Proposed Research

In order to examine the thin film equation, I must first gain working knowledge of lubrication theory and of the body of literature on the subject. I will then attempt to characterize the behavior of the film for certain types of boundary conditions using numerical simulations with L. Zhornitskaya and A. L. Bertozzi’s work in [2] as a guideline, and using the energies of the film. Finally, I will search for new energies of the film that may provide further insight into its behavior.

3 Prior Research

It has been theoretically demonstrated that for $n \geq 3.5$, finite-time singularities (ruptures) cannot occur in Equation 1. It appears that there may be a critical value $n_*$ of $n$ below which rupture may occur and above which it may not. However, this value and any dependence it may have on boundary conditions are unknown. [1]
References
