Research Proposal:
Pebble Sets and Triangulations

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1 Introduction

After attending the colloquium by Randall Maddox entitled “Placing Points in Polygons”, Professor Su and I became interested in how pebble sets could be used to improve lower bounds of minimal triangulations of polytopes.

2 Proposed Research

In his lecture on October 20, 2004, Randall Maddox proved that any chamber in the pebble set of a polygon must have exactly three sides. I plan to examine pebble sets in polytopes to attempt to generalize these requirements for higher dimensions. In particular, I will consider the hypothesis that any chamber in a pebble set must be a simplotope. I plan to first classify pebble sets in three-dimensional objects, beginning with the cube.

In 2002, Su, de Loera, and Peterson [3] proved that any polytope with \( n \) vertices and \( d \) dimensions has a pebble set of size \( n - d \), which means \( n - d \) is a lower bound on the minimal triangulation. I hope to find pebble sets that are larger than \( n - d \) for certain classes of polytopes by using the polytopal generalization of Sperner’s lemma, thus improving the lower bound. I plan to begin by studying simplotope, the minimal triangulation of which was studied in 2004 by Seacrest and Su [2]. I will also consider the minimal triangulation of cubes in \( d \) dimensions. In 2003, Bliss and Su [1] studied the minimal triangulations for the 3-cube and the 4-cube, but the 5-cube is so complex that new methods are needed to feasibly improve upon the lower bound for its minimal triangulation, which is currently 60.

References
