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"Modern Mathematics" at Sonoma State University

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SUMMARY

This essay is a report on a liberal arts course designed to meet the "Quantitative Reasoning" component of the general education requirement as implemented at Sonoma State University. This particular course is also included in the procedure by which the university accommodates students who are certified as being learning disabled in mathematics.

ENROLLMENT

Between Spring '90 to Fall '93 with Summer '92 and '93, I taught fourteen classes of Math 141. There were 445 students in these classes including 136 "special-admission" students who were enrolled through the Sonoma State University Disability Resource Center. The nature of this special-admit program will be discussed later in this paper.

GOALS

Mathematics 141 (Modern Mathematics) is an experimental course originally entitled "Liberal Arts Mathematics" and was designed to provide students with a liberal arts mathematics course which promotes:

- An understanding of logic, generality, abstraction and proofs in mathematics—specifically the construction of indirect proofs.
- An awareness of current significant problems and ideas of interest to the international mathematics community.
- An awareness of the humanity and the limitations of mathematics.
- The development of students' skills in writing about mathematics.

The variety of problems in this course made demands upon the following five skills: Numerical, Algebraic (generalized numerical), Logical, Spatial, and Verbal.

When this course was first developed, there was no single text that could be used to accomplish these goals, so I planned to create an anthology composed of reprints of articles from about twenty-five books and to develop some problem sets to go along with these articles. It turned out that there was considerable difficulty in getting copyright permission for reprinting the articles, so as an alternative to creating an anthology, I gave students a reading list of books containing the articles and a set of corresponding problems, with no explanatory text.

This was not adequate since there were not enough copies of the books for all the students. Eventually, as the course developed over several semesters, I started adding my own descriptive text to the problem set, dropping books from the reading list, and writing my own text. It is now published as *Math Odyssey 2000* [1], Stipes Publishing, 1994.

MEANS OF GOAL ACHIEVEMENT

Here are some sub-goals and specific ways in which these goals were accomplished.

Logic, Generality, Abstractions, and Proofs

- Presentation of logical paradoxes, Aristotelian law of the excluded middle, universal and existential quantifiers, examples of direct and indirect proofs.
- Study of the rational and irrational numbers.
- Euclid's proof of the infinitude of primes.
- The uncountability of the irrationals.
- The concept of measure of sets of reals.
- Presentations of abstract axiomatic systems, and components of a logico-deductive system.
- Examples of abstractions—extending the interpretation of various axiomatic systems to "non-mathematical" entities, such as people, words and oranges.

Current Problems

This included the introduction to long-standing problems either just recently solved or still unsolved. The solved problems include the Four Color Theorem, the Continuum Hypothesis, Gödel's Theorem, and Fermat's Last Theorem. The unsolved problems include Goldbach's Conjecture and the infinitude of twin primes.

Humanity and Limitations of Mathematics

Throughout the text there are small anecdotes and references to various mathematicians; typically, one out of eight homework problems required an answer based upon such a reference (see Note 1). One entire chapter is devoted to biographies of ten interesting mathematicians (five men and five women) with every problem in the exercises asking questions about

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the content of these biographies. In the text and in the reading assignments, we studied the events of the early 20th century relating to Frege, Hilbert, Russell, Gödel, and the constructivists. We studied applications of Gödel's Theorem.

Skills in Writing About Mathematics

An outside book of readings is assigned (in the first semester it was a book list; later I used Guillen's *Bridges to Infinity* [2] until it went out of print in 1992. Then I used Paulos' *Beyond Numeracy* [3]) (see Note 2). Students wrote essays based on readings in these books; I rearranged the order of the authors' chapters to coincide with the material in my text as we covered it in class. The essays, each 5 to 7 typewritten pages in length, were to reflect three things:

1. "Critical Thinking"-type thoughtful summaries of the authors' writing, with insights and personal analysis;

2. discussion of the connection between the readings and our class work, or any other experience; and

3. their personal reaction, in the style of a journal, to both the material from our text and that from the reading assignments.

Essays were graded on these three aspects and were corrected in grammar and spelling. Their authors were chastised for "un-cited", not quoted, "lifted" passages. The students quickly learned to use quotation marks when appropriate.

MEASURING GOAL ACHIEVEMENT

In the latest classes I used 4 exams, 3 midterms plus the final exam (counting 65%), 32 homework assignments (counting 10%), and five essays based upon outside reading (counting 25%).

For some of the earlier classes I used the four exams (60%), 15 weekly journals (5%), 4 essays (25%) and 32 homework assignments (10%). One semester, I gave two classes (60 students) 9 essays, which meant that I ended up grading 540 essays that semester! Big mistake.

SPECIAL ADMISSIONS:

"Learning Disabled" (LD) Students

Some students in this class had not completed our prerequisite of Math 50 (intermediate algebra), had not achieved the equivalent score on the ELM and were admitted through SSU's special program for students with learning disabilities, which is as follows:

1. If a student passes Math 40 (Elementary Algebra), but cannot pass Math 50 and cannot attain an equivalent score on the ELM (Entry Level Mathematics exam) even after repeating, then that student may be eligible to be tested for learning disability.

2. The student is interviewed by a learning disability specialist in the Disability Resource Center to determine whether or not testing is appropriate.

3. If the student is tested and certified as learning disabled, then he or she can be admitted to Math 141 as a special admission.

This admission provides for extra tutorial assistance and certain other arrangements specific to the student's needs. These arrangements apply mostly to exams: isolation during exams, or extra time on exams.

Writing Assignments

Because the regular students had made better grades than the special admits (see Table 1), I had the impression that the regular students in Math 141

TABLE 1: Grade Point Averages (A = 4.00, B = 3.00, etc.)

	Regular Students	Special Admits	Total
Number of Students	309	136	445
Grade Point Average	2.61	2.07	2.44

were, in general, better essay writers than the LD students. I thought I would test this hypothesis by examining the data. My opinion was wrong. Table 2 is of the results for six classes for which I had comparable data (prior to Summer '92 I had used a different type of writing assignment).

Why was there this discrepancy between the Academic year grades and the summer grades? Among the regular students in my summer classes I had at least 3 high school math teachers, a professional advertising writer, and an SSU valedictorian. (She was the student speaker at graduation, June '93.) Also, I believe that Math Anxiety turns into Math Panic in the summer's faster pace.

Why Special Admits Succeeded

Of the 136 special admit students, 17 (12%) made A's, 29 (21%) made B's, 48 (35%) made C's; this is evidence of success in this class. Why was there this much success?

- Much of the material was more "verbal" than usually found in math classes.

- The best of the special-admit students were just plain GOOD STUDENTS. They attended every class and always had their homework ready to turn in on time; they could and did read and write well. They often re-read the reading assignments and had rough drafts of their papers written well ahead of the deadline date. They made changes to the drafts as they gained more insights from our classroom discussions. They participated in these discussions, asked questions about what they didn't understand, and formed study groups.

- The Sonoma State University Learning Center supplies a mathematics major as a student assistant (not a "reader" to grade papers) for this class. This student sits in class and then holds review sessions for the special-admits, though other students may also attend. I periodically met with the student assistant to discuss teaching and studying strategies and to indicate which topics needed to be emphasized in the review sessions.

- Some students did request a separate room for taking exams, but not many. Of the 136 "special-admit" students, only about 12 (9%), actually requested a separate room for taking exams (either my office or a supervised room in one of the administrative offices in "The Village"). The maximum amount of extra time needed on any exam was about ten minutes; most of them completed the exams within the regular time.

Why Were There Failures Among the Special Admits?

There were 12 F's (about 9%). Most of these students were absent a lot, usually didn't read the assignments, didn't get their essays written on time or at all, didn't seek tutors, and didn't study for exams. In about half of these cases, they seemed to be students who had given this class their lowest priority. Some (about 4%), however, did do all the work and tried desperately to overcome an extremely severe disability, but still fell short of passing.

Why were there D's among the special admits?

I could detect two reasons. First, there were the C students who had lost their intensity too soon or for some other reason (pressure from other courses, etc.) dropped down to a D ("C's gone sour"). The other D

TABLE 2: Grades on Essays Based Upon Reading Assignments Maximum possible grade=25.

	Special Admits		Regular Admits		Totals	
Class	Number	Average	Number	Average	Number	Average
Fall '93	7	22.5	36	19.4	43	19.9
Spring '93, sec.1	9	21.3	17	19.2	26	19.9
Spring '93,sec.2	9	20.2	21	18	30	18.6
Fall '92	8	20.9	31	21.3	39	21.2
Summer '93	7	19.7	11	22.5	18	21.4
Summer '92	8	18.7	20	23.5	28	22.1
TOTAL	48	20.5	136	20.4	184	20.4
Academic year	33	21.2	105	19.6	138	20.0
Summer	15	19.2	31	23.1	46	21.8

students were "breakthrough F's". They did attend all classes, did all the homework while getting a lot of help from study groups, and tried very hard on every exam, getting most of the verbal problems and almost half of the computational ones. They might not have passed had it not been for the writing assignments.

"Severe" Learning Disabled, a Judgment Call

Through a very subjective (but not pejorative) judgment on my part I thought I was able to identify some of the students as "severe" and some "not severe" in the difficulty they were having with this material. I did not assign these labels to students while they were in my classes, but only while going back over my grade books, after the fact. Table 3 compares the grade distribution among three groups.

"Severity" had very little relationship to numerical or algebraic skills. They could work with calculations involving specific numbers, and they could substitute specific numbers into formulas. They also "knew" (had memorized) the basic rules of algebra. And severity definitely had nothing to do with verbal skills. Their ability to communicate ideas verbally, to insert personal analysis, to write coherent sentences and well organized paragraphs was more than adequate.

It now seems to me that severity is related to logic in the following sense: a failure to appreciate the intricacy of detail and the "literalness" of a mathematical statement, suggesting a "glossing-over" of the fine points of a logical process, perhaps in the sense of "fuzzy logic". Whatever "severity" is, I thought a student's disability was "severe" if I perceived that student to be unsuccessfully struggling with many of the concepts.

By struggling I mean:

- spending a lot of time on these problems,
- getting help from every available source,
- requiring a variety of explanations and completely misunderstanding every one of them (especially the generality of words such as "any" or "every", as in "prove that the square of every odd number is odd").
- turning in a paper for every homework assignment, even if some were just token papers.

A token paper was one that contained all the solutions to those problems the students could work, plus solutions which were either paraphrased or even copied from the answers or the hints given in the text, *Math Odyssey 2000*. Every problem in this text has a hint and more than half of the problems have answers in the back of the book.

I encouraged and even demanded token papers; the justification for this is based upon the theory that "mathematics enters the body through the fingertips". In addition, it gave me one more chance to concentrate on a particular difficulty that might be revealed by an incomplete copy or an inaccurate paraphrase and to write a comment aimed at resolving this particular difficulty. By the way, sometimes, an incomplete copy was one in which a small word such as "not" was omitted!

To put it another way, not struggling means:

- giving up,
- failing to attend class, and
- failing to turn in even token homework papers.

From Table 3, we see some very interesting results. Some non-severe students got F's (12%), usually because they failed to turn in their homework and essays in on time, skipped exams, or failed exams even when they could work the hardest problems. Of course, some severe students also got F's.

TABLE 3: Grade Distribution Among Three Student Groups

Student	A	B	C	D	F	Total
Non-severe	15(18%)	21(25%)	29(35%)	13(15%)	6(7%)	84
Severe	2(4%)	8(15%)	19(37%)	17(33%)	6(12%)	52
LD Total	17(12%)	29(21%)	48(35%)	30(22%)	12(9%)	136
Regular	82(26%)	96(31%)	79(26%)	34(11%)	18(6%)	309
Overall	99(22%)	25(28%)	127(29%	64(14%)	30(7%)	445

TABLE 4: Comparison of Correct Answers on Final Exams by "Regular", "Severe" LD, and "Non-severe" LD Students on Three KeyFinal Exam Problems

Student	Number	MAP	FERMAT	PRIME
Regular	103	49(47%)	29(28%)	56(54%)
LD(Severe)	19	7(37%)	4(21%)	7.5(39%)
LD(Nonsevere)	21	12(57%)	2(9%)	7.5(36%)
LD(Total)	40	19(47%)	6(15%)	15 (37%)
All Students	143	68(47%)	35(24%)	71(50%)

Some severe students (4%) got A's because they got perfect grades on their essays, memorized proofs for the exams, got lots of help on homework, and were perfect on all the other, easier, exam problems.

Key Problems

Intuitively, I thought I could see an easy way to identify "severe" LD students as ones who had trouble with certain key concepts involving logic, generalization and geometry. I noticed that some students had particular difficulties with the following problems:

- Stating denials and contrapositives
- Construction of a direct proof
- Understanding Fermat's Last Theorem
- Coloring a plane map properly (understanding the four color theorem).

I did a grade analysis on the following final exam problems for which I had comparable data (see Table 4):

MAP: A planar map with instructions to color it properly

FERMAT: A pair of problems; prove FLT for (say) n = 15, given it is true for (say) n = 5 and prove FLT among the odds for (say) n = 3. (Here n is the expo nent in the equation x^n + y^n = z^n.)

PRIME: Prove that there is no largest prime number.

Conclusion

My subjective identification as "severe" failed to distinguish between severe and nonsevere because of the results on FERMAT and PRIME. The funny thing is that the nonsevere LD's had more learning ability than the regular students on MAP.

Perhaps map coloring was sufficiently unlike any math problem they had had that they didn't believe that they couldn't do it. Or maybe they were more desperately in need of a correct solution on the exam and did not treat this problem as cavalierly as the "regular" students.

STUDENT REACTION

Here is a sample of the data that I had at hand; it is representative of the student responses to this class. The written comments were from both summer classes and Spring '92; the machine graded responses were from Spring '92 and Summer '92.

Written comments: (N=65) 75.4% positive, 15.4% mixed, and 9.2% negative. Using 1 for positive, 0.5 for mixed, and 0 for negative, we had a weighted rating of 83%. Machine graded: (N=94) (Summer school 7-point scale transformed to 4.0 scale) 3.46 out of 4, or 86%.

ANOTHER APPROACH TO LIBERAL ARTS MATHEMATICS: NLA, THE SLOAN FOUNDATION'S "NEW LIBERAL ARTS"

Between 1982 and 1990 the Alfred P. Sloan Foundation has spent over $21 million to finance curricular changes (in science, mathematics, humanities, fine arts, and social and behavioral sciences) at 36 liberal arts colleges throughout the U.S. by providing faculty leave grants, course development funds, and financial support for the publication of a variety of texts, monographs, pamphlets, and other classroom material. They have also provided financial support of conferences for faculty members from a great many other colleges all over the U.S. The purpose of this massive effort is to redefine the concept of "Liberal Arts" and to institute the "New Liberal Arts" (NLA) as replacement for the traditional liberal arts curriculum (see Note 3).

...mathematics enters the body through the fingertips.

According to the NLA 1990 report [4] , "...the Sloan Foundation's New Liberal Arts program is based upon the belief that a liberal education for our time should involve undergraduates in meaningful experi-

ences with technology and with quantitative and mathematical approaches to problem solving in a wide range of subjects and fields". They further state that students should understand "...the nature of modern technology, the scientific and cultural settings in which engineers work, and also the impacts (positive and negative) of technology on individuals and society" ([4], Page 1).

They continue to argue that students should be much more comfortable than they currently are with calculations and "reasoning with numbers", and with the application of modern mathematical and physical models not only in the sciences, but in the social sciences and humanities as well.

The report also tells us that a goal of the NLA program is to direct undergraduate curriculum toward "Quantitative Reasoning" which it defines as "engineering thinking", involving "... some numerical, graphical, algebraic and other quantitative or mathematical problem solving technique" [4].

The NLA program strongly promotes mathematical modeling as a crucial part of the curriculum, because they say a mathematical model "... is neutral by its nature and thus the very same piece of mathematics can be variously interpreted to fit applications that at first blush appear quite unrelated".

They speak of "case study" components of the curriculum developed at some places such as at the University of Chicago and at Mount Holyoke College, where the computer modeling software STELLA is extensively used.

Comparison

I like the way the Sloan program is trying to educate our students for today's technological world, but their approach seems to negate the non-technical and non-professional quality that we think of as being the flavor of liberal arts. To me the NLA in the Sloan Program stands for "Non-Liberal Arts", and I think that our Math 141 course in Modern Mathematics is actually closer to the traditional definition of liberal arts and still satisfies even the Sloan Foundation's definition of "Quantitative Reasoning".

NOTES

- You can't expect students to learn anything about the biographies if you don't ask questions about the biographies. In calculus and differential equations, I have tried using texts with biographies sprinkled throughout the text, but these never "worked" in getting students to learn anything about the biographies since they seemed to be off-hand and gratuitously thrown in.

- The two books I used had received negative reviews (Guillen after being in print for 7 years and Paulos after 2 years in print) in the Mathematical Monthly. The criticisms of these books were based upon claims of "errors" and "misleading" oversimplifications and "inaccurate" representation of complex ideas. The critics were saying that students should not be allowed to read these books unless they read them as examples of falsehoods in mathematics.

- The original seven liberal arts (from about 50 BC) were Astronomy, Arithmetic, Geometry, Music, Rhetoric, Grammar and Dialectic– a curriculum for a free people. Today by "liberal arts" we mean intellectual, non-technical and non-professional areas of study such as literature, philosophy, and history.

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