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Workshop on Beauty and Explanation in Mathematics

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Summary

On March 10–12, 2014, Umeå University in Sweden will host a workshop on mathematical beauty and explanation. The goal of this workshop is explore the question of whether beauty and explanation are related in mathematics. The workshop will bring together top researchers from fields such as mathematics, philosophy, and mathematics education for which this topic is relevant. Many of these researchers have till now worked within their own discipline boundaries on related topics, but have not met or worked with each other. We hope the workshop not only develops the programs of these established researchers, but also sparks interest in young researchers and encourages others to contribute to this specific question, or other related questions about the nature of mathematics. Registration is currently open, and will remain so until the capacity, of around 30 participants, is filled.

Participants

Some of the invited participants and speakers currently registered for the conference include:

Alan Baker, Swarthmore College (Philosophy);
Marcus Giaquinto, University College London (Philosophy);
Gila Hanna, University of Toronto (Mathematics Education);
Juliette Kennedy, University of Helsinki (Philosophy and Logic);
Marc Lange, University of North Carolina Chapel Hill (Philosophy);
Hendrik Lenstra, Leiden University and UC Berkeley (Mathematics);
Nathalie Sinclair, Simon Frasier University (Mathematics Education);
Tord Sjödin, Umeå University (Mathematics);
Mark Steiner, Hebrew University (Philosophy).

Detailed description and goals

The purpose of this workshop is to bring together philosophers, mathematicians, and mathematics educators to study a question which is both relevant and timely for all three groups, namely whether mathematical beauty and mathematical explanation are related. This question has its roots in an ancient distinction, made since at least the time of Aristotle, between proofs that explain and proofs that merely demonstrate. Explanatory proofs, whatever 'explanation' actually consists in, tend to be highly sought-after in mathematics. Even if a demonstrative proof has been found, mathematicians will continue to search for an explanatory one, indicating that explanatory proofs are in some way privileged over merely demonstrative ones. However, it is not clear in what way these proofs are privileged. Are they nicer because they convey some sort of understanding? Or do they have some sort of aesthetic appeal, above and beyond the cognitive benefits? This is the question at the center of the workshop: whether explanation and beauty are linked, and whether in trying to articulate what this link might be (or not be), we can deepen our understanding of what explanation and beauty are.

Explanation and beauty, and the kinds of questions posed above, are topics of central importance in the practice of mathematics. Although beauty is difficult to characterize, mathematicians often cite it as a (sometimes primary) motivation for what they do (see, for example, [15, 6]). And explanation, with its ties to sense-making, is commonly discussed among mathematics educators and philosophers (with growing interest in the last several years) as an important component in mathematical understanding (see, for example, [10, 5]).

While no research, as far as we know, has attempted to connect beauty and explanation, we have some reasons to believe they might be related. For instance visual proofs, such as the famous representation of the sum $1 + 2 + 3 + \ldots + n = n(n + 1)/2$ shown in Figure 1, are often claimed to be explanatory and, perhaps, more aesthetically pleasing than inductive or purely algebraic accounts. However, we have yet to establish a bridge between theories of what explanation consists in (see for instance [12, 8]) and what beauty might be (as in [1, 19]).

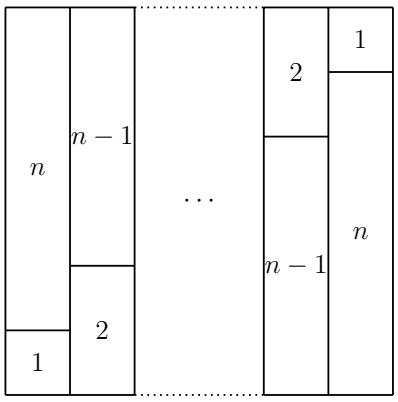

Figure 1: A visual proof.

This is a difficult task, and to make progress we will benefit from interdisciplinary collaboration. The approach taken in this workshop will be largely empirical, built around the examination of concrete examples of proofs commonly held to be beautiful and/or explanatory, to see if links can be found between the two. Pilot data, drawn from a two-year-long seminar, out of which this workshop grew, will help guide our choice of examples. Initial analyses of these data indicate that the relationship between beauty and explanation is plausible, but not at all clear. We have collected examples of beautiful proofs that are explanatory, and some that are not. We also have examples of non-explanatory proofs that might be considered beautiful. To understand the possible relation between explanation and beauty, we need to make progress on a number of thorny issues, including the objectivity/subjectivity of aesthetic judgments in mathematics.

In summary, the specific goals of the workshop are to:

- Improve our understanding of mathematical beauty and explanation, and the extent to which they are related;
- Develop a standard set of examples relating to explanation and beauty;
- Bring together top researchers in different, but complementary, disciplines; and
- Disseminate results of study across discipline boundaries.

The last two of these goals, in our opinion, are particularly important. It is an explicit goal of this project to help break down disciplinary boundaries, which happen to be fairly rigid, and facilitate the exchange of examples, theories, data, and ideas.

Connections to contemporary research

The main theoretical issues involved in analyzing these examples draw on three different but related areas: explanation, visualization, and aesthetics. We discuss them briefly below.

Explanation

In contemporary discussions of mathematical explanation, two standard models are often cited as the main competing views of explanation. The first, put forward by Steiner [20], asserts that explanation comes about when the proof exploits the

'characterizing property' of the class of entities covered by the theorem. This view has been attacked by some contemporary philosophers of mathematics, for instance Hafner and Mancosu [4], on the grounds that it is hard to specify what a 'characterizing property' is and that several mathematical proofs apparently fail to fit Steiner's model. A second view, attributed to Kitcher [7], is a so-called unification theory, which asserts that the arguments that are found to be explanatory are those that unify. This view, while interesting, especially given that there are connections to explanation in science, is also seen as problematic. For instance, Tappenden [21] points out that some uniform patterns of argument are discarded.

Recent work has tried to either build on one of these two views or suggest alternatives to account for explanation. The account of most interest at the workshop is one developed by Lange, which suggests that explanation arises from three properties: unity, salience, and symmetry. The suggestion of symmetry includes a large variety of mathematical relations, such as in the complex plane the fact that i and $-i$ behave the same algebraically (both yield -1 when squared). Lange has suggested six examples of pairs of proofs, one explanatory and one not, that he claims differ along these three dimensions, and one topic of discussion will be whether these three are necessary and sufficient for explanation.

Visualization

A second topic, related to the first, is visualization. This is a hot topic, both within mathematics and science education (see for instance [16, 23]), philosophy (as in [3, 2, 9]) and computer science (see for example [14]). A central question in this area deals with the role of visualization in mathematical understanding. It is widely believed that visualization aids in understanding, and recent research suggests that some types of proofs, namely diagrammatic ones, might be rigorous ([11, 13]) but the question of whether any given proof in a standard mathematical context with a visual representation might be rigorous is still contentious.

This question, in turn, might be connected to beauty. Must beauty involve, in some way, a rigorous and detailed understanding of a particular mathematical object, such as a theorem? If so, does visualization add to or detract from the aesthetic appreciation? We can also ask whether visualization might be related to explanation, which is also a tricky question. If a proof's visualizability helps to make it explanatory, then how do proofs that are not visualizable also manage to explain? Do they explain in the same way as visualizable proofs? One possible path into these difficult questions is to examine the role of symmetry in both our perceptions of beauty and explanatoriness. These kinds of questions form a second theme of the workshop.

Aesthetics

The third topic, aesthetics, has a long history and is difficult to summarize. In particular the definition of beauty has changed over time [22], and given that much of this history involves beauty in the context of the arts, it is hard to provide a concrete formulation for what beauty would mean in mathematics. There are some attempts to apply characterizations of beauty in the arts to mathematics, such as the notion that beauty is essentially compelling [18], or that beauty involves fit [17]. In recent years, Sinclair [19] has suggested moving the discussion away from characterizations of proofs or theorems towards a broader conception of mathematical practice. Most of the attendees of our workshop (both from education and philosophy) subscribe to this view. On the other hand, any careful discussion of beauty in mathematical practice must be consistent with the account of beauty of the mathematical objects contained therein. The reconciliation between these two accounts of beauty will be a third theme of the workshop.

More Information

For more information, please see our webpage: `http://mathbeauty.wordpress.com/wbem/`. Or contact Manya at `manya.sundstrom@matnv.umu.se`.

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