

## A System of Equations: Mathematics Lessons in Classical Literature

Valery F. Ochkov

*Moscow Power Engineering Institute (National Research University)*

Andreas Look

*Friedrich-Alexander-Universität, Erlangen-Nürnberg*

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# A System of Equations: Mathematics Lessons in Classical Literature

Valery F. Ochkov

*Moscow Power Engineering Institute (National Research University), Moscow, RUSSIA*  
ochkov@tw.t.mpei.ac.ru

Andreas Look

*Friedrich-Alexander-Universität, Erlangen-Nürnberg, GERMANY*  
Andreas.Look@en-technik.stud.uni-erlangen.de

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## Synopsis

The aim of this paper is to showcase a handful of mathematical challenges found in classical literature and to offer possible ways of integrating classical literature in mathematics lessons. We analyze works from a range of authors such as Jules Verne, Anton Chekhov, and others. We also propose ideas for further tasks. Most of the problems can be restated in terms of simple mathematical equations, and they can often be solved without a computer. Nevertheless, we use the computer program *Mathcad* to solve the problems and to illustrate the solutions to enhance the reader's mathematical experience.

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**Keywords:** *mathematics; classical literature; system of equations; Mathcad*

## 1. Introduction

The emergence of powerful portable mobile devices such as smartphones and tablets has changed our daily lives. We can lie on the couch, ride the bus, fly on a plane, or sit in a cafe and still continue to have a close connection with the world's cultural and scientific life. For example, we can simultaneously read a book, listen to the text of the book, watch films about this book, surf on the internet seeking other people's opinion about this book, log in to a forum to leave our own opinions about the book, and solve problems ... found in the book. The boundaries between different parts of our lives

start to blur. Poetry becomes a part of science, religion becomes a part of poetry. In such a world, it is important to work and teach in interdisciplinary ways. With this perspective, this paper aims to embed classical literature in mathematics lessons and also mathematics lessons in classical literature.

We can find a lot of mathematical problems in classical literature. Typically, these problems are quite simple to solve, often requiring only some simple mental calculations. Most of the remaining problems can be restated in terms of simple mathematical equations and can be solved using “computers”, those that have also been available to the heroes in these literary works or at least to the authors of these works: pen and paper. Occasionally however, there are more difficult and complex mathematical challenges. In the following we survey a range of such challenges and approach them uniformly with the aid of the computer package *Mathcad*.<sup>1</sup>

## 2. Anton Chekhov’s *The Tutor*

The first problem we analyze is also the simplest one. It is a mathematical task found in Chekhov’s short story *The Tutor* [1].<sup>2</sup> The task is the following:

“If a merchant buys 138 yards of cloth, some of which is black and some blue, for 540 rubles, how many yards of each did he buy if the blue cloth cost 5 rubles a yard and the black cloth 3? Repeat what I have just said.” [...]

The tutor looks in the back of the book and finds that the answer is 75 and 63 [1].

Such a simple problem can be solved without a computer or even without a calculator. However following our own determination to use *Mathcad* in all these challenges, we will proceed that way.

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<sup>1</sup>*Mathcad* is a powerful software package which can solve rather complex mathematical problems. It can be easily installed on every smartphone or tablet PC. You can quickly learn how to use the program and will not lose your skills to work with this package after a forced break. Readers may find more on our thoughts about using *Mathcad* and other mathematical software packages in the mathematics classroom in an earlier essay [4].

<sup>2</sup>All classical references cited are old enough to be in public domain. Interested readers should be able to locate online versions easily. An English version of Chekhov’s story, for instance, is available at <http://www.eldritchpress.org/ac/tutor.htm>, accessed July 24, 2015.

There are two equations and two unknowns. Figure 1 shows how the problem is solved in *Mathcad*. By using the command `solve`, we can solve linear and non-linear equations.

$$\left[ \begin{array}{l} \text{Blue Cloth} + \text{Black Cloth} = 138 \text{ Yard} \\ 5 \frac{\text{Ruble}}{\text{Yard}} \cdot \text{Blue Cloth} + 3 \frac{\text{Ruble}}{\text{Yard}} \cdot \text{Black Cloth} = 540 \text{ Ruble} \end{array} \right] \xrightarrow{\text{solve,}} \left[ \begin{array}{l} \text{Blue Cloth} \\ \text{Black Cloth} \end{array} \right] \rightarrow [63 \text{ Yard} \quad 75 \text{ Yard}]$$

Figure 1: Chekhov's mathematical problem in *Mathcad*.

Here the number of equations is equal to the number of unknowns. But in many problems set in the real world, the number of equations will usually not equal the number of unknowns. We will see that it is possible to find such more complex problems in classical literature too

### 3. Dostoyevsky's *The Gambler*

The mathematical problem which can be found in Dostoyevsky's *The Gambler* [2] is much more difficult than the previous one. It deals with exchange rates of different currencies. In order to describe the problem, we have taken quotes from different parts of the book; dispersed in this manner, the problem is not as clearly formulated as the previous one.

"But you will soon be in receipt of some," retorted the General, reddening a little as he dived into his writing desk and applied himself to a memorandum book. From it he saw that he had 120 rubles of mine in his keeping.

"Let us calculate," he went on. "We must translate these rubles into thalers. Here—take 100 thalers, as a round sum. The rest will be safe in my hands." [...]

There is what is owing to you, four friedrichs d'or and three florins, according to the reckoning here.

Of course, we began by talking on business matters. Polina seemed furious when I handed her only 700 gulden, for she had thought to receive from Paris, as the proceeds of the pledging of her diamonds, at least 2000 gulden, or even more.[...]

“That has nothing to do with it. Listen to me. Take these 700 florins, and go and play roulette with them. Win as much for me as you can, for I am badly in need of money.”

I began by taking out five friedrichs d’or (fifty gulden) and putting them on the even.

“Yes, I have won twelve thousand florins,” replied the old lady. “And then there is all this gold. With it the total ought to come to nearly thirteen thousand. How much is that in Russian money? Six thousand rubles, I think?” However, I calculated that the sum would exceed seven thousand rubles—or, at the present rate of exchange, even eight thousand.

“Oui, Madame,” was the croupier’s polite reply. “No single stake must exceed four thousand florins. That is the regulation.”

“I cannot, Madame. The largest stake allowed is four thousand gulden.”

She was to receive exactly four hundred and twenty friedrichs d’or, that is, four thousand florins and twenty friedrichs d’or.

“Polina,” I said, “here are twenty-five thousand florins—fifty thousand francs, or more. Take them, and tomorrow throw them in De Griens’ face” [2].

We can find the exchange rate against the ruble with the help of all these excerpts. Following equations can be set up:

$$\begin{aligned}
 120 \text{ ruble} &= 100 \text{ thalers} + 4 \text{ friedrichs} + 3 \text{ florins}; \\
 700 \text{ gulden} &= 700 \text{ florins}; \\
 5 \text{ friedrichs} &= 50 \text{ gulden}; \\
 13000 \text{ florins} &= 8000 \text{ ruble}; \\
 4000 \text{ florins} &= 4000 \text{ gulden}; \\
 420 \text{ friedrichs} &= 4000 \text{ florins} + 20 \text{ friedrichs}; \\
 25000 \text{ florins} &= 50000 \text{ francs}.
 \end{aligned}$$

If the number of equations (seven) exceeds the number of unknowns (five), then the system of equations is called *over-determined*. The problem of *The Gambler* is then over-determined. Now such problems can be solved without a computer, and this particular one can too. For instance we can one by one calculate the rates of the individual currencies (1 *gulden* = 1 *florin*, 1 *friedrichs* = 10 *gulden*, and so on) and evaluate them against the ruble. But once again, we will use **Mathcad** to solve the problem.

$$\left[ \begin{array}{l} 120 \text{ ruble} = 100 \text{ thaler} + 4 \text{ Friedrichs} + 3 \text{ florin} \\ 700 \text{ gulden} = 700 \text{ florin} \\ 5 \text{ Friedrichs} = 50 \text{ gulden} \\ 13000 \text{ florin} = 8000 \text{ ruble} \\ 4000 \text{ florin} = 4000 \text{ gulden} \\ 420 \text{ Friedrichs} = 4000 \text{ florin} + 20 \text{ Friedrichs} \\ 25000 \text{ florin} = 50000 \text{ franc} \end{array} \right] \text{ solve, } \left[ \begin{array}{l} \text{thaler} \\ \text{Friedrichs} \\ \text{florin} \\ \text{gulden} \\ \text{franc} \end{array} \right] \longrightarrow [0.94 \text{ ruble} \quad 6.2 \text{ ruble} \quad 0.62 \text{ ruble} \quad 0.62 \text{ ruble} \quad 0.31 \text{ ruble}]$$

Figure 2: Dostoyevsky's mathematical problem in **Mathcad**.

Figure 2 shows the mathematical challenge with its solution in **Mathcad**. Like in the previous task we use the command **solve** to solve the problem. However, the problem here is much more complicated and therefore we also use the **float** operator. Using **float**, it is possible to decide how many significant digits the solution has (here two). As it can be seen by means of the solution one *thaler* was worth 94 *kopecks*, one *florin* or *gulden* 62 *kopecks*, one *franc* 31 *kopecks* and one *friedrich* was equal to six *rubles* and twenty *kopecks*. So in this story the character Babulenska won 7930 *rubles* in the roulette game and the general paid the teacher 117.73 *rubles*.

#### 4. Jules Verne's *Twenty Thousand Leagues Under the Sea*

Next let us look for a mathematical problem in Jules Verne's novel *Twenty Thousand Leagues Under the Sea* [6]. Indeed we can find one which deals with the size of the submarine Nautilus. The following quote describes the problem:

Here, M. Aronnax, are the several dimensions of the boat you are in. It is an elongated cylinder with conical ends. It is very like a cigar in shape, a shape already adopted in London in several constructions of the same sort. [...]

Its area measures 1011.45 square metres; and its contents 1500.2 cubic metres; that is to say, when completely immersed it displaces 1500.2 cubic metres of water, or 1500.2 metric tons [6].

Basically, the submarine's structure can be described by three geometrical shapes. The main part of the submarine is the cylinder. The other part of the submarine consists of two identical cones, which are placed on the bottom and top of the cylinder. Additionally some information is given about the surface and volume. Therefore it is possible to set up two equations, but the problem is described by three unknowns. Figure 3 illustrates the problem.

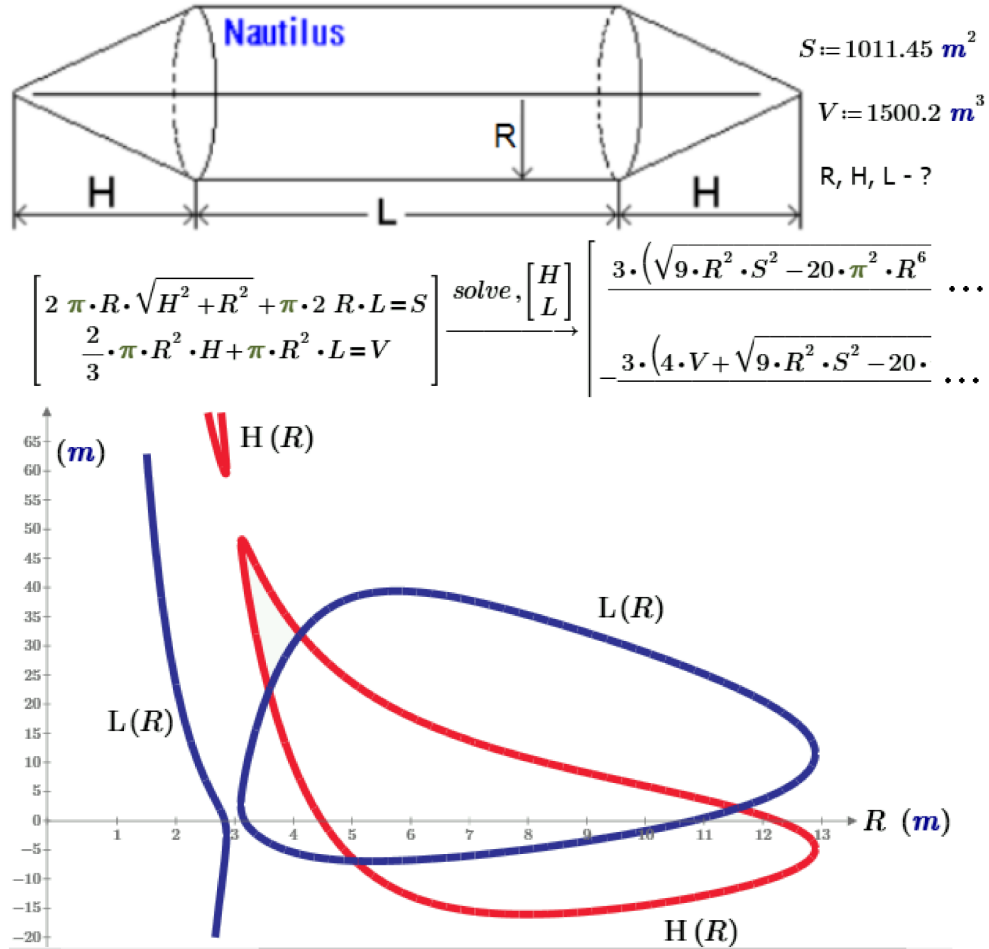


Figure 3: Jules Verne's mathematical problem in Mathcad.

Again, **Mathcad** is used to approach the problem, and like in the previous tasks, we use the **solve** command. However, this time, **Mathcad** returns no specific numerical solution for the task; the system of equations is under-determined. Instead, it returns a function, called **Size**, with the argument

$R$ . So the values of the height ( $H$ ) and length ( $L$ ) both depend on the value of the radius ( $R$ ). The system of our two non-linear equations has two solutions and so the operator returns a matrix consisting of two rows (length  $L$  and height  $H$ ) and two columns (two solutions). To save space, we display only one solution in the matrix above, but both solutions are displayed graphically in Figure 4 below. It is clear that not all solutions of the mathematical problem are possible real solutions—some of the solutions yield negative lengths and this is, of course, not possible.

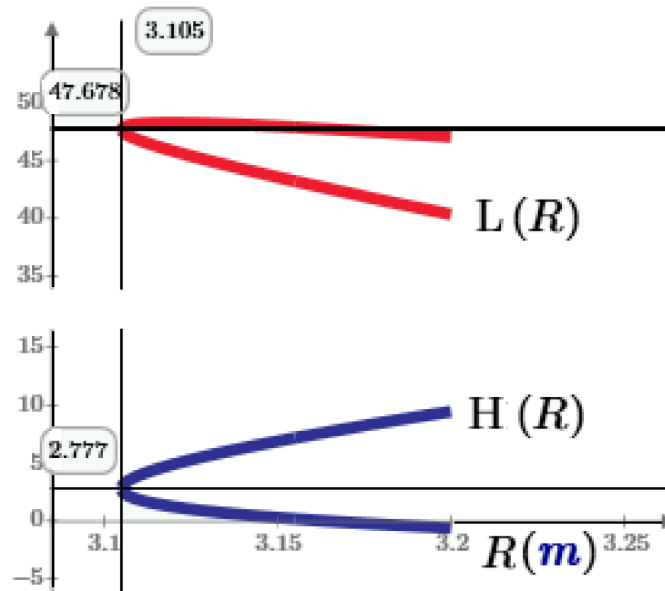


Figure 4: Solution of Jules Verne’s mathematical problem.

Assuming that the radius  $R$  of the cylindrical section of the submarine is 3.105 m, the length  $H$  of its conical parts is equal to 2.777 m and the length  $L$  of the cylindrical central portion 47.678 m. The overall length of the boat will then be equal to 53.232 m. All in all only one of the two solutions can be considered acceptable. Using the “bad” solution would result in an impractical form—the bow would be too blunt, which would increase the water resistance of the boat—and furthermore the resulting shape would not match the image of Nautilus as it is illustrated in the books of Jules Verne.

The Nautilus setup has much potential for creating further tasks. Some examples of problems which can be assigned to university students are given below:



1. Determine the minimum value of  $R$  for which the problem has a unique solution (see left-hand end of the curves).
2. Which of the two solutions is the most likely one and why?
3. Determine the radius  $R$  for which the boat turns into a cylinder without cones ( $H = 0$ ) or into two cones without a cylinder part in the middle ( $L = 0$ ).
4. Determine the dimensions of the submarine with the volume equal to 1011.45  $m$  and a minimum surface.
5. Determine the dimensions of the submarine with the surface equal to 1500.2  $m$  and a maximum volume.

This is but a sample of all possibilities. We believe readers can come up with many others.

## 5. Molière's *Le Bourgeois Gentilhomme*

DORANTE: Do you remember well all the money you have lent me?

MONSIEUR JOURDAIN: I believe so. I made a little note of it. Here it is. Once you were given two hundred louis d'or.

DORANTE: That's true.

MONSIEUR JOURDAIN: Another time, six-score.

DORANTE: Yes.

MONSIEUR JOURDAIN: And another time, a hundred and forty.

DORANTE: You're right.

MONSIEUR JOURDAIN: These three items make four hundred and sixty louis d'or, which comes to five thousand sixty livres.

DORANTE: The account is quite right. Five thousand sixty livres.

MONSIEUR JOURDAIN: One thousand eight hundred thirty-two livres to your plume-maker.

DORANTE: Exactly.

MONSIEUR JOURDAIN: Two thousand seven hundred eighty livres to your tailor.

DORANTE: It's true.

MONSIEUR JOURDAIN: Four thousand three hundred seventy-nine livres twelve sols eight deniers to your tradesman.

DORANTE: Quite right. Twelve sols eight deniers. The account is exact.

MONSIEUR JOURDAIN: And one thousand seven hundred forty-eight livres seven sols four deniers to your saddler.

DORANTE: All that is true. What does that come to?

MONSIEUR JOURDAIN: Sum total, fifteen thousand eight hundred livres.

DORANTE: The sum total is exact: fifteen thousand eight hundred livres. To which add two hundred pistols that you are going to give me, which will make exactly eighteen thousand francs, which I shall pay you at the first opportunity [3].

Here we have six different measuring units: *louis*, *livre*, *sol*, *denier*, *pistol* and *franc*. Again although it is possible to solve the problem completely without a computer, we use **Mathcad** to avoid mental stress. The relationship between *louis* and *livres* is quite easy to understand:  $460 \text{ louis} = 5060 \text{ livres}$ . Therefore  $1 \text{ louis} = 11 \text{ livres}$ . For the second equation, which can be set up, the whole text has to be quoted. It is a balance of the whole text (see the first column in Figure 5).

$$\left[ \begin{array}{l}
 (200 \cdot \text{louis} \\
 + 120 \text{ louis} \\
 + 140 \text{ louis} \\
 + 1832 \text{ livres} \dots \\
 + 2780 \text{ livres} \dots \\
 + 4379 \text{ livres} + 12 \text{ sols} + 8 \text{ deniers} \dots \\
 + 1748 \text{ livres} + 7 \text{ sols} + 4 \text{ deniers} \\
 (200 + 120 + 140) \cdot \text{louis} = 5060 \cdot \text{livres} \\
 20 \text{ sols} = \text{livre}
 \end{array} \right] = 15800 \cdot \text{livres}$$

$$\text{solve, } \left( \begin{array}{l} \text{louis} \\ \text{livres} \\ \text{sols} \end{array} \right) \rightarrow (2640 \text{ deniers} \quad 240 \text{ deniers} \quad 12 \text{ deniers})$$

$\text{sol} = 12 \text{ deniers}$

$\text{livres} = 20 \text{ sols}$

$\text{louis} = 11 \text{ livres}$

Figure 5: Molière's mathematical problem in **Mathcad**.

There is one additional equation in this figure which cannot be found in the text:  $20 \text{ sols} = 1 \text{ livre}$ . Assuming that the exchange rates have to be natural numbers, 20 *sols* have to be one *livre*. Other assumptions would yield

non-natural numbers for the exchange rates. But why could we make such an assumption? Like the Nautilus task, this problem is under-determined. Thus imposing arbitrary but helpful constraints is perfectly fine!

The relation between *pistols* and *francs* seems quite complicated at the start. We need a little bit more work:

$$\left( \begin{array}{l} 15800 \text{ livres} + 200 \text{ pistoles} = 18000 \text{ francs} \\ \text{pistoles} = 11 \text{ livres} \end{array} \right) \text{ solve, } \left( \begin{array}{l} \text{livres} \\ \text{pistoles} \end{array} \right) \rightarrow (\text{francs} \quad 11 \text{ francs})$$

livres = francs      pistoles = louis

Figure 6: Molière’s mathematical problem in *Mathcad*; Part II.

Putting together all our work, the following exchange rates can be found:

$$\begin{aligned} 1 \text{ franc} &= 240 \text{ deniers}; \\ 1 \text{ livre} &= 240 \text{ deniers}; \\ 1 \text{ sol} &= 12 \text{ deniers}; \\ 1 \text{ louis} &= 2640 \text{ deniers}; \\ 1 \text{ pistol} &= 2640 \text{ deniers}. \end{aligned}$$

We see, after all this computation, that one *louis* is equal to one *pistol*. In fact, *louis* and *pistol* are the same thing. Maybe in the future, in a hundred years, scientists will also forget that “Dollars” and “Bucks” are the same and have to calculate the exchange rate to figure it out.

## 6. Sergej Aleksandrovich Rachinskij’s *1001 Tasks for Mental Calculation*

Finally we look at Sergej Aleksandrovich Rachinskij’s *1001 Tasks for Mental Calculation* [5]. Originally published in the 19<sup>th</sup> century, this book was recently revived in the form of a website (<http://www.1001task.ru>). Smartphone owners can download problems from the book for free and try to solve them. All problems should be solved purely mentally, without using any tools such as paper, pen, calculator, and so on.

Rachinskij presents many interesting problems in this book. Through them, it is possible to learn how people lived in the 19<sup>th</sup> century in Russia, what they did, what they sold, and how high the prices were. Like in the previous tasks most problems in this book are also mainly just financial calculations, but there are also some other types of problems.

Here is one of the tasks from Rachinskij's book:

A coppersmith had 8 pieces of copper, each weighing 1 *pound* and 8 *lots*. He made from these pieces some copper kettles, each of the kettles has a weight of 1 *pound*, 21 *lots* and 1 *spool*. How many kettles did he make? Answer: 6 [5].

Many years ago, every Russian student, even illiterate Russian peasants, knew the relation between *pounds*, *lots*, and *spools*; these were all commonly used units of mass. Nowadays nobody knows anymore the conversion factors between these units. Of course, one could just “look it up” on the internet, but we will calculate the required relations between these units with **Mathcad**. This way the problem becomes much more difficult and also more interesting.

$$\begin{array}{l}
 \text{Answer} := \left| \begin{array}{l}
 \text{pound} \leftarrow 1 \\
 i \leftarrow 0 \\
 \text{for } n\_lot \in 1, 2 \dots 1000 \\
 \quad \left| \begin{array}{l}
 \text{for } n\_spool \in 1, 2 \dots 1000 \\
 \quad \left| \begin{array}{l}
 \text{if } 8 \cdot \left( \text{pound} + 8 \cdot \frac{\text{pound}}{n\_lot} \right) = 6 \cdot \left( \text{pound} + 21 \cdot \frac{\text{pound}}{n\_lot} + \frac{\text{pound}}{n\_lot \cdot n\_spool} \right) \\
 \quad \left| \begin{array}{l}
 M^{(i)} \leftarrow \begin{bmatrix} n\_lot \\ n\_spool \end{bmatrix} \\
 i \leftarrow i + 1
 \end{array} \right. \\
 \quad \left. \left. \begin{array}{l} \\ \\ \\ \end{array} \right. \right. \\
 M
 \end{array} \right. \\
 \end{array} \right. \\
 \text{Answer} = \begin{bmatrix} 32 & 34 \\ 3 & 1 \end{bmatrix} \quad \begin{array}{l} \text{pound} = 32 \text{ lots} = 96 \text{ spools} \\ \text{lot} = 3 \text{ spools} \end{array}
 \end{array}$$

Figure 7: Seeking the relations between old measuring units in **Mathcad**.

Figure 7 shows one of the possible solutions of the problem. The solution is based on trying all possible variants for the different relations between the measuring units (try everything from 1 *pound* = 1 *lot* to 1 *pound* = 1000 *lots* and also from 1 *lot* = 1 *spool* to 1 *lot* = 1000 *spools*). As almost all of Rachinskij's tasks involve natural numbers only, using this method for calculating the relation between the different measuring units does make sense.

Of course, we broke the golden rule of Rachinskij and did not solve this problem in our mind. Instead we created a special computer program for the problem. What would you say about that? What would Rachinskij say? Creating computer programs is now the new way of mental training. Maybe, if Rachinskij knew about computers, he would even recommend solving his problems with computers!

## 7. Conclusion

In this article, we argued with concrete examples that many interesting mathematical challenges exist in classical literature. There are many more examples out there, of varying difficulty. Indeed there are good challenges for each level of mathematical maturity—it is possible to integrate literature in mathematics classes from first grade up to the university level. We encourage the readers to find other good examples and bring them to their own classrooms.

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