1-1-2013

Review: Cyclicity Results for Some Antianalytic Toeplitz Operators Acting on H^p

Stephan Ramon Garcia
Pomona College

MR2986295 (Review) 47B35 (30H10 47A16)

Cassier, Gilles (F-LYON-ICJ); **Choukrallah, Réda** (3-CSTJ)

Cyclicity results for some antianalytic Toeplitz operators acting on H^p. (English summary)

Extracta Math. **27** (2012), *no. 1,* 31–58.

In this paper, the authors use certain lacunary decompositions of H^p functions to obtain families of cyclic functions for conjugate-analytic Toeplitz operators $T_{\overline{b}}$: $H^p \to H^p$ $(1 < p < \infty)$ where b is a finite Blaschke product. In addition, they also describe the structure of some of the invariant subspaces for such operators.

Building upon some of their previous work [G. Cassier, in *Operator theory and Banach algebras (Rabat, 1999)*, 51–71, Theta, Bucharest, 2003; MR2006314 (2004i:47001); R. Choukrallah, *Lacunarité et vecteurs cycliques pour les semi-groupes de shifts adjoints*, thèse de doctorat, Univ. Bordeaux, 2006, http://grenet.drimm.u-bordeaux1.fr/pdf/2006/CHOUKRALLAH_REDA_2006.pdf], the authors establish two general decomposition results for H^p functions which may be of independent interest. A key tool is Theorem 2, which asserts that if u is an inner function and f belongs to H^p $(p > 1)$, then $f = \sum_{k=0}^{\infty} f_k u^k$ where f_k is in $K_u^p = H^p \cap (u\overline{H_0^p})$ and the series is norm convergent in H^p. The authors also demonstrate (Theorem 3) that if $(e_k)_{k\in J}$ $(J \subseteq \mathbb{N})$ is a normalized unconditional basis for K_u^p, then any function f in H^p for $p \geq 2$ can be uniquely represented in the form $f(z) = \sum_{k\in J} \widehat{f}_k(u(z))e_k(z)$ where $\sum_{k\in J} \|\widehat{f}_k\|_2^2$ is finite. Both of these decompositions hold for general inner functions u and their proofs occupy the bulk of the paper.

A lacunary decomposition in H^p $(1 \leq p < \infty)$ associated with an inner function u is a function f in H^p which enjoys a decomposition of the form $f(z) = \sum_{k\geq 0} f_k(z)u^{n_k}(z)$ where the exponents form a lacunary sequence (i.e., $n_{k+1}/n_k \geq d > 1$ for $k \geq 1$ where $d > 0$ is a constant). For functions f having such decompositions with respect to some finite Blaschke product b, the authors provide several conditions which imply that f is cyclic for $T_{\overline{b}}$. The settings $2 \leq p < \infty$ (Section 2) and $1 < p < 2$ (Section 3) require somewhat different tools.

Reviewed by *Stephan R. Garcia*