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Review: Cyclicity Results for Some Antianalytic Toeplitz Operators Acting on H^p

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AMERICAN MATHEMATICAL SOCIETY

MR2986295 (Review) 47B35 (30H10 47A16) Cassier, Gilles (F-LYON-ICJ); Choukrallah, Réda (3-CSTJ)

Cyclicity results for some antianalytic Toeplitz operators acting on *H^p***. (English summary)** *Extracta Math.* **27** (2012), *no. 1*, 31–58.

In this paper, the authors use certain lacunary decompositions of H^p functions to obtain families of cyclic functions for conjugate-analytic Toeplitz operators $T_{\overline{b}}: H^p \to H^p$ (1) where bis a finite Blaschke product. In addition, they also describe the structure of some of the invariantsubspaces for such operators.

Building upon some of their previous work [G. Cassier, in *Operator theory and Banach algebras* (*Rabat, 1999*), 51–71, Theta, Bucharest, 2003; MR2006314 (2004i:47001); R. Choukrallah, *Lacunarité et vecteurs cycliques pour les semi-groupes de shifts adjoints*, thèse de doctorat, Univ. Bordeaux, 2006, http://grenet.drimm.u-bordeaux1.fr/pdf/2006/CHOUKRALLAH_REDA_2006.pdf], the authors establish two general decomposition results for H^p functions which may be of independent interest. A key tool is Theorem 2, which asserts that if u is an inner function and f belongs to H^p (p > 1), then $f = \sum_{k=0}^{\infty} f_k u^k$ where f_k is in $K_u^p = H^p \cap (uH_0^p)$ and the series is norm convergent in H^p . The authors also demonstrate (Theorem 3) that if $(e_k)_{k \in J}$ ($J \subseteq \mathbb{N}$) is a normalized unconditional basis for K_u^p , then any function f in H^p for $p \ge 2$ can be uniquely represented in the form $f(z) = \sum_{k \in J} \hat{f}_k(u(z))e_k(z)$ where $\sum_{k \in J} \|\hat{f}_k\|_2^2$ is finite. Both of these decompositions hold for general inner functions u and their proofs occupy the bulk of the paper.

A lacunary decomposition in H^p $(1 \le p < \infty)$ associated with an inner function u is a function f in H^p which enjoys a decomposition of the form $f(z) = \sum_{k \ge 0} f_k(z) u^{n_k}(z)$ where the exponents form a lacunary sequence (i.e., $n_{k+1}/n_k \ge d > 1$ for $k \ge 1$ where d > 0 is a constant). For functions f having such decompositions with respect to some finite Blaschke product b, the authors provide several conditions which imply that f is cyclic for $T_{\overline{b}}$. The settings $2 \le p < \infty$ (Section 2) and 1 (Section 3) require somewhat different tools.

Reviewed by Stephan R. Garcia

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