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Review: On Some Bergman Shift Operators

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On some Bergman shift operators. (English summary)

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The authors study an operator identity satisfied by the shift operators $S_n f(z) = zf(z)$ on the standard weighted Bergman spaces $A_n(\mathcal{E})$. To be more specific, they consider operators T in $\mathcal{L}(\mathcal{H})$ which are injective, have closed range, and satisfy

$$(T^*T)^{-1} = \sum_{k=0}^{n-1} (-1)^k \binom{n}{k+1} T^k T^{*k}.$$

They show that the subspace $\mathcal{H}_0 = \bigcap_{k \geq 0} T^k(\mathcal{H})$ is reducing for T and that the restriction $T|_{\mathcal{H}_0}$ is characterized by the property that the operator $(T|_{\mathcal{H}_0})^*$ in $\mathcal{L}(\mathcal{H}_0)$ is an invertible n -isometry in the sense of Agler-Stankus. They also show that an injective operator with closed range satisfying the identity above and which is *pure* in the sense that $\bigcap_{k \geq 0} T^k(\mathcal{H}) = \{0\}$ is unitarily equivalent to S_n acting on $A_n(\mathcal{E})$. The paper concludes with several applications to the study of shift invariant subspaces of $A_n(\mathcal{E})$.

Reviewed by *Stephan R. Garcia*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.