

1-1-2008

Review: Shift-Type Invariant Subspaces of Contractions

Stephan Ramon Garcia
Pomona College

Recommended Citation

MR2321044 (2008d:47016) L'aszl'o K., Shift-type invariant subspaces of contractions, *J. Funct. Anal.* 246 (2007) 281 - 301.
(Reviewer: Stephan R. Garcia)

This Review is brought to you for free and open access by the Pomona Faculty Scholarship at Scholarship @ Claremont. It has been accepted for inclusion in Pomona Faculty Publications and Research by an authorized administrator of Scholarship @ Claremont. For more information, please contact scholarship@cuc.claremont.edu.

MR2321044 (2008d:47016) [47A15](#) ([47A45](#))

Kérchy, László (H-SZEG-B)

Shift-type invariant subspaces of contractions. (English summary)

J. Funct. Anal. **246** (2007), no. 2, 281–301.

Using the Sz.-Nagy-Foias theory of Hilbert space contractions, the author proves a factorization theorem for asymptotically non-vanishing, absolutely continuous contractions. To be more specific, recall that a contraction T on the Hilbert space \mathfrak{H} is an absolutely continuous (a.c.) contraction if the spectral measure of the unitary component of T is a.c. with respect to normalized Lebesgue measure m on the unit circle \mathbb{T} .

Let $n \in \mathbb{N} \cup \{\aleph_0\}$ denote a cardinal number and let \mathfrak{G}_n denote an n -dimensional Hilbert space. The author shows that if the spectral-multiplicity function of the unitary asymptote of the contraction T is at least n on the Borel subset γ of the unit circle \mathbb{T} , then the natural embedding of the Hardy space $H^2(\mathfrak{G}_n)$ into $\chi_\gamma L^2(\mathfrak{G}_n)$ defined by $Jf = \chi_\gamma f$ can be factored as $J = ZY$ where Y intertwines the unilateral shift S_n on $H^2(\mathfrak{G}_n)$ with T , and Z intertwines T with the unitary operator $M_{n,\gamma}$ of multiplication by the independent variable on $\chi_\gamma L^2(\mathfrak{G}_n)$. Furthermore, it is proved that for each $\varepsilon > 0$, Y and Z may be chosen so that $\|Y\| < 1 + \varepsilon$ and $\|Z\| < 1 + \varepsilon$. These norm estimates improve upon previous work of the author [*Acta Sci. Math. (Szeged)* **61** (1995), no. 1-4, 443–476; [MR1377377 \(97b:47007\)](#)] and they require substantial modifications of the original proof.

The paper concludes with several observations relevant to the study of invariant subspaces. In particular, it is proved that if the residual set of a contraction covers the whole unit circle, then those invariant subspaces where the restriction is similar to the unilateral shift with a similarity constant arbitrarily close to 1 span the whole space. Moreover, the hyperinvariant subspace problem for asymptotically non-vanishing contractions is reduced to this special case.

Reviewed by [Stephan R. Garcia](#)

References

1. B. Beauzamy, *Introduction to Operator Theory and Invariant Subspaces*, North-Holland, Amsterdam, 1988. [MR0967989 \(90d:47001\)](#)
2. H. Bercovici, C. Foias, C. Pearcy, *Dual Algebras with Applications to Invariant Subspaces and Dilation Theory*, CBMS Reg. Conf. Ser. Math., vol. 56, Amer. Math. Soc., Providence, RI, 1985. [MR0787041 \(87g:47091\)](#)
3. H. Bercovici, C. Foias, C. Pearcy, On the hyperinvariant subspace problem. IV, *Canadian J. Math.*, in press. [MR2423456 \(2009f:47009\)](#)
4. J.B. Conway, *A Course in Functional Analysis*, Springer-Verlag, New York, 1990. [MR1070713 \(91e:46001\)](#)
5. J. Dixmier, *Von Neumann Algebras*, North-Holland, Amsterdam, 1981. [MR0641217 \(83a:46004\)](#)

6. C. Foias, C. Pearcy, On the hyperinvariant subspace problem, *J. Funct. Anal.* 219 (2005) 134–142. [MR2108362 \(2005m:47010\)](#)
7. C. Foias, S. Hamid, C. Onica, C. Pearcy, On the hyperinvariant subspace problem. III, *J. Funct. Anal.* 222 (2005) 129–142. [MR2129768 \(2007g:47010\)](#)
8. S. Hamid, C. Onica, C. Pearcy, On the hyperinvariant subspace problem. II, *Indiana Univ. Math. J.* 54 (2005) 743–754. [MR2151232 \(2006j:47006\)](#)
9. K. Hoffman, *Banach Spaces of Analytic Functions*, Dover, New York, 1988. [MR1102893 \(92d:46066\)](#)
10. L. Kérchy, A description of invariant subspaces of C_{11} -contractions, *J. Operator Theory* 15 (1986) 327–344. [MR0833215 \(87f:47005\)](#)
11. L. Kérchy, On the spectra of contractions belonging to special classes, *J. Funct. Anal.* 67 (1986) 153–166. [MR0845196 \(87i:47012\)](#)
12. L. Kérchy, On the residual parts of completely non-unitary contractions, *Acta Math. Hungar.* 50 (1987) 127–145. [MR0893253 \(88g:47020\)](#)
13. L. Kérchy, Isometric asymptotes of power bounded operators, *Indiana Univ. Math. J.* 38 (1989) 173–188. [MR0982576 \(90f:47050\)](#)
14. L. Kérchy, Injection of unilateral shifts into contractions with non-vanishing unitary asymptotes, *Acta Sci. Math. (Szeged)* 61 (1995) 443–476. [MR1377377 \(97b:47007\)](#)
15. L. Kérchy, Operators with regular norm-sequences, *Acta Sci. Math. (Szeged)* 63 (1997) 571–605. [MR1480500 \(99i:47011\)](#)
16. L. Kérchy, Hyperinvariant subspaces of operators with non-vanishing orbits, *Proc. Amer. Math. Soc.* 127 (1999) 1363–1370. [MR1600097 \(2000d:47016\)](#)
17. L. Kérchy, On the hyperinvariant subspace problem for asymptotically nonvanishing contractions, in: *Oper. Theory Adv. Appl.*, vol. 127, Birkhäuser, Basel, 2001, pp. 399–422. [MR1902813 \(2003e:47013\)](#)
18. W. Rudin, *Real and Complex Analysis*, McGraw-Hill, New York, 1966. [MR0210528 \(35 #1420\)](#)
19. B. Sz.-Nagy, C. Foias, *Harmonic Analysis of Operators on Hilbert Space*, North-Holland/Akadémiai Kiadó, Amsterdam, Budapest, 1970. [MR0275190 \(43 #947\)](#)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

© Copyright American Mathematical Society 2008, 2013