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Review: Certain Shifts on Banach Spaces of Formal Power Series

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Certain shifts on Banach spaces of formal power series. (English summary)
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Let $p>0$ and let $\{\beta(n)\}$ denote a sequence of positive numbers such that $\beta(0)=1$. The author considers the space of all sequences $f=\{\hat f(n)\}$ such that

$$\|f\|_p^p=\sum_{n=0}^{\infty}|\hat f(n)|^p\beta(n)^p<\infty.$$

The formal notation $f(z)=\sum_{n=0}^\infty \hat f(n)z^n$ is used (regardless of whether the series converges) and the space $H^p(\beta)$ is defined to be

$$H^p(\beta)=\left\{f\colon f(z)=\sum_{n=0}^{\infty}\hat f(n)z^n,\ \|f\|_p<\infty\right\}.$$

The author studies the representation of certain shifts on a Banach space as the operator M_z on $H^p(\beta)$.

Specifically, a bounded linear operator A on a Banach space $\mathfrak{Y}$ is called a shift if: (i) A is injective, (ii) the range of A has codimension 1, (iii) $\bigcap_{i=1}^\infty A^i y=\{0\}$. Under these circumstances, there exists $y_0\in\mathfrak{Y}$ of unit norm such that $\mathfrak{Y}=\langle y_0\rangle\oplus A\mathfrak{Y}$ [R. M. Crownover, Michigan Math. J. **19** (1972), 233–247; MR0361843 (50 #14288)] (here $\langle y_0\rangle$ denotes the one-dimensional subspace spanned by the unit vector y_0). From this, one obtains the so-called Taylor formula for y:

$$y=\hat y(0)y_0+\hat y(1)Ay_0+\cdots+\hat y(n)A^ny_0+A^{n+1}y_{n+1},$$

where $\{\hat y(n)\}_{n=0}^\infty$ is a numerical sequence and $\{y_n\}_{n=0}^\infty$ is a sequence of vectors in $\mathfrak{Y}$. The main results of the paper are:

Theorem 2.1: Let A be a shift on the Banach space $\mathfrak{Y}=\langle y_0\rangle\oplus A\mathfrak{Y}$. Suppose that there exists $1\le p<\infty$ such that for every $y\in\mathfrak{Y}$:

(a) $\|\sum_{i=0}^n\hat y(i)A^iy_0\|^p=\sum_{i=0}^n\|\hat y(i)A^iy_0\|^p$ for all nonnegative integers n, where A^0 is the identity operator, and

(b) $\sum_{n=0}^\infty\|\hat y(n)A^ny_0\|^p<\infty$;

then A is unitarily equivalent to M_z on $H^p(\beta)$ for a suitable choice of β. Moreover, such p is unique.

Theorem 2.3: Two operators $(M_z,H^p(\beta))$ and $(M_z,H^p(\alpha))$ are unitarily equivalent if and only if there exists a constant c such that $\beta(n)=|c|\alpha(n)$, for every nonnegative integer n.

(Note that the author's definition of $H^p(\beta)$ requires that $\beta(0)=1$ and that $\beta(n)>0$ for all nonnegative integers n and likewise for $H^p(\alpha)$.)

Theorem 2.6: The operator $(M_z,H^p(\beta))$ is similar to $(M_z,H^p(\alpha))$ if and only if there exist positive constants C_1 and C_2 such that $0<C_1\le\beta(n)/\alpha(n)\le C_2$ for every nonnegative integer

n.

A corollary (Corollary 2.8) of the preceding theorem asserts that $(M_z, H^p(\beta))$ and $(M_z, H^p(\alpha))$ are similar if and only if they are quasisimilar.

Reviewed by *Stephan R. Garcia*