Journal of Humanistic Mathematics

Volume 5 | Issue 1 January 2015

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Recommended Citation
Amy Shell-Gellasch Dr, "The Schilling Kinematic Models at the Smithsonian," *Journal of Humanistic Mathematics*, Volume 5 Issue 1 (January 2015), pages 167-179. DOI: 10.5642/jhummath.201501.09. Available at: https://scholarship.claremont.edu/jhm/vol5/iss1/9

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The Schilling Kinematic Models at the Smithsonian

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Synopsis

The kinematic models manufactured by the German firm of Martin Schilling were used in the late 19th and early 20th centuries to depict mathematical curves. The Smithsonian Institution owns twelve Schilling models. As a volunteer researcher in mathematics at the Smithsonian National Museum of American History, the author has chosen a few of her favorite models as an introduction to this interesting set of kinematic models.

In 2012, I moved to the Washington DC area. Move number 4 in ten years of marriage, that's what I get for marrying a career Army officer.

Before I even looked for a job, I contacted my colleague Peggy Kidwell, Curator of Mathematics at the Smithsonian's National Museum of American History (NMAH) to see if we could work together on a project. She confessed that she had no funding. I replied that I would be happy to volunteer, as long as I could have a Smithsonian nametag. So that is what I have!

More specifically, I research and prepare text for groups of mathematical objects that are going into the online collections of NMAH. Museums of the 21st century are striving to digitize most of their holdings and make them accessible to people all over the world. NMAH is where all the Smithsonian's mathematical and computing objects are housed because much of the original collection consisted of American inventions and U.S. patent models.

I don't view what I do at the Smithsonian as work so much as play time, but with white gloves on. As an historian of mathematics with a strong bent towards all things mechanical, working with (playing with) these items is such fun! In this article I share with readers one group of objects I have been working with to give them a good feel for what NMAH has to offer for people with mathematical interests.

An Invitation to the Schilling Kinematic Models

My first project (or object group) was the Schilling Kinematic Models. The relevant online exhibit can be accessed through my blog post on the NMAH blog site "Oh Say Can You See" at http://blog.americanhistory.si.edu/osaycanyousee/2014/03/the-spirograph-and-kinematic-models-making-math-touchable-and-pretty.html, posted on Pi Day 2014, or by searching the NMAH collections online. Information on other online object groups at the Smithsonian and how to use them in the classroom can be found in [1].

In the late 19$^{\text{th}}$ and early 20$^{\text{th}}$ centuries, physical models were widely used by mathematics educators to depict a wide range of mathematical ideas in three dimensions. Most models were constructed in Europe, in particular Germany, for use in schools and colleges and became popular in the United States. They were used to show mathematical surfaces and curves as well as other mathematical concepts useful to mathematicians, engineers, and scientists. Depending on their use, the models could be static or articulated. As more and more American institutions added mathematical models to their collections, American firms grew up to supply this growing demand. By World War I, interest in these physical models waned, and most were relegated to closets and basements. But with the advent of computer graphics systems, the interest in physical mathematical models is returning.

The vast majority of mathematical models made around the turn of the twentieth century were static and made of wood, paper, or plaster. The paper models of American mathematician A. Harold Wheeler are a prime example. A common static model still used in many high schools is the wooden or plaster cone that shows how the conic sections (circle, ellipse, parabola, hyperbola) arise from slicing a cone at different angles to its axis. See Figure 1 on the next page for an example.

One famous group of articulated models are the Olivier String Models, see [11]. Less common articulated models are kinematic models. These devices depict mathematical concepts that involve motion. The models can be manipulated through the use of cranks and hinges and were usually constructed of metal, most often brass. Interested readers may view the whole NMAH collection of kinematic models at http://americanhistory.si.edu/collections/object-groups/kinematic-models.

Figure 1: Conic Section Model, ca. turn of 20th century. Shows an ellipse, parabola and hyperbola. Cutting the cone parallel to the base would produce the fourth conic section, a circle. Smithsonian object 1979.3002.021.

One of the most prominent producers (known as publishers) of models for commercial use was the firm of Ludwig Brill of Darmstadt, Germany. In 1899 Martin Schilling took over and expanded the business from Brill. Originally located in Halle, Germany, the firm moved to Leipzig sometime after 1903. The firm produced numerous types of mathematical models, including twelve kinematic models of which the Smithsonian presently has ten. The kinematic models were designed by German mathematician Frederick Schilling (1868–1950), professor of mathematics at Gttingen, who became the scientific director of the company. I have yet to determine if these two Schillings were related.

The kinematic models are listed in the firm's 1903 and 1911 *Catalog mathematischer Modelle*. This catalog lists 377 items divided into forty series or types of models. Series XXIV consists of Kinematic models (Kinematische Modelle) and is divided into 4 groups:

- Group 1 (models No 1-4): Trochiods (Epitrochoid and Hypotrochoid),
- Group 2 (models No 5-7); Cyclic curves (including cycloids),
- Group 3 (models No 8, 9): Twin cranks,

- Group 4 (models No 10-12): Inversors by Peaucellier, Hart, and Sylverster and Kempe.

Though Schilling categorized the models into the four groups, it is easier to think of them in two groups: linkages and trochiods. I will showcase one or two of my favorite models from each group.

Linkages

The most widely used kinematic model of the time was the linkage. A linkage is a device made of hinged armatures linked at pivot points to allow the whole assembly to move and deform in order to transform one type of motion into another. The Schilling models of linkages fall roughly into two subgroups: bar linkages and gear linkages.

Inversors:

The most common and useful linkages are those that transform circular movement into linear movement, known as *inversors*. Circular motion is easy to produce using any sort of wheel device. However, true linear or straight line motion is not. To create linear motion, the circular motion of a crankshaft needs to be transformed into straight line motion. A common example of the conversion between linear and rotary motion is found in internal combustion engines. The piston's linear (up and down) motion is converted into the rotary motion of the crank shaft. A simpler linkage to visualize is the wheels of a steam locomotive, the rods move back and forth horizontally pushing the wheels to rotate. And the simplest linkage of all would be a set of fireplace tongs.

The Smithsonian has three bar linkages: The *Peaucellier Inversor*, the *Hart's Inversor* and the *Sylvester-Kempe Inversor*. The Peaucellier Inversor is one of the most common types of linkages and is the easiest to envision. See Figure 7. In 1864 French engineer Charles Nicolas Peaucellier (1832-1913) created a seven-bar linkage which succeeded in producing pure linear motion. His discovery was the first solution of what was referred to as the problem of parallel motion: converting rotational to linear motion using only "rods, joints and pins". The linkage was first exhibited by Lipkin in 1873 at the International Exhibition of Vienna [4]. Since then, such seven-bar linkages are often referred to as Peaucellier cells or a Peaucellier's inversor.

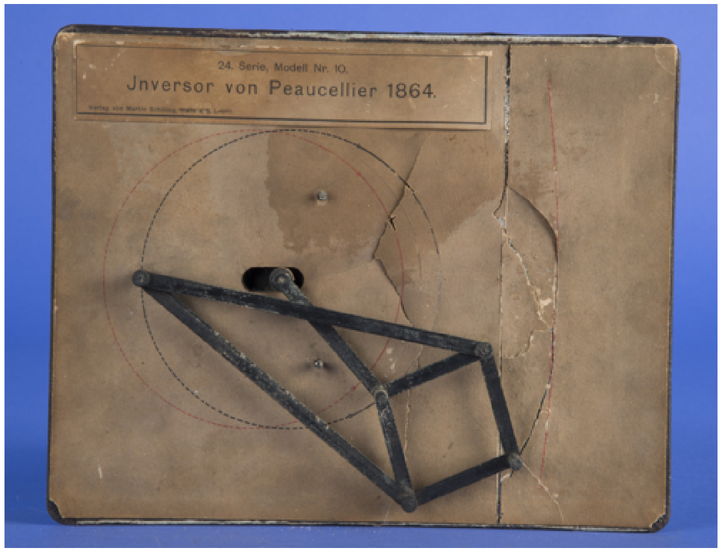

Figure 2: Schilling Model Number 10, Peaucellier Inversor, SI image DOR2013-50203.

In the model above, the tail of the kite (far left vertex of the linkage) is fixed at the side of the circle. The bar from the center of the linkage to the center of the circle is attached to a crank underneath the base plate. As this crank is turned, the central point of the linkage follows the black circle as indicated in the image. (Only the right semicircle is able to be traced.) This causes the top (far right vertex) of the kite to trace back and forth along the vertical line on the far right of the model. (In the model, the tracing point has worn through the covering of the base plate.) This is more easily seen in the diagram in Figure 3 on the next page.

Point C in the figure is fixed on the far end of the diameter of the circle that is perpendicular to line L. As point A rotates around the circle, the rhombus (diamond) at the top of the kite shape is compressed horizontally as A approaches the top of the circle and then stretched vertically as it approaches the side of the circle. This motion in turn keeps the point B on the line L [7]. This motion is oscillatory and will produce a line segment. Adjusting the relative lengths in the linkage will result in segments of various lengths based on the result desired.[1]

[1]The mathematical theory behind all of the linkages can be found in [4, 6, 7, 8, 9] and is readily available on the web now.

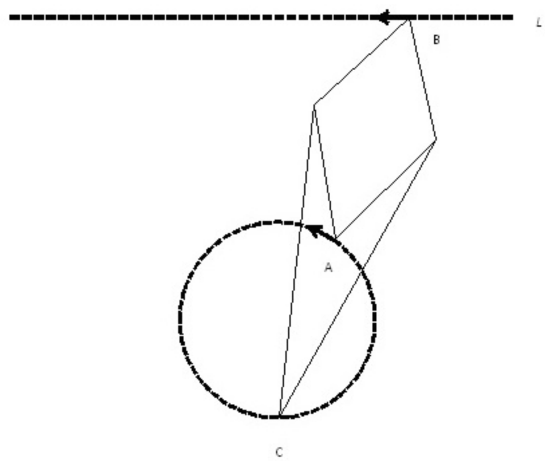

Figure 3: Diagram of the 7-bar linkage.

Gear Linkages:

The Smithsonian owns two linkages that can be thought of as gear linkages: The *Twin Hyperbolic Gear* (Figure 8) and the *Twin Elliptic Gear* (Figure 6).

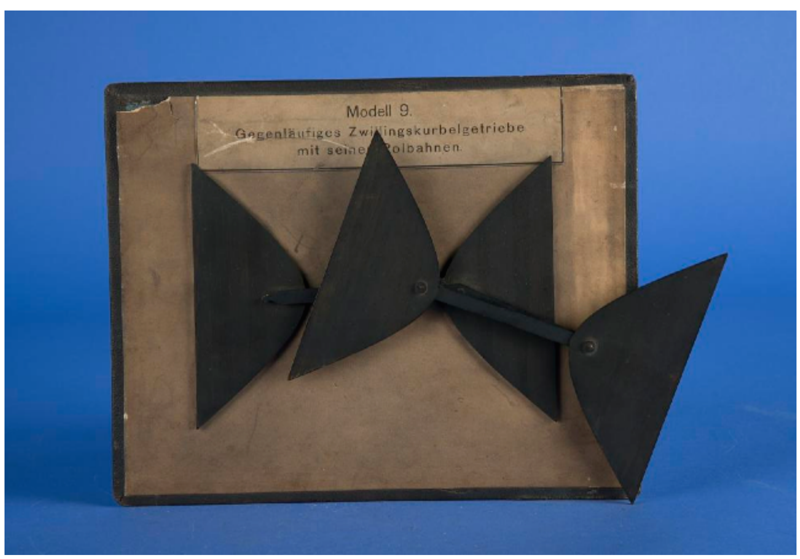

Figure 4: Schilling model Number 9, Twin Hyperbolic Gears, SI image DOR2013-50206.

The Twin Hyperbolic Gears is an example of a Watt's linkage. In 1784, Scottish engineer James Watt (1736–1819), developed a system of rods and pins that allowed the pistons in his steam engines to exert force on both the downward and upward stroke of the piston (as opposed to in just one direction as in previous designs). Producing straight line motion was an important component of many machines. But producing true linear motion is very difficult and one area of research during the 19th century was to use linkages to produce linear motion from circular motion. A Watt's linkage is a three-bar linkage in which two bars of equal length rotate to produce congruent circles. The ends of these two radii are joined by a longer crossbar. As the radii counter-rotate, the midpoint of the crossbar traces out a Watt's Curve, a curve related to the lemniscate (see Figure 5). As the midpoint of the crossbar traces the region of the lemniscate where the curve crosses itself, the motion is approximately linear. In the Schilling model shown (No. 9), the curve is generated by the far right vertex of the hyperbolic curve on the top "bowtie" as it follows a figure-eight path about the other "bowtie".

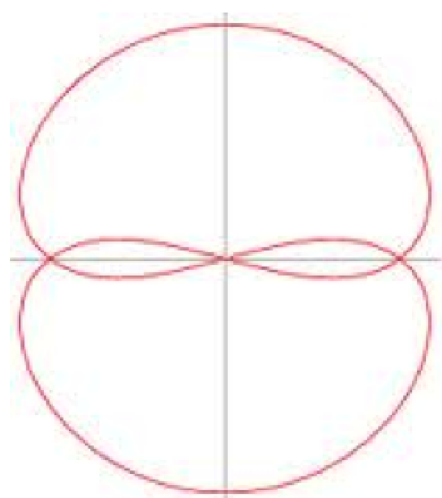

Figure 5: Watt's Curve. Image from [12].

The other gear style linkage in the collection is the Twin Elliptical Gears. Many machines need to produce a back and forth motion, such as the back and forth motion of the rods of a locomotive that drive the wheels. This back and forth motion is achieved by converting circular motion (produced by the pistons of the steam engine) to linear motion (of the rods). One way of

achieving this in a smooth way is through a quick return mechanism. In this model, two ellipses rotate about each other while held in constant contact producing an "elliptical gear" (No. 8).

The rod in the image keeps the fixed focus of one ellipse and the free focus of the other at a constant distance. The brass knob on the rod rotates one ellipse around the other and traces out a barely visible circle on the paper overlay. Rotating the knob at a constant speed causes the ellipse to rotate at a variable speed. The speed of rotation increase as the ellipses move towards a side-by-side orientation, and slows as the ellipses move towards an end-to-end alignment. Thus the velocity increases and decreases periodically. The velocity ratio of the rotating gear is the portion of the length of the top arm over one ellipse divided by the remaining length (over the other ellipse). Mathematically this velocity ratio varies from $e/(1-e)$ to $(1-e)/e$ where e is the eccentricity of the (congruent) ellipses [7]. The cyclic nature of the velocity of this motion explains why it is known as a "quick-return" mechanism, which converts rotational motion into reciprocating or oscillating motion.

Figure 6: Schilling Model No. 8, Twin Elliptical Gears, SI image DOR2013-50208.

Curve Drawing

The second general category of kinematic models produced by Schilling are those models that generate curves associated with circles, in particular, those that generate involutes and those the produce trochoids.

An involute of a circle is a curve that is produced by tracing the end of a string that is wrapped around a circle as it is unwound while being kept taut. It is the envelope of all points that are perpendicular to the tangents of a circle. In the model in Figure 7 below, a toothed circular gear is mounted on the base plate and can be turned via a crank on the underside of the baseplate. This forces the dark metal toothed bar past the circular gear while rotating around it. Perpendicular to the bar is a thin clip with three small colored balls. A blue ball is attached at the edge of the bar where the bar touches the circle and traces the involute of the circle in blue on the glass. A green ball is behind the toothed side of the bar and traces a "stretched" involute in green. A red ball is placed in front of the toothed side of the bar and produces an even more "stretched" involute in red.

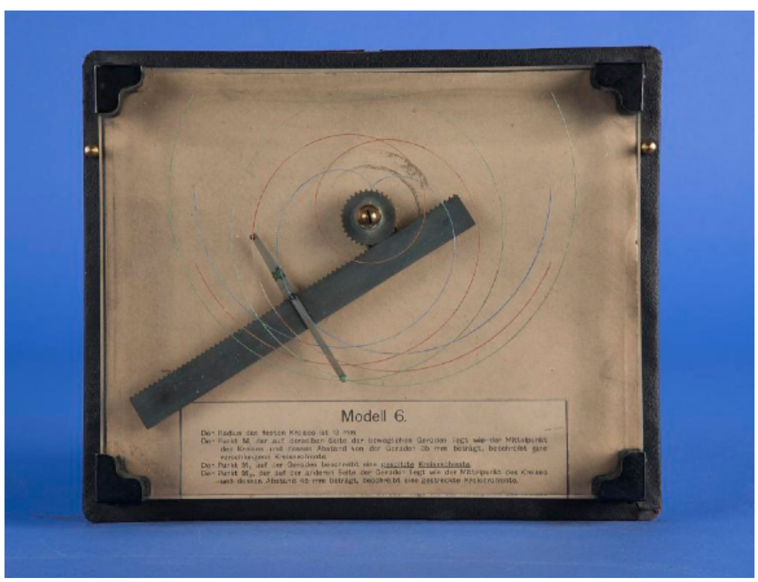

Figure 7: Schilling Model Number 6, Involutes of Circles, SI image DOR2013-50220.

Though the involute has been studied for several centuries, starting in the 17th century, one of the most studied types of curves has been the trochoids.

These are curves generated by tracing the motion of a point on the radius of a circle as it rolls along another curve. Included in the family of trochoids are the cycloids (a circle rolling along a straight line), epitrochoids (a circle rolling around the outside of another circle) and the hypotrochoids (a circle rolling around the inside of another circle). To give a concrete example that you may have experienced, the popular later 20$^{\text{th}}$ century toy the Spirograph produces various trochiods.

These types of curves have applications in engineering with respect to the workings of gears and motors, as well as answer interesting questions in mathematics and physics. In particular, it was found in the eighteenth century that gear teeth shaped using cycloids as well as involute curves reduce friction and torque, allowing gears to rotate more efficiently. As a side note, surprisingly there are many applications of non-circular gears, such as elliptical, triangular, and quadrilateral gears; see for instance [2, page 160], [3, page 9], or [5, page 69].

Of the four trochoidal models in the Smithsonian's collection, my favorite is the model entitled *Hypotrochoids* (Figure 8).

Figure 8: Schilling Model Number 3, Hypotrochoids, SI image DOR2013-50214.

The name hypotrochoid comes from the Greek word *hypo*, which means under, and the Latin word *trochus*, which means hoop. Thus *hypotrochoids* are curves formed by tracing a point on the radius or extension of the radius of a circle rolling around the inside of another stationary circle. An infinite number of hypotrochoids can be formed, depending on the distance of the tracing point from the center of the rolling circle. Hypotrochoids, for which the tracing point is on the extension of the radius, form curves that resemble petalled flowers and are called roses.

The model in Figure 8 consists of a stationary toothed metal ring (with teeth on the inner edge of the ring). A toothed metal disc is attached to a brass arm which can be rotated by turning a crank below the base plate. As the arm is rotated, the disc rolls around the inside of the ring. Three points lie along the radius of the disk and trace corresponding curves, or roulettes, on the glass overlay. The blue point on the circumference of the disc traces a blue five-pointed star shape referred to as a hypocycloid. The green point on the radius of the disc traces a green curve inside the ring known as a curtate hypotrochiod, and the red point on the extension of the radius of the disc traces a curve that extends past the radius of the ring and produces a prolate hypotrochoid.

Conclusion

All the Schilling models housed at the Smithsonian were sold around the turn of the last century in order to train the growing number of engineers and machinists needed as the industrial age gained momentum. Now we simply command Maple or Mathematica to generate a given curve or pull a desired curve off a drop-down menu in a drafting package. However, there is something special about physically producing these curves using one of these beautifully crafted, high precision devices. Modern technology has given us so much and let us create things unimaginable just a few years ago. But, as a math historian, I wonder if something is also being lost when we no longer touch our mathematics. Mathematics is a wholly humane endeavor, and as humans, we love to physically explore our world. Even worlds we created in our minds, like mathematics. And I am honored to be able to work with these items that so clearly bring to the fore the tangible dimensions of mathematics.

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