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I met Paul Erdös shortly after his 40th birthday in April 1953 at Purdue University in West Lafayette, Indiana. Hewas already a living legend because of his substantial contributions to the theory of numbers, the theory of sets, what is now called discrete mathematics, as well as to many other areas of mathematics. (For example, although he had little interest in topology, his name appears in most topology texts as the first person to give an example of totally disconnected topological space that is not zero-dimensional.) I was a 26-year old instructor in my first year at Purdue. Many of my colleagues knew him well. He had been a visiting research associate at Purdue for a couple of years during World War II, and had visited so many universities and attended so many conferences that he was well known to most of the others. Those that were active in research admired his mathematical accomplishments, while others on the faculty were amused by his eccentricities. What I remember most clearly is his announcement to everyone that "death begins at 40".

I am not qualified to write a biography of Erdös, but some background seems necessary. There is an excellently written and accurate obituary of him by Gina Kolata in the Sept. 21, 1996 issue of the New York Times, beginning on page 1. An interview conducted in 1979 which reveals much of his personality appeared in the volume *Mathematical People* edited by D.J. Albers and G.L. Alexanderson (Birkhauser 1985). The Mathematical Association of America (MAA) sells two videos of Erdös, and Ronald Graham, a long time collaborator, has edited together with Jarik Nesetril two volumes on his mathematical work and life. (Both volumes have been published by Springer-Verlag and were available in January 1997. They include a detailed biographical article by Bella Bollobas.)

Erdös was born in Budapest in 1913 of parents who were Jewish intellectuals. His brilliance was evident by the time he was three years old. For this reason, and perhaps because two older sisters died of scarlet fever shortly before he was born, his parents shielded him almost completely from the everyday problems of life. For example, he never had to tie his own shoelaces until he was 14 years old, and never buttered his own toast until he was 21 years old in Cambridge, England. In return for the freedom to concentrate almost exclusively on intellectual pursuits, he paid the price of not learning the social skills that are expected of all of us and usually acquired in childhood.

He became internationally famous at the age of 20 when he got a *simple* proof of a theorem that was originally conjectured by Bertrand and later proved by Tchebychev: For every positive integer n, there is a prime between n and $2n$. Tchebychev's proof was quite hard! Erdös completed the requirements for the Ph.D. at the University of Budapest about a year later, but had no chance of getting a position in Hungary because he was a Jew living under a right wing dictatorship allied with Nazi Germany. He spent some time at Cambridge University in 1935. There, his life as a wandering mathematician began. In fact, he had visited Cambridge three times the year before. He liked traveling and had no trouble working while doing so. He liked people, and except for those who could not tolerate his ignorance of the social graces, they liked him. He tried his best to be pleasant to everyone and was generous in giving credit and respect to his collaborators.

I do not know when he first came to the United States, but he spent the years of World War II here, two of them at Purdue. Nor can I give a list of the many universities he visited for any substantial length of time. By the time I met him, he had written joint papers with many mathematicians most of whom had established research reputations before working with Erdös. The only Erdös collaborator who worked with him unwillingly was Atle Selberg. In the late 1940s, both of them, working independently, had obtained

"elementary" proofs (meaning: proofs that did not use complex analysis) of the prime number theorem. The theorem states that the number of primes less than or equal to (a positive real number) x is asymptotically equal to $x/\log(x)$. This had been conjectured by Gauss and Legendre based on empirical data, but it had only been proved many years later, by two French mathematicians, Jacques Hadamard and Charles de la Vallée Poussin (also working independently). Both proofs depended heavily on complex analysis. What Selberg and Erdös did in their "elementary" proofs was to avoid using complex analysis (the proofs were in no sense "easy"). In those pre-email days, the fastest courier of mathematical news was Paul Erdös. He told anyone who would listen that Selberg and he had devised an elementary proof of the prime number theorem.

Almost every number theorist knew of Erdös, while few had heard of the young Norwegian Selberg. So when the news traveled back to Selberg, it appeared that Erdös had claimed all the credit for himself. The ensuing bitterness was not healed by the two of them writing a joint paper. Selberg later published another elementary proof on his own,and went on to a brilliant mathematical career, eventually becoming a permanent member of the Institute for Advanced Study in Princeton, the Valhalla for mathematicians. Erdös had been a visitor there earlier, but was not offered a membership. Exactly what happened is controversial to this day, and reading the article by Bollobas will shed more light on this matter than this short summary can.

Erdös spent the academic year 1953-54 at the University of Notre Dame in South Bend, Indiana. Arnold Ross, the chairman of the Mathematics Department, had arranged for him to teach only one (advanced) course, and supplied an assistant who could take over his class if he had the urge to travel to talk with a collaborator. Erdös had rejected organized religion as a young man, and had been persecuted in Roman Catholic Hungary. So we teased him about working at a Catholic institution. He said in all seriousness that he liked being there very much, and especially enjoyed discussions with the Dominicans. "The only thing that bothers me," he said, "there are too many plus signs." He came by bus to West Lafayette fairly often for short periods because he had so many friends there and because he liked the mathematical atmosphere.

At that time, Leonard Gillman and I were trying to study the structure of the residue class fields of rings of real-valued continuous functions on a topological space modulo maximal ideals. We had learned quite a bit about them, but had run into serious set-theoretic difficulties. Erdös had little interest in abstract algebra or topology, but was a master of set-theoretic constructions. Without bothering him with our motivation for asking them, we asked him a series of questions about set theory, which he managed to answer while we could not.

He was not terribly interested when we supplied him with the motivation, and I have often said that Erdös never understood our paper; all he did was the hard part. This paper by Erdös, Gillman and Henriksen was published in the *Annals of Mathematics* in 1955. Without any of us realizing it in advance, it became one of the pioneering papers in nonstandard analysis, and was often credited to Erdös, et al.
Erdös got an offer allowing him to stay indefinitely at Notre Dame on the same generous basis. His friends urged him to accept. "Paul", we said "how much longer can you keep up a life of being a traveling mathematician?" (Little did we suspect that the answer was going to turn out to be "more than 40 years.") Erdös thanked Ross, but turned him down. As it turned out, he would not have been at Notre Dame the next year whatever his answer had been.

The cold war was in full swing, the United States was in the grip of paranoia about communism, and many regarded unconventional behavior as evidence of disloyalty. Erdös had never applied for citizenship anywhere he lived, and had acquired Hungarian citizenship only by accident of birth. He belonged to no political party, but had a fierce belief in the freedom of individuals as long as they did no harm to anyone else. All countries who failed to follow this were classified as imperialist and given a name that began with a small letter. For example, the U.S. was **samland** and the Soviet Union was **joedom** (after Joseph Stalin). He talked of an organization called the f.b.u—a combination of the F.B.I and O.G.P.U (which later became the K.G.B) and conjectured that their agents were often interchanged.

In 1954, Erdös wanted to go the International Con-

gress of Mathematicians (held every four years), which was to be in Amsterdam that August. As a non-citizen leaving the U.S. with plans to return, he had to apply for a re-entry permit. After being interviewed by an INS agent in South Bend in early 1954, he received a letter saying that re-entry would be denied if he left the U.S. He hired a lawyer and appealed only to be turned down again. No reason was ever given, but his lawyer was permitted to examine a portion of Erdös' file and found recorded the following facts:

- He corresponded with a Chinese number theorist named Hua who had left his position at the University of Illinois to return to (red) China in 1949. (A typical Erdös letter would have begun: Dear Hua, Let p be an odd prime...)

- He had blundered onto a radar installation on Long island in 1942 while discussing mathematics with two other non-citizens.

- His mother worked for the Hungarian Academy of Sciences, and had had to join the communist party to hold her position.

To Erdös, being denied the right to travel was like being denied the right to breathe, so he went to Amsterdam anyway. He was confident that he could easily obtain a Dutch and an English visa. The Dutch gave him a visa good for only a few months, and England would not let him come, likely because if they chose to deport him, the only country obligated to accept him was communist Hungary. By then, Erdös was a member of the Hungarian Academy of Sciences, but he would go to Hungary only if his friends could assure him that he would be permitted to leave. At this point, he swallowed his pride and obtained a passport from israel (note the punctuation) which served to give him freedom to travel anywhere in western Europe. He was permitted to return to the United States in the summer of 1959 on a temporary visa to attend a month long conference on number theory in Boulder, Colorado. He stopped at Purdue on his way back to Europe to give a colloquium talk. When I picked him up at the airport, what struck me first was that he had a suitcase! For many years, he traveled only with a small leather briefcase containing a change of socks and underwear in addition to a wash-and-wear shirt, together with some paper and a few reprints. About a year later, the United States government lost its fear of Erdös and gave him resident alien status once more. He never had trouble going in or out of the U.S. again. Erdös had lived from hand to mouth most of the time until the late 1950s. When the Russians sent Sputnik into orbit and the space race began, there was a vast increase in government support of research. This made it possible for his many friends and co-authors to give him research stipends. This had little effect on his lifestyle. His suitcase was rarely more than half full, and he gave away most of his money to help talented young mathematicians or to offer cash prizes for solving research problems of varying degrees of difficulty. (The cash prizes were not as costly as he had expected. The winners would often frame his checks without cashing them. Solving a $1000 problem would make you internationally famous, and being able to say that you solved any of his prize problems enhanced your reputation.) Around 1965, Casper Goffman concocted the idea of an Erdös number. If you had written a joint paper with him, your Erdös number was 1. If you had written a joint paper with someone with Erdös number 1, your Erdös number is 2, and so on inductively. There is now an Erdös Number Project home page on the web where you can see a list of all who have an Edos number of 1 (there are 462 of us) and 2 (all 4566 of them, including Albert Einstein). All in all, Erdös wrote about 1500 research papers, and 50 or so more will appear after his death.

While we did no more joint research, we often met at conferences or when we were both visiting the same university. Sometimes I could hardly talk to him because he was surrounded by mathematicians eager to ask him questions, but when I could, he inquired about mutual friends and asked about follow-up work on our paper and progress about solving the open problems we had posed. While he devoted his life to mathematics, he was widely read in many areas and I almost always learned a great deal talking to him about many non-mathematical ideas. I saw him last in Budapest last Sept. 4. He attended the first half of a talk I gave about separate vs. joint continuity. He apologized in advance about having to leave early because he had made an appointment he could not break before he knew I would be speaking. Even then, he made two helpful comments while present. Before I left the Academy of Sciences, I stopped to say goodbye and saw him going over a paper with a young Hungarian mathematician. He died in Warsaw of a

heart attack on Sept. 20. He worked on what he loved to do to the last!

Erdös had a special vocabulary that he concocted and used consistently in his speech. Some samples are:

- **Children** are **Epsilons**
- **Women** are **Bosses**
- **Men** are **Slaves**
- **Married Men** have been **Captured**
- **Alcoholic Drinks** are **Poison**
- **God** is **The Supreme Fascist** or **SF**
- **Music** is **Noise**.

Examples:

I asked Barbara Piranian (President of the League of Women Voters in Ann Arbor, Michigan in the early 1950s) "When will you bosses take the vote away from the slaves?" Answer :"There is no need; we tell them how to vote anyway."

"Wine, women, and song" becomes "Poison, bosses, and noise".

Erdös said that the SF had a Book containing elegant proofs of all the important theorems, and when a mathematician worked very hard, the SF could be distracted long enough to allow her or him to take a brief peek. Particularly elegant proofs were described as fit to be placed in the Book.

There are many Erdös stories that were embellished over the years and made more delightful than the truth. For example, consider the story about blundering into a radar installation in 1942:

- **Embellished version:** Erdös, Hochschild (a German) and Kakutani (a Japanese) drove a car out onto Long Island and held an animated mathematical conversation in German. They walked onto a radar installation and were apprehended by a guard who was convinced that he had caught a group of foreign spies. They were questioned closely by military intelligence and released with a warning when they promised never to do such a thing again.

- **Actual version:** The car was driven by Arthur Stone (an Englishman). Hochschild was supposed to come,but did not because he had a date. They were speaking English because it was their only western language understood by Kakutani. The guard was satisfied as soon as they presented proper identification, and they were visited individually and briefly a few days later by military intelligence agents.

Erdös liked to tell many stories about himself. In particular, when he grew older, he claimed to be two billion years old because when he was in high school, he was taught that the earth was two and a half billion years old—but now we know it is four and a half billion years old.

Because he seemed to be in a state of Brownian motion, it was often hard to locate him at any given time. Erdös visited Claremont twice in the 1970s and could often be found at UCLA. For many years the way to contact him was to call Ron Graham of Bell Labs on the east coast, Paul Bateman of the University of Illinois, or Ernst Strauss at UCLA to find out where he was. Strauss died in 1983 and was replaced by Bruce Rothschild. Paul Bateman retired. Although Ron Graham himself traveled a great deal, until the end he was the person most likely to know of Erdös' whereabouts.

With Erdös' death we have lost one of the great mathematicians and free spirits of this century and it is hard to imagine that we will see anyone like him again. I feel fortunate to have had the privilege of knowing and working with him.