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Illumination and Geometry in Islamic Art

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A form of art was created in the Islamic world between the 10th and the 13th centuries that integrated rhythmic geometric patterns, calligraphy, and illumination. The dynamic geometric forms, infused with light, were created by artists working in collaboration with mathematicians. All Islamic monuments, spreading from Egypt to Spain, were decorated with these patterns. The mathematical basis of infinitely repeating geometric patterns includes the use of symmetry transformations like rotations, reflections, translations and glide reflections. Tessellations were created from these geometric transformations and an infinite variety of patterns resulted from the use of such elegant and simple mathematical laws. Oleg Grabar, a scholar of Islamic Art, describes this art as follows:

> After the 10th century a second type of ornament appears alongside the earlier one, emphasizing polygons and stars. It makes geometric pattern almost the only pattern of decoration.... From evidence which is only now being discovered, it seems that this art was made possible by a conscious attempt on the part of professional mathematicians to explain and to guide the work of the artisans.[1]

The Alhambra is a walled fortress and one of the masterpieces of Islamic architecture built during the 13th century in the city of Granada in Spain. Scholars of Islamic art have referred to the Alhambra as a "geometer's odyssey" due to the rich variety of illuminated geometric designs that decorate the walls and ceilings of the building.[2] Islamic art is for the most part abstract and geometric; illuminated geometry became an important feature of Islamic art and architecture and resulted in an art form that has been characterized as "transcendental."[3] Techniques of illumination evolved over time and included the use of special colors embedded into the building material, luster painting, marble inlays and the use of colored and stained glass. In some architecture no medium was used except abstract geometric forms that allowed the penetration of sunlight. The stars and polygons dispersed the incoming light into geometrical shapes and projected the illuminated forms into the space enclosed by the building. The rays of the sun created an interplay of light and geometry. This subtle design feature allowed one to experience a sublime interior light within the enclosures of sacred buildings. Such buildings as the Alhambra were made radiant with light in this manner.[4] Figure 1 is a scene from the interior of the tomb of Itimad al-Dawa built in 1628 in Agra, India.[5] It illustrates the use of illuminated geometric patterns and the interplay of light and geometry in Islamic art.

Exploring visual images of geometric art from the Islamic world is now possible through the Internet. The Rotch Visual Collections at the MIT library (**http://nimrod.mit.edu/rvc**) has a World Wide Web page on Islamic art and architecture subtitled "The Aga Khan Visual Archives" which contains a selection of 167 images representing historic and contemporary art and architecture from 26 countries. The images can be indexed by geographic location and architectural components. A tour through this archive provides an extensive visual experience of various art forms developed in the Islamic world. Another impressive web site is Jan Abas' Islamic Patterns page (**http://www.bangor.ac.uk/~mas009/islampat.htm**) which contains a mathematical treatment of the subject.

The Dutch artist M.C. Escher studied Islamic art when he visited the Alhambra palace in 1935. He declared that the Moorish majolica mosaics in the palace made a profound impression on him and realized that the Moors had used exclusively abstract mathematical motifs for their decorations.[6] Escher then decided to study the mathematical laws that formed the basis of these patterns. It was pointed out to him by the mathematical community that the regular division of the plane into mathematical congruent figures is part of the study of geometric crystallography including symmetry transformations and the theory of plane symmetry groups. He began to do his artistic compositions based on these mathematical laws. What the Moorish artists had done with abstract geometric figures, Escher began to do with figures from nature including bird, animal, human and architectural motifs. In his book *Escher on Escher: Exploring the Infinite*, Escher declares that the Moorish artists as well as the mathematicians, and in particular the crystallographers, have had a considerable influence on his work of the last twenty years. With this new style Escher tried to "approach infinity as purely and as closely as possible" and expressed by his compositions "the idea of endlessness, represented on a piece of paper: where our eyes reach the border of the print, the imagination must assume the task of the eye."[6]

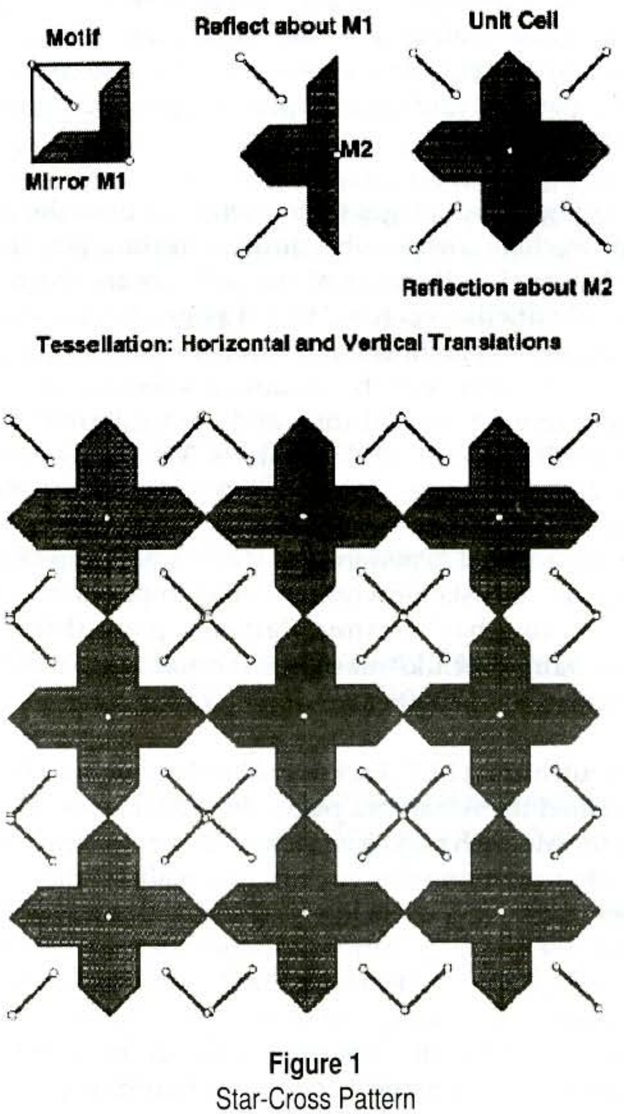

Figure 1
Star-Cross Pattern

The use of two dimensional transformation geometry in creating infinitely repeating geometric patterns was most pronounced in Islamic art. Its application forms the basis for the creation of rhythmic crystallographic patterns. This feature of Islamic art makes it a valuable tool for teaching mathematical topics such as symmetry, transformation and tessellation. It also allows the possibility of learning and integrating multiple disciplines including mathematics, art, computer graphics and Islamic culture in a secondary school curriculum.

The mathematical process involved in creating these patterns can be realized by applying symmetry transformations on a two dimensional motif that is sometimes referred to as the generating motif for the pattern. Symmetry transformations include rotations, reflections, glide reflections and translations together with their compositions. Doris Schattschneider introduces the idea of a plane symmetry group into which infinitely repeating geometric patterns can be classified.[7] A plane symmetry group is a group of symmetry transformations including rotations, reflections, glide reflections, translations and their compositions. Mathematically it has been proven that such patterns can be classified into seventeen distinct plane symmetry groups. All seventeen pattern types were utilized by the Moors in their decoration of the Alhambra palace although there is no documented evidence that the artists and mathematicians during that time were actually familiar with the theory of symmetry groups.[2]

The practical steps involved in creating these patterns include: drawing a 2-dimensional generating motif, applying a symmetry transformation or a combination of symmetry transformations to create a unit cell, and then applying further symmetry transformations like translations in the plane to create a tessellation. All these operations can be performed by a compass and straightedge. In recent years the advances in computer technology and geometry software have made possible the implementation of symmetry transformations and tessellations in fascinating ways. One such geometry program is *The Geometer's Sketchpad*.[9] It allows the exploration of ideas in transformation geometry interactively and with ease. The program allows the student to apply the concept of transformation for creating tessellations and to learn the corresponding mathematical terminology in the process. Since the program performs the necessary repetitive

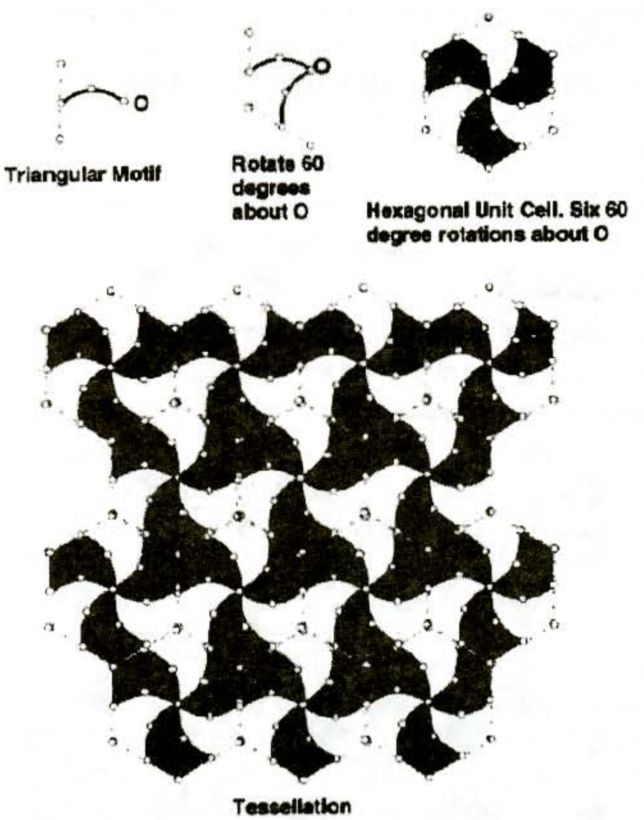

Figure 2
Andalusian Pattern

steps involved in creating the unit cells and tessellations the students are able to explore more geometrically complex tessellations and investigate the necessary mathematical ideas that form the basis of Islamic geometric art and the art of Escher. The following illustrations describe some creations that are possible with *Sketchpad*.

The Star-Cross Pattern:

The star cross pattern is a frequently used pattern in Islamic art. It is found in many areas of the Middle East as well as in Spain. The pattern exhibits four-fold symmetry with four-fold rotations and reflections. The generating motif is a simple polygon and the unit cell is a square. The tessellation is obtained by translating the unit cell horizontally and vertically. The stars are often decorated with poetry and verses from the *Quran* and illuminated with luster paint. Islamic artists often integrated symbols like the cross from other religious traditions into their art. The technique of creating these patterns with *Sketchpad* is illustrated in Figure 1.

The Andalusian Pattern

This pattern is commonly found in Islamic monuments in Spain. It displays hexagonal symmetry and the generating motif is composed of a single arc in a triangular shape. The unit cell is hexagonal and is formed by performing a six-fold rotation of the generating motif. The tessellation is obtained by translating the unit cell horizontally and vertically or by rotation and translations. The technique of creating this pattern with the program is illustrated in Figure 2.

The Star-Hexagon Pattern

This is another repeating pattern that uses stars. The star is a popular design element in Islamic art and plays an important role in illuminating the art. The stars signify heaven; many artists used stars to decorate the ceilings and domes of Islamic monuments. In this way they created a vision of heaven through their art. The star hexagon pattern has a six-fold double hexagonal symmetry with rotations and reflections performed on a simple generating motif. The steps involved in creating this pattern with the program are illustrated in Figure 3.

Sketchpad also allows one to decorate these patterns with the use of color. Most Islamic patterns are filled

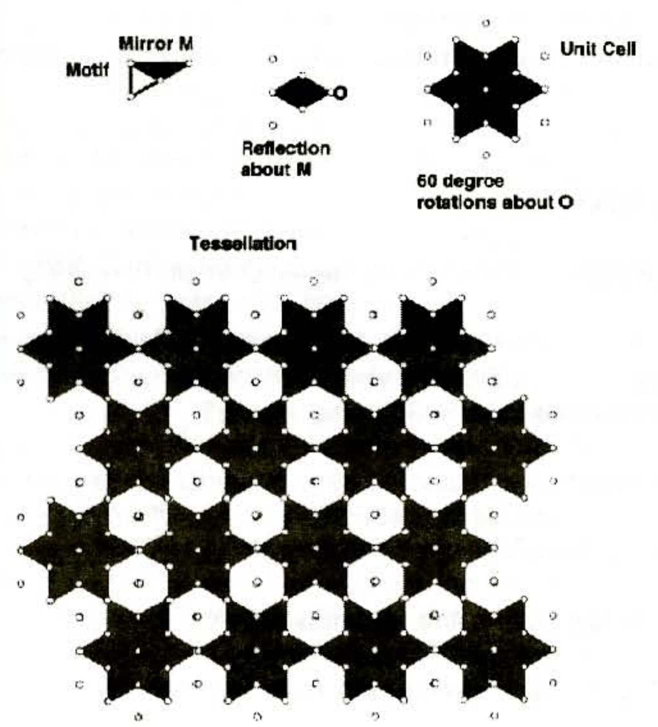

Figure 3
Star Hexagon Pattern

with brilliant colors dominated by blue and gold. Blue is a symbol of the infinite and gold symbolizes the glory of the creator. The patterns were often glazed with turquoise and cobalt, a common technique of illumination, and decorated with divine inscriptions and poetry that encompassed divine compassion.

Variations of Patterns and Original Designs

Simple variations of the above patterns can be implemented easily by changing the original motif and creating the unit cell and tessellation with this new motif. For example, the Andalusian pattern can be given a different expression by changing its motif. The resulting pattern is a tonal version of the pattern and shows how altering the generating motif and applying the same symmetry transformations gives a different expression to the pattern. The result is shown in Figure 4.

New designs can also be created by generating an original motif to create the tessellations. This process of creating unit cells and tessellations allows the student to explore ideas in transformation geometry with relative ease and since the results are interactively and visually generated a deeper understanding of the mathematical concepts becomes possible. Learning about transformations is an important part of mathematical understanding since transformations connect ideas from geometry and algebra as well as probability theory and statistics. This approach also allows the possibility of learning multiple disciplines including mathematics, art, computer graphics, and Islamic culture.

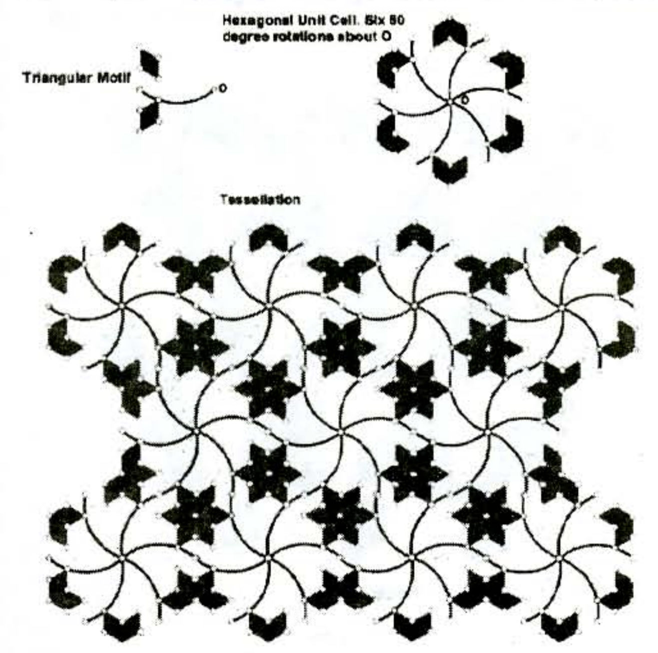

Figure 4
Andalusian Variation

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