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Cover Page Footnote

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Using IBL in a History of Mathematics Course: A Skeptic's Success

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Abstract

A college professor, who is highly skeptical of change, but sensing a need for teaching in a more inspiring and engaging way, implements an inquiry-based learning (IBL) approach to teaching the history of mathematics. The first author (Matthews) worked with an experienced IBL colleague mentor (Hodge) on the course. Some student data was collected to document the effects of the class on the students. The approach taken for the course is described in detail including how the students of the course learned about and used IBL in key peer-to-peer teaching about historical mathematics (with a primary focus on the mathematics). An innovative rubric is described to evaluate IBL teaching. Results indicate that the students were engaged in the course and inspired about mathematics, seemingly more than previous semesters. The results also imply that the students learned at least as much mathematics and history as in previous semester. Finally, the students, most of which were future high school teachers, began to seriously reflect on their own teaching and appreciated the IBL approach.

Note from the first author:

If it ain't broke don't fix it.

-Bert Lance

That old cliché fits my approach to life in general. I'm extremely slow at trying anything new. For example, I got my first cell phone about two years

ago. I still haven't tried out Facebook. I love oldies and listen with skepticism to music produced by someone who won a TV contest show. That is just the way I approach things. When change is suggested, if it's necessary, I'll adopt it. But, to avoid change is my first instinct. It has served me well in life. The ups and downs of hundreds of popular trends have come and gone, and by simply waiting, I've avoided all sorts of mistakes that my friends have made. In the 80s, I wasn't revolting against the demise of the 8-track player and the Beta video player. In the 90s, I wasn't wondering what I was supposed to do with all of my neon clothes. In the 2000s, I wasn't still lamenting my dead Napster account. Today I don't have a bookshelf at home littered with Atkins' diet books and PDA (Personal Digital Assistant, sort of an iPad predecessor) manuals.

I've taken the same approach to teaching. I adapt slowly and carefully. I'm skeptical. Is this a passing fad? Is some corporate greed driving this idea instead of real research? Is this really helpful and useful?

But my teaching is very different than most elements of my life for several reasons. First, I'm passionate about teaching. Deep down in my heart, I want to teach well, to make a difference. This is not true about my choice of entertainment, clothes, or efficient communication. I'm OK with being "good enough" in those fields. Second, I realize, starkly, that I'm merely an adequate teacher at times. I'm just "good enough." This rankles me and unlike most of what I do in my life, I'm really NOT ok with being somewhere between adequate and good. I want to be an incredible teacher. I want to be an inspiring teacher. I ran across a quote by William Arthur Ward that epitomizes my hopes for improvement in my teaching.

The mediocre teacher tells

The good teacher explains

The superior teacher demonstrates

The great teacher inspires

So ... unlike almost everything else in life, I am more open to trying new ideas when it comes to teaching. I've tried different teaching approaches throughout my career and have slowly morphed away from being a very traditional teacher who lectured exclusively, to one who tried more activity-based lessons once in a while until I've settled into a teacher-centered hybrid of lecturing and activities. So when I started learning about inquiry-based learning (IBL), my first reaction was "eh... just another teaching idea."

Then something happened that changed my mind. I started getting glimpses of how it seemed to impact the students. Intrigued enough to listen more, I attended an inquiry-based learning workshop that the second author of this paper helped organize for our faculty. I walked away with enough interest piqued that I decided to learn more and give it a try. IBL seemed to promise something that I wanted. It seemed to inspire the students. I committed to trying it with a class that I had already decided I needed to revise anyway, my history of mathematics class, a university class populated primarily by future high school teachers. I applied for and was awarded a small grant from the Academy of Inquiry Based Learning to support my time as I delved into what it meant to teach with an inquiry-based learning approach. The second author of this paper was my mentor as we tried this approach, as she has experience with IBL in various mathematics classes. Together (using her knowledge of IBL and my experience with the course), we use this paper as a way to share my journey as a skeptic trying a new approach to teaching.

In this paper, we will describe my approach to IBL, describe the efforts that I undertook to learn about IBL, and my efforts to adapt it to a history of mathematics classroom. These efforts include developing materials for the students to use in presentations, developing a rubric that would encourage the students of the course to use IBL concepts during their presentations, and thinking of ways to effectively evaluate student presentations. We will also describe how the students reacted to the IBL-based history of mathematics course. Finally, we will discuss what we have learned and future plans.

1. Background

Although there are several publications (for example, the entire *Journal of Inquiry Based Learning* (<http://www.jiblm.org>); [4]; and [6]) that give perspectives on inquiry-based learning, we highlight the one IBL publication that had the greatest influence on Matthews's re-designing of his history of mathematics course. We do not wish to discount other literature, but this book guided Matthews in how he structured his course and the aspects of inquiry-based learning that he felt were most important to focus on when revamping his usually traditionally taught course on the history of mathematics.

Specifically, the book *The Moore Method: A Pathway to Learner-Centered Instruction* [2] cited several aspects of inquiry-based learning that are vital to the education of future mathematics teachers (and important to all in the learning of mathematics):

- Empowerment: Students feel empowered in their learning.
- Self and Peer Critique: Students critique each other and themselves as they take responsibility for their own learning.
- Authentic experiences: The whole environment of the class needs to make the educational experiences as authentic as possible. The reasons for learning need to be the same as in real life settings beyond the classroom.
- Environments that are trusting: Failure and not getting it right is seen as part of the norm. Students are given the chance to fix what they need to. Effort is valued not just the results, because effort is seen as leading to results.
- Expectations of Greatness: Students are held to high, rigorous standards, and more than that, they are all expected to reach those high achievements. Teachers set the bar too high and expect and anticipate their students making it there.
- Enjoyment: Students and teachers enjoy the subject. They see this as a responsibility and something to work for when necessary.
- Lots of work on their own: Students are expected to work on their own outside of class to accomplish their purposes. This will take many hours more than normal homework at times because the experiences are authentic.
- Students have the ability to correct and revise
- Peer teaching: students teach and learn from each other.

These aspects, when brought together, create an inquiry-based learning classroom environment according to Coppin *et al* [2].

Research provides suggestions on how to be successful in IBL courses such as linear algebra [1], abstract algebra [4], and real analysis [3]. However, how does one successfully bring these features into a history of mathematics class? Why not try to have the students teach the class to each other with the professor as a guide? In fact, why not also expect the students to employ IBL techniques themselves while peer teaching? Since history of mathematics classes on many campuses including ours are predominantly populated by pre-service secondary mathematics teachers, this also seemed to be a fitting way for the future teachers to practice teaching. That is, teach an IBL course on the history of mathematics, by having the students teach the history of mathematics using IBL techniques. The majority of this paper will be spent discussing the details of how Matthews revamped his history of mathematics course to allow just this (students to teach IBL lessons to their peers).

2. The Nuts and Bolts

The class was populated by nine undergraduate students and three graduate students and met two times a week for seventy-five minutes. The majority of the students were undergraduates who were interested in becoming secondary mathematics teachers. However, two of the students were undergraduate mathematics majors who had no interest in teaching at this point in their career trajectory. Three of the students were graduate students currently teaching with a middle school endorsement and were taking the class as coursework to acquire a state certificate/endorsement to teach secondary mathematics (grades 7-12). One student was an elementary teacher taking the class for personal growth along with other graduate mathematics courses. All of the undergraduate students were under 25 years old. There were five females and seven males in the class. The graduate class members were all middle aged.

In order to guide the class, Matthews selected 52 historical mathematics-rich problems and theorems/concepts to be given to the class. Since he had taught the course several times in the past, he had an idea of what topics he definitely wanted to cover and included those in the list. The problems/theorems were meant to be authentic, challenging, and accessible with work. They also represented many different eras and cultures. For example, here is a selected list of the problems/theorems: Euler's lines, Constructing regular 17-gon (Gauss), Space Filling Curves (Peano), Different Infini-

ties (Cantor), Delian Cube Doubling, Mercator Projections, Al-Khayammi's solving cubic equations using conic sections, Finding square roots from Nine Chapters of the Mathematical Art, and Non Transitive Paradoxes of Charles Dodson (Lewis Carroll).

Thirty of the problems/theorems were identified as quickly accessible and more appropriate for a shorter lesson than the others. The other twenty-two problems/theorems were identified as richer and more in-depth.

Groups of four were selected randomly at the beginning of the class. Each class member was responsible for teaching three lessons, a one-day lesson taught individually only to their fellow group members, and two week-long lessons taught as a group to the entire class. To avoid confusion in the future, we will refer to the students as an IBL facilitator when they act in the role as a teacher for the class. We will refer to the class members as students when they are being taught by their peers. The professor who taught the course will be referred to as the instructor or Matthews. The individual lessons covered one of the thirty quicker/more accessible problems/theorems. The individual lessons started on the third day of class (the first day was an introduction and the second day was a day in which the instructor taught some mathematics from some early cultures such as the Babylonians). The two other week-long lessons were taught by the foursomes and came from topics from the more advanced twenty-two problems/theorems. For each lesson, the IBL facilitator (or a group of facilitators):

- a) picked one of the problems or theorems to present (pending instructor approval),
- b) assigned a historical reading due for the first day of class, usually about 1 hour worth,
- c) designed, taught (or perhaps better said, facilitated) lessons focusing on the mathematics primarily on the problem/theorem
- d) assigned homework/gave feedback on the homework
- e) wrote test questions/retaught as needed
- f) gave a self critique of their own teaching/gave critiques of their peers' teaching when they were students, and met with the instructor for a final reflection.

3. The Rubric

The following rubric was used to guide the IBL facilitators in the course as they prepared and evaluated their lessons. The rubric was modeled after the IBL approach and captures what Matthews sees as the essence of IBL.

The TEACHER rubric: 7 Guiding Principles

- 1. Trustworthy:** Reputable documents sources: You need to rely heavily on known, trustworthy sources. I don't mean that you should avoid Wikipedia. I mean that it should be obvious that you have done a lot of reading from a variety of good sources. Teach the students to learn to use these sources themselves to find answers to questions (your questions and their own questions).
- 2. Enjoy:** You enjoy teaching the material. You teach it in such a way that helps others learn to enjoy it also. You are creative/different.
- 3. Adapt:** You adapt the lesson to fit the needs/wants of the students that you teach. So you must know them. You must know what they like, what they struggle at, what their strengths are, etc.
- 4. Challenge:** Expect great things from them. Trust that they can. Give them opportunities to do great things in what you expect from them. Ask probing questions that cause the students to think deeply. Be sincerely interested in their answers and respond appropriately. Assign stimulating and demanding homework.
- 5. Help:** Hold office hours. Not just sitting back and watching them work. Making and seeing learning as a joint venture. Helping each other during the class. Provide good feedback on homework. When students are stuck provide just enough help, but not too much.
- 6. Engage the students in the teaching of the material:** Facilitate their teaching of each other. Encourage opportunities to ask their own questions of you and of each other. Invite and encourage them to explain.
- 7. Ready:** You must know the material backwards and forwards yourself. You have sought and received multiple layers of feedback on what you are planning to do.

This rubric was used mainly as a way of challenging the IBL facilitators to teach using IBL techniques. During the first immediate lesson taught during the first weeks of class, the TEACHER rubric was only used as a goal. For the next two group lessons, the rubric was also used for evaluation purposes. It provided a framework for our efforts to discuss teaching in context of this class. The students gave the IBL facilitators feedback using the TEACHER rubric after each class during a five-to-ten minute wrap-up and reflection section. The IBL facilitators also used the TEACHER rubric as a guideline as they self-evaluated their own teaching and as they evaluated their group as a whole. Their students also gave the IBL facilitators feedback as a group via a Blackboard administered survey that the students filled out after the lessons were finished. Because the results of the survey were rich and helpful in terms of evaluating the teaching and providing helpful focus for teaching improvement, the survey is described in detail next.

4. Self-evaluation of the Guiding Principles

For both group presentations (with the first presentation, which was thought of as a sort of a warm-up), the IBL facilitators completed self-evaluations on each of the seven guiding principles from the TEACHER rubric. They had a choice of saying that they either *Excelled* at the principle, *Met Expectations* for the principle, *Approached Expectations* or *Fell Somewhat Short of Expectations* of the principle. The guidelines on the following page were provided to the IBL facilitators as they considered their self-evaluation.

Furthermore, the students evaluated the entire group that presented the lesson(s) using the same rubric with questions reformatted from their point of view. To make the evaluations go quickly, the evaluation was set up using a Hot Spot question type on Blackboard, where each student could click on emoticons that represented each evaluation choice for each item of the TEACHER rubric.

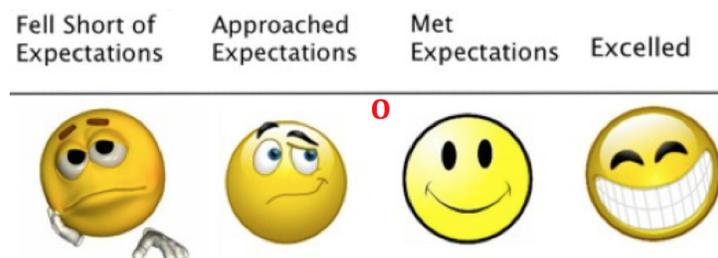
The Hot Spot question type is one of the types of questions that one can use when creating tests or surveys on Blackboard. More traditional question types include multiple choice, free response, fill in the blank, and true/false. Hot Spot is a unique question type that allows a test taker to click on an image to answer the question.

Facilitator Guidelines.

Below are the four rating levels that you can rate how well you did on each guiding principle, along with a definition of the level and some statements that might be said under each level. There are statements related to planning, statements related to what you learned upon reflection, and statements related to how you think your implementation of the guiding principle impacted the lesson and learning.

	Excelled at the Guiding Principle	Met Expectations for the Guiding Principle	Approached Expectations	Fell Short of Expectations
Definition	This principle was one that I excelled at and believe that I really succeeded at incorporating it.	I focused on meeting the expectations of this principle and feel like I covered all of the major points	I came close to meeting expectations of this principle and feel like I had some good uses and some that could be improved upon.	I think I fell short of the expectations of this principle and feel like I need to improve on it for my next teaching experience.
Planning	Specifically planned how to incorporate the principle in what I taught or how I prepared.	Specifically planned how to incorporate the principle in what I taught or how I prepared.	Specifically planned on using it but realize I could have done so more intentionally.	I didn't really plan on how to incorporate this principle.
Reflection	As I taught or reflected, I overtly noticed several ways in which the principle was working well in action.	I overtly noticed it working well a few times.	Noticed some ways in which I should have used the principle and ways in which it worked well.	I frequently saw ways I could have used or improved my use of this principle.
Impact	The principle had a major impact on how I taught or how well the teaching went.	It had a moderate impact.	The principle might have had an impact, but it was minimal if it did.	It probably had no impact on the lesson/learning.

For example, the IBL facilitators and their students were given the following image as they evaluated each guiding principle. To answer, they simply clicked on the image or on the line. For example, the O below would represent a student clicking there and it would have been scored as between approached expectations and met expectations.



The IBL facilitators and the students were also given a text box for which to enter any comments about the teaching overall.

There was over 90% return rate on the student evaluations even though it was not officially part of their grade. We attribute this to the innate interest in the subject and the ease of the evaluation process, due mostly to the ease of use of the Hot Spot tool.

5. Pre-service Secondary Mathematics Teachers

Each IBL facilitator gave themselves an overall presentation grade with a rationale justifying their choice. Their students' evaluations and the instructors' observations were used to adjust the grade as needed. This was almost always adjusted, as those who taught were extremely hard on themselves, likely a product of the experience being so authentic. For feedback, what their students/peers said in general comments and average "scores" that their students gave them on each guiding principle were compiled. This overall feedback was provided privately through email, but there was a final meeting of the entire group and the instructor to discuss what they learned. Ownership of your own learning appears to be a vital element of IBL teaching and this seemed like a fair compromise between the student completely self-evaluating and the normal teacher-dominated evaluation.

6. The Modification

About half of the way through the 15-week semester, there was a decision to spend some time discussing what went well and what did not go well as a class. About ten minutes after each class was spent discussing how the class went, using the TEACHER rubric to guide our discussion. This really helped with the peer-critique and self-critique. As the semester went on, the students and IBL facilitators were much more willing to share ideas about teaching and mathematics for correction purposes, not just for a pat on the back or a “good job.” After a few weeks this helped to make the classroom a safe environment for learning.

7. The Assessment

At the end of every presentation cycle, class members took a test made from questions created by their IBL facilitators. The questions were based on the presentations that the IBL facilitators gave and the history readings and mathematical homework that the IBL facilitators assigned. Here is where Matthews made the most controversial decisions for the course. The decision was made that every class member’s test scores were slightly impacted by how well their students did on the portion of the test that they taught. For example, let us assume that class member A taught students B, C, and D as an IBL facilitator. Then class member A’s overall test grade was calculated as 80% of their score +20% of the average of B, C, and D on the topic that A taught. To encourage good/appropriate questions, the questions that each IBL facilitator wrote for the test were also evaluated. If questions were too easy, then it would lower their test score some, and if they were challenging, it might raise their scores some (regardless of how their students did).

This grading policy was intended to help the experience become even more authentic and as a way to expect greatness from the class members. Noted by Matthews, “In the past I have asked class members to teach parts of the history of mathematics class, but they have at times done so without much sense of what I am teaching matters.’ I wanted it to matter.”

Now, back to one more of the nuts and bolts. There was one day set aside for review before each test. The idea was that the IBL facilitators could reteach, based on how their students did on homework and be available for questions.

This did not go as well as it could have at first because the last IBL facilitator in the first round of teaching got the homework back from his/her students on the day of the review. There should have been an extra day put in before the review. After getting feedback from the students, this extra day will, in the future, be spent on making historical connections, giving the students a sense of scope and timeline of how the problems/theorems/and mathematicians fit together chronologically. Questions that could be answered during these days include:

- (a) Where does this problem fall in the history of the time better?
- (b) Who were contemporaries of the problem and the mathematicians involved? and
- (c) What mathematics later was based on solutions to the problem?

8. Conclusion

Well, what conclusions can we make from this experience? Of course the reader should accept, if it wasn't already made painfully clear before, that Matthews is a skeptic. That skepticism applies to everything, including his evaluation of his own research and teaching. So our conclusions reflect that caution.

First, we report a non-finding. Did the students learn the history of math as well as they would have in another circumstance? Regarding learning some historical mathematics deeply, the IBL facilitators/groups chose a wide variety of topics. A few chosen topics could be considered interesting, but were more quickly accessible (for example, Chinese Methods of finding Square Roots). These less challenging topics were only available for teaching during the first one-day teaching experiences. However, most of the topics were intriguing and challenging, especially for those in the course who had never been exposed to any of the thinking in the topic (for example, Cantor's proofs of different infinities). The grades on the tests in the course were distributed around a tight Bell curve almost with B average and seem to agree with what the class members said, that they "learned a lot." It seemed to be that the students of the class were challenged and engaged in the material. They said as much to Matthews in person and in the course reviews. But did this course do better than other potential variations of the course? We say frankly, "we don't know."

Next we report a cautious finding. Are the class members walking out inspired? From Matthews's perspective the class members appeared to meet the expectations he was hoping to achieve. That is, the class members seemed to leave the class inspired to some extent. He wanted them to learn deeply some mathematics related to the history of mathematics, but he also wanted to expose them to IBL techniques (using IBL techniques in doing so).

Related to being inspired and using IBL techniques, Matthews felt like the class members really took to the ideas embedded in the TEACHER rubric. They certainly appeared to try to hit all seven guiding principles while they taught. The conflict that most often arose was the conflict between engaging the students and challenging them. The fact that the IBL facilitators were trying to reach this delicate balance is perhaps an essence of good teaching (IBL or not). This seems to be in the spirit of Vygotsky's Zone of Proximal Development [7]. Also, the lessons usually seemed to employ IBL ideas. The lessons were definitely not simply lectures. Often students were engaged and were being taught mathematical ideas in an IBL fashion. The class members certainly rose to the challenge of meeting the guiding principles on the TEACHER rubric in unique and creative ways.

Hodge, the second author and the mentor of Matthews in his efforts to use IBL, was able to meet with the students near the end of the semester for a focus group and talk with them about their experiences in this history of mathematics course. From her perspective, Matthews was correct. The students were inspired. They were challenged and at first a bit out of their comfort zones, but they felt like they really learned the material. They were able to learn both mathematics and valuable teaching skills at the same time. They expressed a new ability and confidence to learn mathematics and were inspired to do so.

We will provide one example as evidence that to us showed how engaged and inspired the group was. The group that decided to teach Cantor's infinities had the one elementary teacher (6th grade in an honors class) as one of the IBL facilitators of the group. One of the guiding principles is the principle of *Ready*. As they were planning, as an effort of getting ready, this group decided to teach the unit to the elementary class of the said IBL facilitator! Not only did they do this, apparently the elementary students in this class understood much of what they were taught. The idea behind *Ready* is that you have prepared and sought for and received feedback. Teaching Cantor

to children, even smart children, in an hour is brave almost to the point of reckless. But it's a great way to think about how to get ready. It reminds us of a quote about teaching by John Taylor:

It is true intelligence for a man to take a subject that is mysterious and great in itself and to unfold and simplify it so that a child can understand it.

The idea is that if you can teach a concept to children, then you really know it backwards and forwards. Was the class inspired? We think yes. Coming from Matthews that is a relatively strong statement. Moreover, he has taught the course recently with similar results. The IBL approach seems to be having success in helping inspire his students. He even attempted implementing an IBL approach in a different mathematics course but found it to be not as successful. But he kept the IBL approach with the history of mathematics course and it has been successful and he is considering ways to implement elements of IBL in his other courses.

Finally, a surprise result that emerged from the experience, one that Matthews would like to see if it will continue to be true in further iterations of the class. For the vast majority of the class members, the teaching experiences in this class was one of their first intensive teaching experiences. They really seemed to relish in the opportunity. But as they reflected on their own teaching and received feedback, it actually led some of the class members to have some doubts in their ability to teach effectively. Some of the students even started to question whether or not they would be good teachers, based on this small sample size of struggling with teaching. This was surprising, but we believe very healthy. Critical reflection often leads to exposure of weaknesses. Exposure to weaknesses drives the desire to change and improve. All of the class members who were planning on becoming teachers took their methods class the following semester. We believe their experiences in this class and their own sense of wanting to improve likely provided significant motivation and context for learning in the methods class.

Of interest, perhaps, to the reader, of the two non-teaching class members, one talented mathematics student decided after taking the course that perhaps he would like to teach at the college level. He is currently in a graduate mathematics degree pursuing that goal. The other enjoyed the class and has moved on to graduate work in the medical field.

9. Future Efforts

As we look forward, Matthews will continue to tinker with his adaptation of IBL in a history of mathematics setting. His experience as someone who is skeptical and slow to change may be helpful to others considering implementing IBL in their classrooms. However, hearing from other new IBL adopters would be helpful to paint a better panorama of the motivations, experiences, and conclusions that new IBL adopters have. From a history of mathematics perspective only, it would be nice to see other IBL adaptations of such a course.

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