E-Brock Bugs©: An Epistemic Mathematics Computer Game

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Abstract

Devlin in [7] argues that video games are an ideal medium for the teaching and learning of mathematics, though he points out that very few 'good' mathematics video games exist.[1] Building on a probabilistic board game developed in the 1980s, we created a mathematics computer game, *E-Brock Bugs*. The design of the game carefully follows Devlin's principles of a good mathematics video game, including a well-developed storyline, the selection of an in-game avatar, and an environment where mathematics arises in a natural and meaningful way. As a result, we argue that *E-Brock Bugs* is an epistemic computer game [1]; it goes beyond teaching basic facts and skills, and may encourage the players' development of mathematical thinking as 'working mathematicians'.

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[1]In the following all references to Devlin are to his 2011 book *Mathematics Education for a New Era: Video Games as a Medium for Learning* [7].

1. Introduction

The use of computer technology in mathematics education has been a topic of interest for quite some time. In fact, due to the widespread use of technology and the push to integrate appropriate technology in mathematics education, several mathematics programs and curricula call for the integration of various information and communication technologies; see for instance the curriculum documents for the Ontario Ministry of Education [17, 18, 19]. Yet, one kind of computer technology, that of video games, is often absent from the lists.

In his book, *Mathematics Education for a New Era: Video Games as a Medium for Learning* [7], Keith Devlin argues strongly in favour of the use of video games in the teaching and learning of mathematics. In general, he presents an extensive use of video games as an obvious way of solving a well-known dilemma. He explains that, on one hand, "[t]eachers complain that many students appear uninterested in [mathematics] and are unmotivated to make the effort necessary to progress in developing computational skills, problem solving ability, or an understanding of basic mathematical concepts" [page 45]. On the other hand, he refers to a study by the Pew Research Center, which found that 97% of American teens aged 12–17 play video games [11]. While many students are eager to spend hours and hours engaged in the challenging worlds of video games, few students are willing to spend the same time and effort on completing their mathematics homework. Devlin therefore suggests that using appropriate video games in mathematics education could be extremely valuable in solving problems related to student motivation and achievement. In fact, this claim has been explored and empirically supported by past research concerning specific video games (see for instance [2]).

But, throughout his book [7], Devlin puts forth even higher expectations for video games: that is, their use in helping students develop their ability to think mathematically, or, to think like mathematicians. Providing several examples of (non-math) computer games and simulators, he argues towards the use of what are called *epistemic mathematics computer games*: games in which the player becomes a (better) mathematician [15, 16].

We should perhaps note that Devlin didn't explicitly use the term *epistemic computer game* in his book nor was he the first to point out the importance that video games may have in enhancing the thinking of young children. In fact, much of what Devlin discusses in [7] can be seen as an

extension of the work done by researchers before him. For instance, the concept of epistemic games has been discussed in a much wider context by other scholars, who see such games as a way to provide learners with the chance to experience and adopt the innovative ways of all kinds of professional practitioners, not just mathematicians (see for instance [23]). Much of Shaffer's work has focused on epistemic games in this wider context (see [1, 21, 22]), while Gee has worked extensively on identifying the general educational nature of video games (see [8, 9, 10]). Devlin devotes an entire chapter of [7] to a discussion of the insight that Gee's work can provide in the context of mathematics education. It is this particular focus on mathematical games that made Devlin's book the most helpful reference for the work that we discuss in this paper. Nonetheless, we will adopt the term *epistemic* from the research as we see it as a natural descriptor of what Devlin is after.

So, how does one design an epistemic mathematical computer game? Devlin proposes several design principles that he believes would allow a video game to go beyond the strict drilling of mathematical skills and actually support the development of mathematical thinking. However, he does not provide an example of such a mathematics game since, according to him, none had been developed at the time of writing his book. He does mention *DimensionM*[1] and *TimezAttack*[2], which he considered to be "two of the best math ed video games on the market" at the time [page 5]. More recently, he has also worked on producing a game, *Wuzzit Trouble*[3] which aims to develop mathematical thinking in its players. In addition, in her reflections about the analysis of video games for mathematics education, Offenholley [15, 16] identifies another game, *Ko's Journey*[4], which she argues to be epistemic.

Within this broader context, inspired and guided by Devlin's (theoretical) principles of designing mathematics computer games that prompt players' development of mathematical thinking, we created a mathematics computer game called *E-Brock Bugs* [5].

We start this paper in Section 2 with a brief description of *E-Brock Bugs*. In Sections 3 and 4 we describe in detail the implementation of Devlin's

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[1] http://www.dimensionu.com/math/, accessed on July 3, 2015.
[2] http://www.bigbrainz.com/, accessed on July 3, 2015.
[3] http://wuzzittrouble.com, accessed on July 14, 2015.
[4] http://www.kosjourney.com/, accessed on July 3, 2015.

principles in *E-Brock Bugs* and the mathematics that players may encounter throughout the game. We argue for the epistemic character of the game in Section 5. Finally, we provide some concluding remarks in Section 6. In response to the variety of instructional games that exist currently in terms of content and design, Offenholley points out that "[t]o truly analyze the effect of games-based learning, we must begin to examine rigorously what each game does for its particular group of learners, and for the particular mathematics we hope it will impart" [15, page 46]. This paper marks the beginning of this process for *E-Brock Bugs*.

2. *E-Brock Bugs*: The Game

E-Brock Bugs, created by Broley in 2013 with contributions from Buteau and Muller, is a computer game inspired by two previous works. Its core gaming mechanism originates from a board game developed by Muller [13], see Figure 1 below. The board game has three levels of play. As an illustration, we describe only the first, which starts with two players taking turns choosing a number from one to fourteen on a board. Once all the numbers
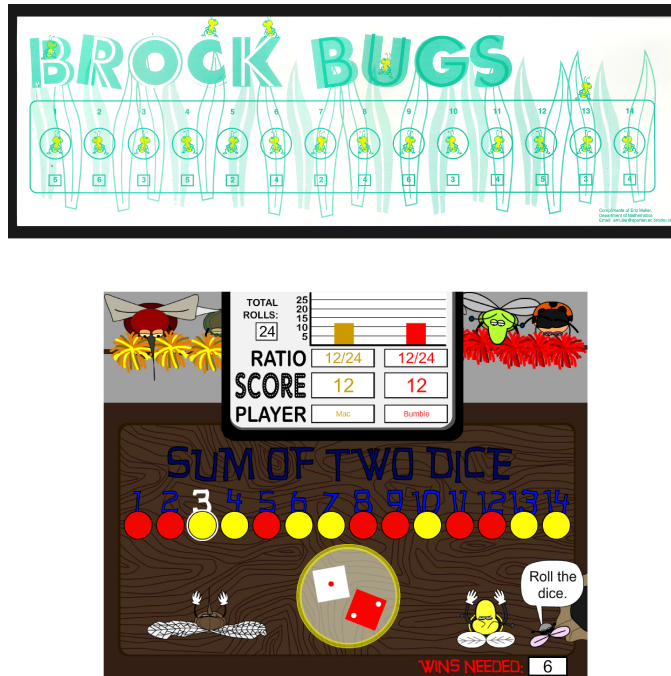

Figure 1: *Brock Bugs* board game (top) implemented in *E-Brock Bugs* (bottom).

have been chosen, the players roll two ordinary dice, determine their sum, and award a single point to the player who chose the number corresponding to that sum. After twenty-five rolls of the two dice, the player with the most points wins. It is through several different computer versions of this kind of game that the player of *E-Brock Bugs* might engage in doing mathematics.

Inspired by a partially developed computer Learning Object [14] version of Muller's board game, *E-Brock Bugs* also contains animated and interactive lessons that support the transition from the various probabilistic games to their related mathematical theory. Ultimately, elements of the board game and Learning Object have been modified, adapted, and embedded into a computer game environment that, we hope, will drive players not only to be engaged in the planned mathematical activities, but also to develop their mathematical thinking related to basic probability. Whether this is achieved or not, *E-Brock Bugs* is first and foremost a game, which offers players the chance to embark on a fun-filled adventure, as described below:

> *Since the beginning of time, Bug City had always been a peaceful place to live, where even the simplest of bugs could feel right at home. Then, one day, the city was swarmed by an evil band of Bullies and their mysterious leader, the all-powerful Dr. P. Darkness quickly spread across all six districts that make up the city, transforming it into the wasteland it is today. But the situation is not entirely hopeless, for the player of* E-Brock Bugs *is the hero that Bug City has been waiting for! To restore the city to its original beauty, the player must journey through each district and defeat all of the Bullies at their probabilistic games. With the help of some friends made along the way, the player may finally convince the citizens of Bug City that they have nothing to fear, and that knowledge should never be used as a weapon.*

3. The Implementation of Devlin's Principles in *E-Brock Bugs*

Though the design of *E-Brock Bugs* was influenced by several different factors, the creation process was mainly guided by Devlin's principles of a mathematics video game that prompts the development of mathematical thinking. In this section, we highlight some of the most important principles and how they were implemented within *E-Brock Bugs*.

3.1. The Back Story

According to Devlin, game designers should think very carefully about their storyline because "the back story is crucial to the success of a game" [page 134]. In other words, games with ill-developed storylines tend to fail. To avoid such failure, *E-Brock Bugs* has a carefully-developed storyline, which is conveyed to players first through an introductory animation and then through various other dialogues and actions within the game. An element of mystery has also been added to make the storyline more interesting; precisely, the existence of Dr. P is kept hidden from the user until they have defeated all of the Bullies, though subtle appearances by the villain at different points in the game may indicate to players that he will eventually play some significant role. Once players have defeated all six of the Bullies, they are captured by Dr. P and taken to his secret lair, where they must use his simulation machine to overcome one final challenge. The aim of the storyline construction is to motivate the player enough to not only want to play, but also reach and conquer this final challenge. In *E-Brock Bugs*, playing the game becomes about saving Bug City and not just about learning mathematics.

3.2. Identification with an In-Game Avatar

The first task a player encounters in *E-Brock Bugs*, as in most other successful video games, is to select and name his/her in-game identity amongst the collection of six pre-made avatars shown in Figure 2. Immediately, the player's experience becomes personalized; the goal is that the player will easily identify with the character and therefore want this character to succeed. To encourage this identification to occur, the avatar is displayed on the screen in front of the player in many different ways (e.g., from behind, from above, from the side). Nonetheless, seeing the avatar on the screen also allows the player to have a sense of being outside the action taking place, which means that "failure is not as personal as it is in real life; the player can mitigate failure by saying 'It was my character who screwed up,' even though the character failed only because the player did" [7, page 88]. As a result, the player may be more likely to take chances and less likely to be embarrassed by mistakes.
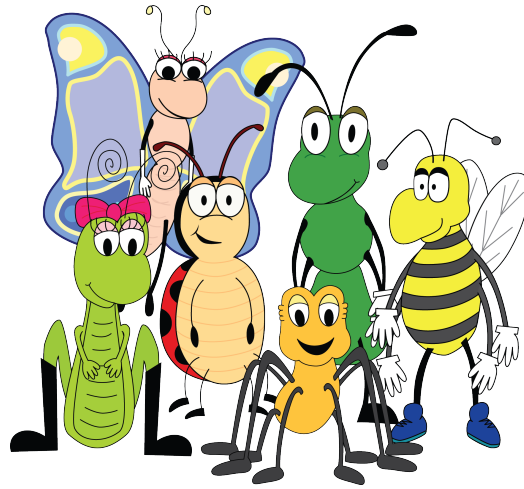
Figure 2: The six *E-Brock Bugs* avatars.

3.3. Balancing Cost and Reward

Even if having an in-game avatar may lessen the blow of failure, determining the costs and rewards for a player's actions in a video game is a delicate balancing act. Devlin's Principle 10 of an Ideal Learning Environment states that "[t]here should be sufficient 'cost' at getting something wrong to motivate correction, but not so great that it leads to the student losing heart and giving up" [page 30].

In *E-Brock Bugs*, there are two ways players can defeat a Bully: either they must win the Bully's game six times, or they must provide the correct answer to a challenge question (of mathematical nature) that is posed by the Bully only once he/she feels threatened enough to do so. When players lose a Bully's game, the only consequence they face is some virtual heckling (see Figure 3 on the next page). The hope is that the players' motivation to beat the Bully who is antagonizing them will be stronger than their fear and frustration so that they keep playing the game until they have mastered it and the mathematics involved. A player who chooses to attempt a Bully's challenge question and answers it incorrectly must face a harsher penalty: having to win the Bully's game two times more than already required. This punishment intends to encourage the player not only to think more carefully about the question and the mathematics involved, but also to want to come back and attempt the question again since it can lead to success much quicker
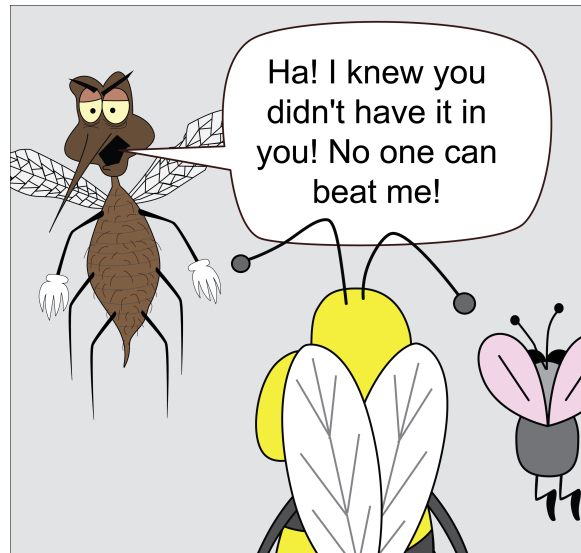
Figure 3: Virtual heckling from one of the *E-Brock Bugs* Bullies, Mac the Mosquito.

and with much more certainty than playing the regular game. This last characteristic of the challenge question justifies the harsher punishment. Just how harsh the punishment should be, however, is a difficult problem that can only be solved after some testing of the game with players.

As mentioned previously, once all six Bullies have been defeated, the player must successfully overcome one more obstacle to officially become a hero of Bug City. More specifically, players are given only five attempts to use Dr. P's simulation machine (see Figure 4) to determine the correct answer to a question he poses about one of the Bullies' games, which has been previously chosen by the player. At this point, the player has already worked through six other challenges and only has one more task to complete before experiencing ultimate success; in other words, the stakes are high. Thus, determining the right penalty for failure is tricky. Nonetheless, to encourage more serious thought about the challenge, a penalty is required. In the end, we decided that if a player does not complete the final challenge successfully, they are brought back to a Bug City where the last Bully they defeated is still in control of his/her district; i.e., only one more Bully must be defeated before the player may make another attempt at the finale. We expect that such a setback would not be too harsh, especially for the player who has already mastered the mathematics behind the Bullies' games.
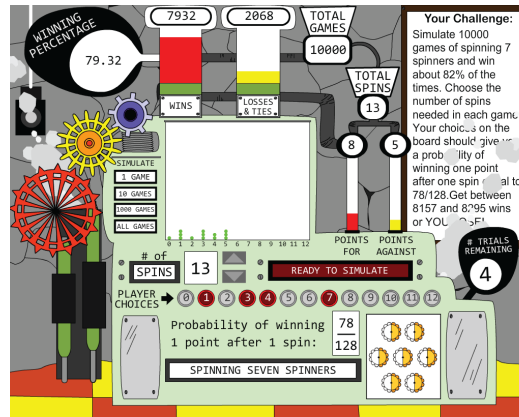
Figure 4: Dr. P's simulation machine.

When it comes to rewards, Devlin's Principle 9 of an Ideal Learning Environment states that "[t]he student should be given immediate positive (and ideally public) feedback for any success that is commensurate with that student's current level of attainment" [page 30]. Furthermore, there must be just the right amount of acknowledgement for the player to feel as though his/her hard work was worthwhile and that he/she is deserving of the praise. To this end, we thoughtfully designed the dialogue and events that occur after a player experiences success in our game. For instance, after defeating each Bully, the player gets to see the transformation of the corresponding district from a dark and destitute wasteland to a happy and bright community, full of new characters, lively animations, and cheerful music. For the players who complete *E-Brock Bugs* in its entirety, an (optional) online Hall of Fame[5] was also created to complement the in-game celebration that occurs when they persevere to the very end, defeat Dr. P, and save Bug City.

3.4. Self-Paced, Multi-Route, Pre-Planned Gaming/Learning Experiences

The navigational aspects of *E-Brock Bugs* were also designed following Devlin's principles. First and foremost, the player of *E-Brock Bugs* is always able "to explore new concepts and practice new techniques at his or her own

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[5]http://www.brocku.ca/mathematics-science/departments-and-centres/mathematics/resources/brock-bugs/e-brock-bugs-video-game/hall-of-fame, accessed on July 3, 2015.

pace", which means that hesitant players can take their time and bolder players can advance as quickly as they would like, in line with Devlin's Principle 8 of an Ideal Learning Environment [page 29]. In fact, unlike in traditional classroom settings, the player is never forced to learn or be evaluated under time constraints, which may alleviate some of the mathematical anxiety that is known to accompany timed testing [3].

In addition, more often than not, the player may direct his/her own path in the game, resulting in a much more personalized experience. The fact that players may take many different paths "allows [them] to make choices, rely on their own strengths and styles of learning and problem solving, while also exploring alternative styles"; this reflects Gee's Multiple Routes Principle [8] as reported in [7, page 95]. Nonetheless, Devlin maintains that it is still important that the student be directed towards "pre-planned learning experiences, some of them in a particular order" (Devlin's Principle 7 of an Ideal Learning Environment [page 29]). Certain design elements have therefore been added to *E-Brock Bugs* primarily to indicate a suggested path of events. For instance, the districts are numbered and placed in a particular order in hopes of obtaining a coherent configuration in terms of the mathematics, the game dynamics, and the storyline, and in many situations, dialogue, animations, and graphics indicate the ideal place to click on next. Without being forced into a certain path, the learner may still become aware of an overall carefully-planned structure.

3.5. Learning Mathematics by Doing/Through Exploration

In addition to being able to experience the game activities at their own pace and in their own preferred order, players of *E-Brock Bugs* are "never put in a position of having to 'learn something' prior to playing the game in order to play the game" [7, page 128]. In fact, players are always prompted to explore probability concepts through play first and, if they choose, are gradually given more guidance as they go on. In other words, the traditional presentation order of mathematical concepts has been reversed so that practice and experience come before theory, and therefore, players are given the opportunity to construct their own understanding of their experiences. Even the more (optional) theoretical activities in the game are built according to this constructivist approach [20, 25], i.e., in such a way that students are given space to explore and develop the concepts themselves. After all, according to Devlin, a student who learns by exploration has more fun, gains

more powerful, usable, and durable knowledge, and takes more ownership of such knowledge [page 99]. He reminds readers of the old saying: "You tell me, I forget; you show me, I remember; you let me discover, and I know".

3.6. *Mathematics Knowledge only On-Demand and Just-In-Time*

A significant part of *E-Brock Bugs*, besides the game play, is the optional and complementary Learning Object component that can be found in Smarty's Shack (see Figure 5). Smarty is the shy, but extremely intelligent bug who has understood all the mathematics behind the Bullies' schemes.
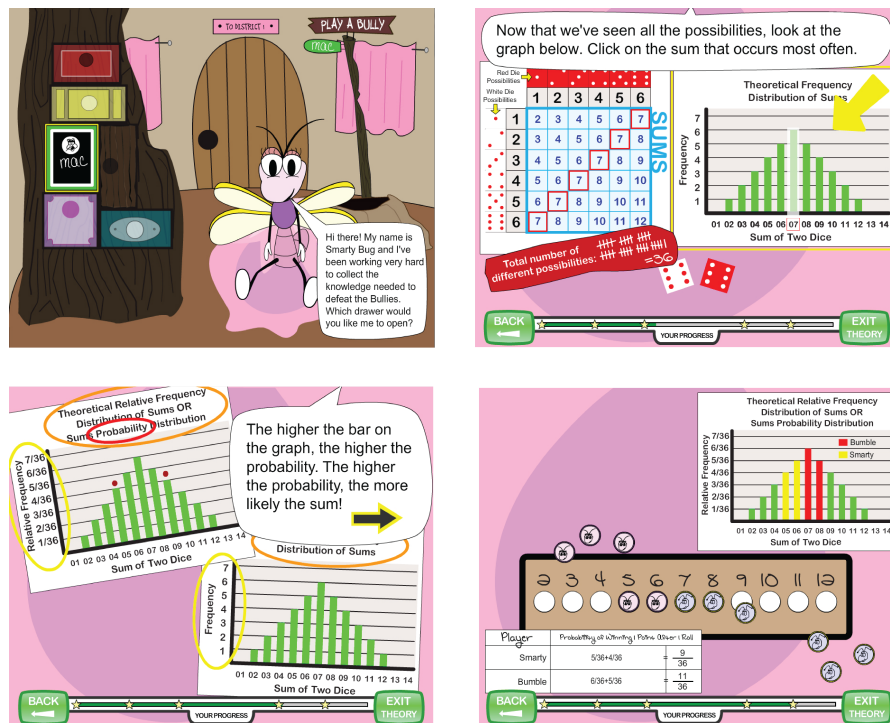
Figure 5: Smarty and her visual lessons in the Learning Object component of *E-Brock Bugs*.

The gathering of all theoretical material in the game into one place was done strategically so to maintain its optional characteristic, for according to Devlin, "[p]utting symbolic expressions in a math ed game environment is to confuse mathematical thinking with its static, symbolic representation on a sheet of paper" [page 6]. Smarty can be seen either as a 'teacher-on-demand',

who would facilitate the use of the game outside of a math classroom, or as an educational assistant, whose work would complement that of a mathematics teacher.

Despite the optional nature of Smarty's Shack, players are still prompted to view certain theoretical sections right when the concepts involved would help them in the game; that is, "[t]he learner is given explicit information both on-demand and just-in-time, when the learner needs it or just at the point where the information can best be understood and used in practice" (Gee's Explicit Information On-Demand and Just-in-Time Principle [8] as reported in [7, page 99]). In order to allow for exploration, a player will likely play against a Bully at least twice before being invited to Smarty's Shack. At this point, the player may decide to learn about the theoretical concepts related to (and strictly needed for) that specific game. Once they have been guided by Smarty to understand how to use mathematics to defeat a particular Bully and are given the opportunity to practice their newly-acquired skills as many times as they wish, players are encouraged to go back to the corresponding game and put the knowledge they gained to immediate use. Ideally, this would perpetuate the view of mathematics as useful, worthwhile, and applicable in the game world.

The most extreme example in *E-Brock Bugs* of just-in-time mathematics occurs during Dr. P's final simulation challenge. Though this final challenge involves a player-selected Bullies' game, it is quite different from anything players would have experienced during regular game play: rather than using a game board, they must operate a machine; rather than playing one game at a time, they are simulating 10,000 games at once. To ease the transition into this new and possibly uncomfortable situation, players are given some directional input from some familiar friends. But, in hopes of encouraging some exploration, they are required to make at least one attempt at the challenge before Smarty can come to the rescue. In terms of the storyline, Smarty and some of the Bullies (who are now allies of the player) were able to escape the wrath of Dr. P that was unleashed upon the city as soon as the player defeated all six Bullies. They have made their way to Dr. P's secret lair and have developed some activities that could give players insight into the challenge they face. Of course, receiving such insight is entirely optional. However, should players choose to accept Smarty's invitation to help, they will be given the opportunity to construct the mathematical concepts needed to defeat Dr. P once and for all.

3.7. Regular Tests

Attempting to complete the final simulation challenge is just one of the frequent 'tests' throughout *E-Brock Bugs* that allow players to "see how well they have mastered the latest facts or skills" [7, page 130]. First, players are told that they must win against each Bully six times in order to defeat him/her entirely. As a result, they have the opportunity to experience a repeating cycle of learning and testing to see if they have understood the best strategy for each game and, possibly, the mathematics involved. This process is magnified greatly in the final three districts, where randomized game elements force players to determine a new strategy with each new game (see Section 4). The challenge question corresponding to each Bully may be seen as a kind of ultimate test of the knowledge players gain over the cycle mentioned above since it requires a mathematical response concerning the best strategy for the specific game. Furthermore, if answered correctly, the challenge question ensures instantaneous defeat of the Bully, a success that signals to the player that they are ready to move on to the next district or task. Since the finale simulation challenge allows for five attempts, a similar cycle of learning and testing may occur at that point in the game, though with more to lose, the player could experience a little more test anxiety. Luckily, the theoretical activities and practice exercises provided by Smarty also allow the player to constantly self-assess their understanding. In the end, as Devlin explains, "it's the enjoyment of taking and passing the 'tests,' often after several failures, that motivates players to learn" [page 130].

3.8. Playing Mathematically Leads to Faster Game Progression

It is important to note that just because frequent 'tests' exist throughout *E-Brock Bugs* it does not necessarily mean that a player will use them in the intended way to develop and test their mathematical abilities. Though it may not be preferable for mathematics educators, a player of *E-Brock Bugs* may successfully complete the entire game using only his/her intuition, mathematical or not. As Devlin suggests, "while conceptual understanding is a goal that educators should definitely strive for, we need to accept that it cannot be guaranteed, and accordingly we should allow for the learner to make progress without fully understanding the concepts" [page 115]. After all, if a game puts a player in a position where they cannot move forward because they are stumped by a problem that is just too difficult, then the game fails at everything it is trying to accomplish in the first place.

This being said, in order to discourage players from avoiding the use of mathematics altogether, *E-Brock Bugs* was designed in such a way that if a user decides to make the effort to think mathematically, then they can progress much more quickly. For instance, a player who looks for patterns and determines the best strategy when playing against each Bully is more likely to win and therefore more likely to progress faster than a player who plays without any mathematical approach. Also, the addition of challenge questions in each district allows mathematical thinkers to defeat Bullies instantaneously! And finally, with only five attempts at Dr. P's Simulation Challenge, the likelihood of winning by chance is relatively low; even the player who has proceeded through the rest of the game on instinct may decide to do some mathematical thinking in this case.

4. The Mathematics in *E-Brock Bugs*

The breakdown of the probability concepts introduced in *E-Brock Bugs* is summarized in Table 1 below.

	DISTRICT	GAME	PROBABILITY CONCEPTS
FIXED DISTRIBUTIONS	1	Sum of Two Dice	probability distribution; addition rule
	2	Sum of Two Fibonacci Dice	asymmetric probability distribution
	3	Sum of Two Drawn Balls	independent/dependent events; product rule
RANDOMIZED DISTRIBUTIONS	4	Sum of Two Spinners	equally/not equally likely events
	5	Spinning Seven Spinners	Binomial distribution (n = 7)
Increased Level of Difficulty & Engagement	6	Sum of Two Dice with Point Values	expected value
	Finale	Simulation Challenge	Binomial distribution (n to be determined)

Table 1: The breakdown of probability concepts in *E-Brock Bugs*.

Should players choose to journey through the districts of Bug City in order, they will find that the mathematical concepts and games gradually increase in both their level of difficulty and engagement, though certain game elements do remain constant throughout. The first three districts each involve a fixed probability distribution (e.g., the sum of two dice in District

1), whereas the last three districts each involve a randomized probability or expected points distribution due to a variable component in the game, i.e., in the experiment (e.g., randomized point values in District 6). The single question involved in the simulation challenge is also randomized each time the player attempts the finale. In order to determine the best strategy to defeat the Bullies in Districts 2, 4, 5, and 6, a player will likely have to rely on more than just intuition; even a professional statistician might need to work outside the game with pencil and paper to calculate the probability or expected points distribution in these cases. Developing a mathematical approach in the finale simulation challenge may require even more careful effort outside of the game play. The player who participates in Smarty's (optional) theoretical activities at this point in the game is provided with a new tool, a Binomial Calculator, to make the calculations more manageable. Ultimately, by taking time out to consider the mathematics involved, the player "not only advances faster or further in the game, she or he also learns the valuable lesson that it is sometimes better in the long run to stop for a while and reflect than to continually press forward in an exploratory mode" [7, page 81].

In addition to the specific probability concepts outlined in Table 1, there are three main probabilistic notions emphasized throughout *E-Brock Bugs* (see Figure 6). The first concerns the creation and use of frequency and relative frequency bar graphs. For example, after playing against a Bully, players have access to the frequency bar graph that represents the activity that took place in that particular game. Should they view the bar graph, they will also encounter Smarty, who makes a quick appearance to point out that "[t]here is a reason why certain bars tend to be higher than others!" Note that the graph in District 6 displays the points received per sum rather than the frequency of the sums.
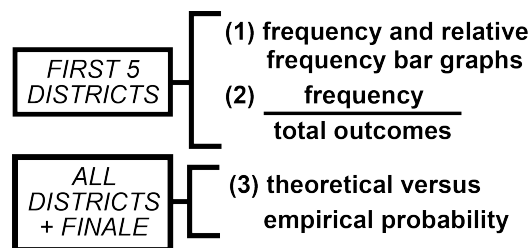

Figure 6: The three main probabilistic notions in *E-Brock Bugs*.

In her theoretical explanations of each game, Smarty visually constructs the corresponding theoretical frequency bar graph and its related relative frequency bar graph, i.e., the probability distribution, or the expected points distribution in District 6 (Figure 5 depicts Smarty's graphs for the Sum of Two Dice game). The construction of these graphs also stresses the second main idea (found only in Districts 1-5), which is the calculation of probabilities of equally likely events by placing their frequency over the total number of outcomes. Even for District 4, where the probability experiment involves not equally likely events, Smarty's explanations are such that the experiment is first transformed to involve equally likely events before the use of the multiplicative rule is addressed. The second main idea is also reinforced in the (optional) practice exercises that are available in Smarty's Shack, where players must use it to calculate probabilities.

The final main probabilistic notion within *E-Brock Bugs* is the difference between a theoretical and empirical probability. For instance, when playing against a Bully, a good game strategy (i.e., a strategy that ensures a probability of winning larger than 50%) does not guarantee a win. In addition to personally experiencing and/or becoming aware of this idea during game play, players may also explicitly learn about it through explanations in Smarty's Shack. Unlike the other two, this theme also extends into the final simulation challenge, where players may select the theoretically correct answer but still experience a 'loss'. Once again, during players' guided theoretical explorations, Smarty explicitly reminds them that "theory is not always 'exactly' reflected in real life events!"

5. The Epistemic Character of *E-Brock Bugs*

Pedagogical approaches and content can vary greatly among computer games that claim to be educational. Of particular interest when developing *E-Brock Bugs* were those instructional games that are called 'epistemic', "where players think and act like real world professionals" [1, page 36]. Offenholley describes epistemic mathematics computer games as games in which "[t]he player becomes a mathematician and problem solver within the context of the game" [15, page 45]. In other words, an epistemic mathematics game is not simply about acquiring basic facts and skills, but rather encourages its players to develop their mathematical thinking as 'working mathematicians'. And, as mentioned previously, this is exactly what Devlin is after in [7]: a

game that is no longer about learning how to do mathematics in the mindless manipulating symbols sense, but is about learning how to be a (better) mathematician. We argue that a combination of selected features of *E-Brock Bugs* make it worthy of this category.

First of all, when players are able to select avatars to represent them in a game, all of the activity is presented not to the players themselves, but to their character identities. In *E-Brock Bugs*, these in-game identities are immediately and consistently provided with tasks and challenges that encourage them to act like mathematicians within the game world. The existence of such an avatar means that the game is no longer about simply learning mathematics; it is about developing (the avatar) into a (better) mathematician. Having chosen the avatar themselves, the hope is that players will easily identify with it and adopt its mathematical identity.

The problem, as Devlin points out, is that it can be difficult to persuade a player to adopt the identity of a mathematically-able person in the computer game world, let alone in the real world. The best solution, he proposes, is:

> to build the entire game world and game around key mathematical learning experiences [...] Thinking mathematically should simply be part of what [the] character does in that world. The mathematics should not be hidden; the players should know they are doing math! But that math should arise naturally in the game, it should have meaning in the game, and it should make sense in the game [7, page 127].

Playing each Bully's game, attempting to answer the corresponding challenge questions, and facing Dr. P's final simulation challenge can be seen as the key mathematical learning experiences in *E-Brock Bugs*. Through these experiences, players may be prompted to engage in 'real-world' mathematical activities, such as the development of probabilistic methods, organization of numerical data, and extension of graphical patterns. Note that this activity and the related mathematical concepts (as outlined in the previous section) are inextricably linked to the proposed games and simulation challenge; that is, rather than being able to place any set of concepts (mathematical or not) within the exact same game mechanisms, the design of the latter cannot be separated from the corresponding probabilistic concepts. Furthermore, whether through experimentation or with the help of Smarty, in a context where the mathematics progression has been designed to carefully build in

complexity, players are encouraged to develop their mathematical thinking in order to save Bug City from the Bullies (and eventually Dr. P), who are using their own mathematical knowledge to dominate the game world. Supported by the game mechanism, optional constructivist theoretical activities, and the storyline, mathematics arises naturally in *E-Brock Bugs*, it has meaning, and it makes sense.

It is important to mention that although Devlin constantly emphasizes the potential benefits of making the game world as close as possible to the real situation in which a player would need to perform the same kind of mathematical thinking, *E-Brock Bugs* is far from being realistic. However, Devlin also recognizes that the game could have little to do with the real world, "provided the learner could be cognitively immersed in it" [page 26]. Van den Heuvel-Panhuizen explains that "the context can be one from the real world but this is not always necessary. The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the student's mind" [24, page 4]. Hence, even if *E-Brock Bugs* is a fictional world full of made-up characters, we expect that certain elements of the game (e.g., the storyline, character selection, and graphics) could make the world real enough for the player to become immersed in it, to identify with his/her avatar, and to adopt the mathematical identity involved.

In addition to the selection of an avatar, naturally arising mathematics, and an immersive game world, Devlin stresses five other features that are crucial in assisting players to develop their mathematical thinking, which is an essential part of the development of a mathematical identity: learning by doing, self-paced learning, progressing through exploration, learning new skills and facts for immediate use, and regular tests of the most recent learning. Section 3 of this paper has already outlined in great detail the implementation of these key features in *E-Brock Bugs*. Self-paced learning means that players can develop their mathematical thinking in a way that is tailored to their own needs. Learning by doing and through exploration means that they may be able to construct more powerful mathematical knowledge. Regular testing means that they will be able to self-assess such knowledge. And new mathematics for immediate use means that players may come to see a purpose in thinking mathematically, a purpose that is enhanced by the fact that thinking mathematically also leads to faster game progression.

Finally, though the addition of Smarty and her theoretical activities was in no way suggested by Devlin's principles, we would argue that she is essential in supporting the epistemic character of *E-Brock Bugs*. First of all, Smarty directly enhances some of the previously mentioned key features for the development of mathematical thinking. For instance, it is through her that the feature of learning new skills and facts for immediate use may become fully effective. Smarty also allows players to regularly test their knowledge in a non-threatening environment and gives them the opportunity to personalize their experience by getting as much or as little assistance as they need or want. More important, however, is the fact that she may serve as a role model mathematician and constant guide in the players' development, who if given the opportunity is willing to pass on her wealth of wisdom to help solidify their ideas, deepen their understanding, or redirect their thinking right when they need it, just as good teachers do. In his book, Devlin emphasizes that the teacher will need to play an important role when it comes to implementing educational video games in the mathematics classroom. Embedding the teacher in *E-Brock Bugs* may allow the game to have the same impact independent of a traditional learning environment.

6. Concluding Remarks

In this paper, we examined the design of *E-Brock Bugs*, a probabilistic computer game, which was founded in Devlin's principles of mathematics video game design. We further argued that *E-Brock Bugs* may prompt players to go beyond the mastery of computational skills, to the development of their mathematical thinking as 'working mathematicians'. As such, *E-Brock Bugs* could be categorized as an epistemic game and this paper could provide a good starting point for the creation and analysis of other epistemic mathematics computer games.

Though we claim to have succeeded in creating an epistemic mathematics computer game, we must point out that Devlin promotes a much more ambitious paradigm than has been implemented in *E-Brock Bugs*. In particular, he envisions a multiplayer role-playing game that allows the user to literally roam through a complex world of intricate storylines, three-dimensional graphics, advanced animations, and constant interaction. Indeed, he uses some of the most popular computer games on the market (such as *World of*

Warcraft[6]) to illustrate his vision. Yet, as mentioned previously, he does not provide any concrete example of a mathematics computer game that meets his ideal; his design principles remain strictly theoretical. Given some of our limitations—the creation of *E-Brock Bugs* was originally linked to an undergraduate Honours Project [4]—it would have been virtually impossible for us to attempt to meet all of Devlin's criteria. Of course, producing a game akin to *World of Warcraft* would require a lot of expertise, time, and money.

E-Brock Bugs is currently available online for free (see [5]). From its launch in October 2013 to today (July 2015), the game has seen over 3400 hits, which indicates that it has already been used and, hopefully, enjoyed by many. A teacher document [6] was also created to support its integration in the classroom. In fact, we have already become aware of its implementation in a few different classrooms, which has opened up some opportunities for research. For example, a small-scale exploratory study was performed to test the impact of *E-Brock Bugs* on Grade 12 students' mathematics achievement and motivation [12]. In the future, some research concerning the implementation of the game within a range of mathematics courses at various levels of education could be extremely useful. We also intend to examine the degree to which players of *E-Brock Bugs* actually develop their mathematical thinking. In the meantime, we hope that *E-Brock Bugs* will continue to be played, whether it is with the purpose of being engaged in mathematics, or simply to have some fun! It is a computer game after all.

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