Journal of Humanistic Mathematics

Volume 5 | Issue 2 July 2015

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Recommended Citation

Byung-In Seo, "Mathematical Writing: What Is It and How Do We Teach It?," *Journal of Humanistic Mathematics*, Volume 5 Issue 2 (July 2015), pages 133-145. DOI: 10.5642/jhummath.201502.12. Available at: https://scholarship.claremont.edu/jhm/vol5/iss2/12

Mathematical Writing: What Is It and How Do We Teach It?

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Synopsis

National Council of Teachers of Mathematics (NCTM) recommends that students be able to communicate mathematics, using correct and appropriate language, by eighth grade [8]. Mathematics teachers at all levels agree that they have the responsibility to teach their students content-specific writing, but many feel that they don't have the tools to do this work. This article offers a foundation and methodologies for different writing assignments that can be used in mathematics classes.

Keywords: *mathematical writing; writing pedagogy; writing instruction; writing methodologies*

1. Introduction

Mathematics teachers believe that they have the responsibility to teach their students content-specific writing; in particular they want their students to write clearly and coherently. Students however do not think much about writing in a mathematics class. They even change their writing styles according to their audience, even if the writing topic is mathematical. For example, when explaining mathematical processes, secondary-level students will use words and mathematical symbols in an English class, but in a mathematics class, their writing will comprise primarily of symbols, using a minimal number of words [12]. Also, students will provide more details to an English teacher, perhaps because they perceive the English teacher as not knowing as much about mathematics [13].

Students need to understand that reading and writing are not only relevant for their English classes. Indeed reading and writing skills taught in English classes can and do apply to mathematics classes. Mathematics teachers can and perhaps should emphasize such connections by explicitly assigning writing assignments. However, there are mathematics teachers who feel that they do not have the tools to teach writing (even within context) or to grade substantial writing assignments properly [10]. In this article, I offer mathematics teachers tools they can use in their classes. These include a variety of writing activities, and I make specific suggestions as to when to use each activity. A sample rubric is also included, to guide teachers in the grading process. Specific examples of student work illustrate the methodologies.

An Instructional Framework for Mathematical Writing

When working with mathematics teachers, I often see their students memorizing and regurgitating facts, figures, and processes. On a rare occasion, a teacher will ask a student to explain a mathematical concept, but the vast majority of class time is spent on calculations. Calculations are of course important, but they shouldn't be the primary activity in a mathematics class.

Writing-to-learn activities have students analyzing their mathematical knowledge, evaluating what is important to know, and creating a piece of writing that conveys the critical thinking that has taken place. Writing is a constructivist activity. Through appropriate writing exercises, students' mathematical knowledge is not only reinforced, but there is also an opportunity for students to construct new meanings from that knowledge [7].

2. Theoretical Framework: What is Mathematical Writing?

In its most basic essence, writing is an organized collection of symbols that conveys meaning, and that meaning is dependent on its context [5, 11]. For example, "−" is a symbol. Without context, it may be seen as only a mark on a surface. However, when it is used in "decision−making", it means "hyphen". In a different context, "3 − 2", the meaning changes. It now means "subtraction".

Symbols delegate complex cognitive tasks to an external environment [14]. Using "−" again as an example, subtraction is a cognitive task. It is a thinking activity of taking away of one value. Before understanding what "−" means in a mathematical context, the reader must understand the concept and activity of subtraction.

Mathematics is based on a system of writing, though this writing is not used to transcribe or represent other languages, such as English or a mother tongue [9]. Modern science, technology, engineering, and mathematics all rely on writing systems [6]. Most of us do not complete mathematics problems only in our minds. Furthermore, symbols allow us to convey these concepts and processes to others.

Mathematical writing consists of more than words. It also involves symbols and images. The intertwining of these three aspects of mathematical writing makes communication possible between the students and their audience. Students who are successful at mathematics are able to understand mathematical language and manipulate and apply mathematical meanings to the symbols, images, and words [11].

Mathematical writing is multi-dimensional, taking on more than one form [3, 4, 12]. While there is no steadfast definition of mathematical writing [7], we can view it as a thematic condensation of terms, symbols, and images to convey mathematical knowledge and meaning. Here we use the phrase *thematic condensation* to signify that "the whole implied activity can be qualified and related to other activities in a highly condensed manner" [6, page 60]. A mathematical example would be the equation $2x + 3y = 4$. In order for condensation to take place, there should be symbols, images, and words whose meanings are agreed upon by everyone in a given community. In mathematics, thematic condensation can exist in several ways: *symbols*, *nominalizations*, and *images*.

Symbols are the essence of math. They are objects that convey meaning, and this meaning is contingent on the social context and accepted discourse. Symbols condense meaning to its smallest component [6, 13]. For example, "x^2" uses two symbols ("x," "2"), and it represents the squaring of an unknown variable. In order to understand this combination of symbols, the reader needs to understand the concepts of variables and exponents.

Nominalizations are words that are used to convey complex processes and concepts [6]. Often used in journalism, nominalizations perform a natural role in mathematics too [1]. For example, the term "z score" "measures the number of standard deviations a raw score x lies from the mean" [2, page 343]. In order for the nominalization to make sense to students, it is necessary for the students to understand the concepts behind the terms.

Images can be diagrams, pictures, graphs, tables, or other forms of representation that are not considered to be symbols or nominalizations. Widely used in mathematics, the most common image type is graphs [9].

3. Writing Activities for the Mathematics Classroom

Here I share a handful of activities I use with my mathematics students. When I use these exercises, the students are often quite vocal about how "this isn't English class." However, writing is everywhere, and students need to be able to transfer the skills from one content area to another one. There might also be some reluctance from mathematics teachers themselves. But before they think of "reminding" the rest of the world that they "aren't English teachers," mathematics teachers might realize that they know what is considered to be "good" or "bad" writing. For example, proper paragraph and essay structures need to be in place. Another important component of good writing is of course clarity, which encompasses a variety of skills from proper use of vocabulary to grammar and mechanics. If any writing is difficult to read, for whatever reason, then it is not clear.

The writing exercises described here have worked in mathematics classes, specifically algebra, geometry, and statistics. For each methodology, I give a short explanation of the nature of the activity. Then I describe the procedures for that activity. At the end of this section is a sample rubric that teachers can use when grading the writing assignments.

3.1. Explanatory Essays

Explanatory essays require the student to explain a mathematical/scientific process in an essay format. For each concept/process, students need to first explain the steps or logical parts of the concept, and then they need to give a rationale for their explanation. Therefore, students need to explain not only the "What" of a process/concept but also the "Why."

- Give students a mathematical problem to solve. This activity works well with algebraic problems.
- Have students solve the problem. At each step of the problem, students need to note, in words the process that they used and the rationale for that process.

- Have the students explain the problem that they just solved in an essay or in long paragraph format. They should refer to their notes.

It is very important that students use transition words such as *first*, *second*, *third*, *finally*, and so on, so that their steps reflect the logical procession. Depending on the ability level of the students, this activity can take place in one 45 minute class period, or it may take two class periods (one day to solve the problem and write notes and one day to write the explanatory essay)

In Appendix A I share an example of an explanatory essay from a seventh grade mathematics student.

3.2. Narrative Essays

Narratives are stories which can be based on fact or fiction. In general, students have a difficult time incorporating their mathematical knowledge into a story. However, once they get started, they get excited to finish it.

- Give students a list of vocabulary words or a mathematical/scientific problem and have them write a story incorporating those words or problem.
  - For vocabulary words, students need to use the mathematical meanings of the words and incorporate those meanings into the text.
  - For the mathematical/scientific problem, students need to write a story that incorporates the problem and its solution.

The topics of narratives do not need to be mathematical or scientific, but the mathematical and/or scientific concepts need to be clearly established within the text. I have found that it is best to have two to three students work on one story. It usually takes 90 minutes, in class, to have a decent story (two 45-minute classes or one 90-minute block).

In Appendix B I share an example of a narrative essay incorporating vocabulary from a geometry class. This narrative from a tenth grade student was four pages long; only the first page is included in the appendix. Note that all of the vocabulary words are underlined.

3.3. Argumentative / Persuasive Essays

NCTM requires students to be able to take a stance and defend their position of that stance [8]. In order to defend a position, a student needs to fully understand the concept and identify the positive and negative aspects of it. In this activity students are asked to argue that one topic or concept is better than another using mathematical reasons.

- Put students in groups of four and give them two mathematical topics/concepts, one topic per two students. It's best if the topics / concepts are similar (for example, Addition Property vs. Multiplication Property).
- Have the two students write a persuasive essay arguing that their topic or concept is better than the other.

This assignment can be conducted as a debate. It usually takes 135 minutes to complete (45 minutes to organize the argument, 45 minutes to write the arguments, and 45 minutes to present the arguments).

3.4. Role Audience Format Topic (RAFT)

In this activity, the *role* is the student who writes to the *audience*. A specific *format* is given, and usually, it is one which is familiar to the students. Finally, the writing exercise needs to focus on a designated *topic*. Depending on the ability level, this activity can take one 45-minute period or two class periods (one day to organize their ideas/concepts and one day to construct the format).

Here are three examples:

Example 1:	This activity can take the place of a VENN diagram.
Role:	t score
Audience:	z score
Format:	Letter
Topic:	Have t score explain to z score why they should go out on date
Example 2:	This example gives a real-life application to a geometric concept.
Role:	You (student)
Audience:	Teacher
Format:	An explanation
Topic:	Explain why it is easier to cut across the gym floor instead of walking around it

EXAMPLE 3:	In this example, students need to understand not only the parts of a geometric proof but also how everything works together.
Role:	Given
Audience:	Image
Format:	Letter
Topic:	Have "Given" explain to "Image" why they are inseparable

In Appendix C I provide a sample of writing for a RAFT exercise from a university student.

3.5. A Sample Rubric

The following is a sample rubric. Students automatically earn 60% if they make an honest attempt at the writing exercise. Then, there is a gradation based on the quality of the student's work. As stated earlier, all teachers know when writing is clear and easy to understand, and when writing is closer to a Soduku puzzle, a collection of letters, numbers, and other symbols that the teacher needs to decipher and put in logical order. They don't need to know grammatical and mechanical writing rules in order to determine if the students are clear in their writing. All teachers know what is incomplete or poor writing, and all teachers know when something is written well.

	0	6	7	8	9	10
MATH CONCEPT/ PROCESSES	Didn't bother trying	Made an attempt, even though it is incorrect		Answer is incorrect, but there is evidence of understanding the concept/problem		Everything is correct. Concept is clear. Problem is solved.
WRITING CONTENT	Didn't bother trying	Made an attempt, even though it is incorrect	50% of content is correct	75% of content is correct.	80% of content is correct.	> 90% of content is correct.
GRAMMAR/ MECHANICS	Didn't bother trying	Made an attempt, even though it is difficult to read.		Has errors, but it is readable and understandable.		Minimal errors. Clearly written and easily understood.

Total: _____/30

4. Implications and Reflections

The methodologies presented in this article are common activities in English classes. Moving these same activities into the mathematics classroom provides continuity for the students. This reinforces not only the writing skills that are taught in English classes but also the mathematical knowledge relevant to the mathematics classes. Across content areas, writing activities allow teachers to gauge their students' content knowledge.

I spent 15 years as a secondary-level English and mathematics teacher. Usually, my students performed at a low-average level and came in with little to no foundational knowledge in mathematics. When I got them as freshmen, I had to start with the basics: Addition, subtraction, multiplication, division. Fast forward 8 years, and now, I'm teaching statistics to doctoral students who haven't seen a mathematics class since their undergraduate days. In both situations, my students needed to learn/relearn the basics before moving to more complicated concepts. These writing exercises combined with calculation problems provided a means to assess their mathematical knowledge. When students, at any level, are able to explain the concept/processes in written form, they are more likely to take ownership of the topic and not keep that information in isolation. (In fact, I would give one of these assignments in lieu of an exam. Frankly, they're more fun to grade. I would much rather read a narrative based on a mathematics concept than a bunch of definitions that have been regurgitated.)

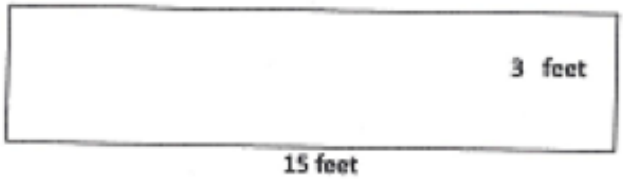
I urge mathematics teachers to get over their fear and insecurities about "grading" writing. We all read journal articles, and each of those journal articles effectively weaves symbols, nominalizations, and images into a coherent whole. Why are we not willing to teach what we read? Using the activities I present here, teachers can start experimenting with writing in their classroom. As mathematicians, don't we occasionally wonder how nice it would have been if someone had taught us the finer points of writing mathematics? Now, we can offer the opportunity to our students.

NCTM states that students need to be able to communicate using appropriate academic discourse in written form. Improved oral and written communication benefits everyone. Teachers and students can have better discourse within their academic communities, and more thoughtful and meaningful literacy can take place.

A. Example of a seventh grade explanatory essay.

Someone asks you to do this problem:

Find the perimeter of this rectangle



Once you find the perimeter, find the area of this rectangle.

Explain how you found your answer. Tell why you took the steps you did to solve the problem. You can use the back of this sheet, if you need more space.

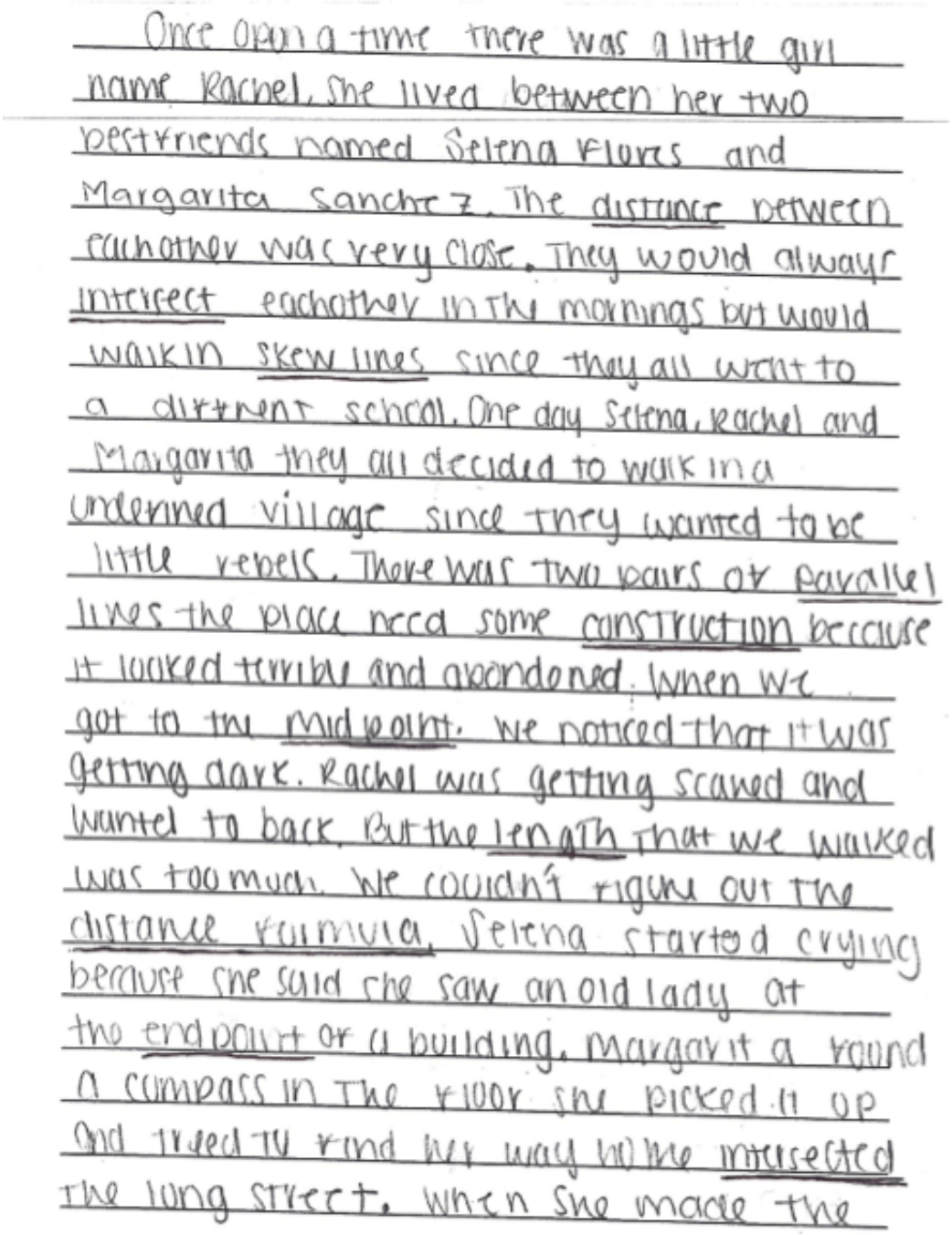
First, I know that the perimeter is all of the sides added together. So next, I also know that if one side of a rectangle is one number, so is the parallell side. So the longer side is 15feet and the shorter side is 3 feet. 15+15=30 3 and 3+3=6. 30+6=36feet. So the perimeter is 36feet.

First, I know area is length multiplied by width. Next, I know the length is 15feet and the width is 3feet. So I will multiply 15x3. So the perimeter is 45feet.

perimeter: 36feet

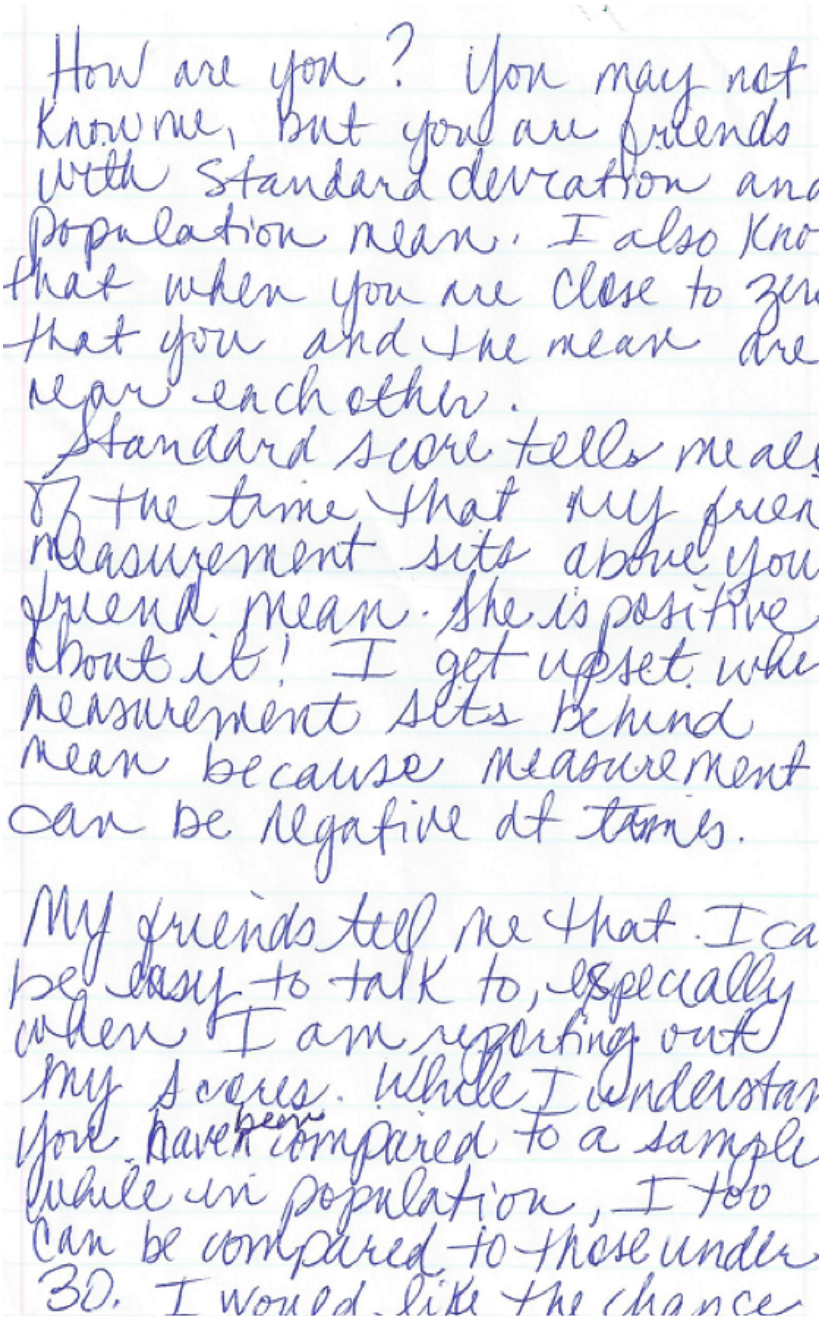
Area: 45 feet.

B. Example of a tenth grade narrative essay



Once Opon a time there was a little girl name Rachel. She lived between her two bestfriends named Selena Flores and Margarita Sanchez. The distance between eachother was very close. They would always intersect eachother in the mornings but would walk in skew lines since they all went to a diffrent school. One day Selena, Rachel and Margarita they all decided to walk in a underined village since they wanted to be little rebels. There was two pairs of parallel lines the place need some construction because it looked terrible and abandoned. When we got to the midpoint. We noticed that it was getting dark. Rachel was getting scared and wanted to back. But the length that we walked was too much. We couldn't figure out the distance formula, Selena started crying because she said she saw an old lady at the endpoint of a building. Margarita found a compass in the floor she picked it up and tried to find her way home intersected the long street. When she made the

C. Example of a university-level RAFT exercise.



How are you? You may not
know me, but you are friends
with standard deviation and
population mean. I also kno
that when you are close to zer
that you and the mean are
near each other.
Standard score tells me als
of the time that my frien
measurement sits above you
friend mean. She is positive
about it! I get upset whe
measurement sits behind
mean because measurement
can be negative at times.

My friends tell me that I ca
be easy to talk to, especially
when I am reporting out
my scores. While I understan
you have been compared to a sample
while in population, I too
can be compared to those under
30. I would like the chance

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