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Review: J.R. Porter and R.G. Woods, Extensions and Absolutes of Hausdorff Spaces (New York, Berlin, Heidelberg, 1987)

Melvin Henriksen
Harvey Mudd College

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WILLIAM ABIKOFF
UNIVERSITY OF CONNECTICUT

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Extensions and absolutes of Hausdorff spaces, by Jack R. Porter and R. Grant Woods. Springer-Verlag, New York, Berlin, Heidelberg, 1987, xiii + 856 pp., $89.00. ISBN 0-387-96212-3

The book being reviewed is designed for advanced graduate students and scholars who want up-to-date information about an important part of general topology. What this latter means is less than clear since the word "topology" is used differently from words like "algebra," "analysis," or "geometry" which also attempt to act as signposts for branches of mathematics. The *Journal of Algebra* publishes articles on groups, rings, and other parts of algebra. Most journals devoted to analysis have space for papers on real analysis, complex analysis, or differential equations, and similar statements can be made about geometry. This kind of integration contrasts with a de facto apartheid as far as topology is concerned.

There are three journals widely circulated in the United States devoted to topology. The journal *Topology* publishes articles in algebraic topology. *Topology and Applications* (formerly *The Journal of General Topology*) and *Topology Proceedings*, on the other hand, contain almost exclusively articles on general or geometric topology. This latter journal is reserved for articles presented at the annual Spring Topology Conference with an attendance of two to three hundred which only rarely attracts an algebraic topologist. There is nothing unusual about a conference attracting only specialists in a particular area, but often, in the case of topology, no qualifying adjectives seem to be used to describe its real nature. Whatever the reason, there seems to be a topological Tower of Babel.

There would seem to be three branches of the topological tree; indeed, until a few decades ago, there were about four—but what used to be called combinatorial topology is now, for the most part, subsumed under the title "graph theory" or absorbed into algebraic topology. At the center one finds geometric topology, the kind you describe to lay people who ask you to tell them what topologists do. While geometry is the study of properties of "objects" that remain invariant under rigid motions, topology is the study of properties that remain invariant under arbitrary one-one bicontinuous transformations. Geometric topologists generally confine their studies to spaces that resemble subspaces of Euclidean spaces or Hilbert spaces at least locally, and they do regard topology as a generalization of geometry. Algebraic topologists cover a lot of the same territory while putting more emphasis on (locally) Euclidean spaces, often with richer structures.

(For example, they study manifolds with differentiable structures.) They associate algebraic structures (usually groups) with the topological spaces they study and use them to get a great deal of information. General (alias point-set) topologists concern themselves more with "big" classes of spaces. They too start out with metric spaces, but soon find themselves working with subspaces of arbitrary products of metric spaces and images of them under various kinds of mappings. They find themselves forced to consider separation axioms; indeed, they worry more about axiomatics than other kinds of topologists.

Topologists are not in high demand in the American academic marketplace today, and most of the time when vacancies occur, it goes without saying that "topology" is shorthand for "algebraic topology." Geometric toplogists are the next most favored, and without doubt, general topology is in bad odor. To me, the most convincing evidence of the latter is that so few American mathematicians have heard of Eric van Douwen, who died suddenly in 1987 at the age of 41 after establishing himself as one of the most brilliant general topologists of this century. Also, in the last fifteen years, I have had experience with two unspecialized mathematical journals whose editors refused to send papers in general topology to a referee, while not advertising this policy publicly. There remain, however, quite a number of American institutions where workers in general topology are welcome, and in Eastern Europe general topologists continue to outnumber their algebraic counterparts, so the field remains alive and reasonably well.

While the book under review is addressed to specialists and does not stress directly applications to other parts of mathematics, it does stress those parts of general topology most applicable to functional analysis. Although it is not mentioned in the introduction, this book is a descendant of Gillman and Jerison's *Rings of continuous functions* **[GJ]** despite the fact that rings play only a small role in its development. This is the case because many of the concepts in the book arise because of the important role they play in the study of the ring $C(X)$ of continuous real-valued functions on a Tychonoff space X. For example, a central notion in the book is an *extremally disconnected space*, i.e., one in which the closure of every open set is open. This concept, which is pathological from the geometric point of view, arose because X is extremally disconnected if and only if every bounded subset of $C(X)$ (as a lattice) has a least upper bound. As a result, quite apart from its main purpose, it is a valuable reference book for those eager to find out the developments in this kind of general topology since the publication of **[Wa and We]** in the early 1970s without conducting an extensive literature search.

A space Y is called an *extension* of a space X if X is a dense subspace of Y. One looks for "nice" extensions of spaces that are easier to study with a view to getting information about the original. Compactifications and realcompactifications of Tychonoff spaces, and H-closed extensions of Hausdorff spaces are among the more important ones considered. (A space is said to be H-closed if it is closed in every Hausdorff space containing it. Both the authors and I adopt the convention that spaces considered are assumed to be Hausdorff unless the contrary is stated explicitly.) In 1958, A. Gleason showed that the projective objects in the category of compact spaces and continuous maps are the extremally disconnected spaces, and given a compact space X, there is a space EX in this category mapping irreducibly onto X. In 1963, S. Illiadis generalized this result to the category of Hausdorff spaces and perfect θ-continuous maps, and the authors call EX the (Illiadis) *absolute* of X. (If one is content to stick to regular spaces, it is enough to consider perfect continuous maps.) EX is a very useful tool enabling one to "lift" many problems from X to EX to take advantage of the special nature of extremally disconnected spaces, and then apply the results obtained to X. The purpose of this book is to develop the properties of extensions and absolutes while showing how valuable they are as a tool.

The authors achieve their goal in this excellent volume written with great care. Its value is increased immeasurably by their decision not to head towards it in the most direct way possible. Chapter 1 gives a review of the topology needed for the sequel, and the reader is encouraged to supplement it with **[Du, En, or Wi]** if necessary. Chapter 2 is about lattices and filters on them; for the most part, the members of the lattice belong to various families of subsets of a topological space. Boolean algebras and their completions are introduced in Chapter 3 to help set the stage for the study of absolutes. It includes a section on Martin's axiom and the continuum hypothesis so that some recent solutions to old problems which require more that Zermelo-Fraenkel set theory and the axiom of choice might be included in the sequel. Extensions are introduced in Chapter 4 with an emphasis on compactifications and H-closed extensions, and Chapter 5 is devoted to *maximum* P-extensions for various topological properties P. T is a *maximum* P-extension of a space with P if T has P, and every continuous map on X into a space Y with P has a continuous extension over T into Y. Chapter 6 is concerned with extremally disconnected spaces and absolutes, and in Chapter 7 the deeper properties of H-closed spaces are explored. The contents of Chapter 8 probe more deeply into absolutes and some generalizations, while in Chapter 9, these are placed in a categorical perspective.

Each chapter is supplemented by between 13 and 34 multiple-part problems in the style of **[GJ]**, which together with thorough historical notes increase the scope of the book greatly. An extensive bibliography and an index are included. (The latter, while better than most, seems frustratingly inadequate in view of the almost gigantic scope of this volume; especially for those of us spoiled by the extraordinarily good index in **[GJ]**.)

In summary, this book contains not only an excellent treatment of extensions and absolutes, but also covers a large amount of important material bordering on them. As a result, it is indispensable to any scholar with a serious interest in general topology.

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MELVIN HENRIKSEN
HARVEY MUDD COLLEGE

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Introduction to Arakelov theory, by Serge Lang. Springer-Verlag, New York, Berlin, Heidelberg, 1988, x + 187 pp., $49.95. ISBN 0-387-96793-1

In the beginning (or shortly thereafter) there were complex projective surfaces. These algebraic subsets of $\mathbf{P}^n(\mathbf{C})$ of complex dimension 2 were extensively studied by Italian geometers, such as Castelnuovo, Enriques, and Severi, during the late nineteenth and early twentieth century. It soon became apparent that the key to understanding such surfaces is to study the curves which they contain. Thus if $X \subset \mathbf{P}^n(\mathbf{C})$ is a smooth surface, one looks at the curves $C \subset X$; and, more generally, one looks at the free abelian group generated by these curves, which is called the *group of divisors on* X and is denoted

$$\operatorname{Div}(X) = \left\{ \sum_{C \subset X} n_C[C] \colon n_C \in \mathbf{Z}, \text{ almost all } n_C = 0 \right\}.$$

Associated to a rational function f on X is its set of zeros and poles; taken with multiplicities, these zeros and poles give a divisor. Two divisors $D_1, D_2 \in \operatorname{Div}(X)$ are called *linearly equivalent* if their difference $D_1 - D_2$ is the divisor of a function.

Given two distinct curves C_1 and C_2 on X, one can count the number of points where they intersect (with multiplicity, if the intersection is not transversal). Extending this intersection index linearly to $\operatorname{Div}(X)$ gives the *intersection pairing*

$$\langle\ ,\ \rangle\colon \operatorname{Div}(X) \times \operatorname{Div}(X) \to \mathbf{Z},$$