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Problems That Matter: Teaching Mathematics as Critical Engagement

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INTRODUCTION

This paper is the result of a struggle to understand what it is about what I do that *really* matters. It started two and one-half years ago when I first became a high school mathematics teacher in Miami, Florida. More specifically, it started with the first of countless times my students asked "Mr. Bohl, why do we have to learn this?" This question has since become the focus of my thinking about mathematics education. In two years of teaching (including summers) I had the opportunity to work with a broad range of students, from classes of advanced college-bound students to a program for ninth-grade repeaters. The question 'Why?' came largely from students in the lower tracks, and I found it disconcerting that I could often not answer this ultimately important question. Ironically (or perhaps not?) those who were succeeding in school, the 'best students,' didn't seem to be as concerned with 'Why?' I wanted all of my students to be inclined to ask 'Why?', and I believed that they all deserved a satisfactory answer whether they asked it or not. This paper is an exploration of how we might reconfigure mathematics education to answer the 'Why?' question for all math students.

STANDARD ANSWERS TO THE 'WHY?" QUESTION

As time passed, I became increasingly frustrated with the onslaught of unanswerable 'Why?'s, the irrelevance of the materials available to me, and my very spotty success designing more relevant curricula. So I began asking other math teachers why they thought certain subjects and concepts were taught. Their answers fell into four categories: tomorrow, jobs, general mental strength, and tests. In the first category are answers of the type: "because they'll need it for tomorrow," "...for the next chapter," "...for next year," etc. When pushed further, this often resulted in "they'll need it for calculus." To some, then, we were teaching mathematics for the sake of other mathematics, with the ultimate goal being a year of calculus by high school's end. When I asked why students needed calculus at all, usually an answer in one of the other three categories was given.

The 'jobs' answer was obviously predicated on the belief that the best jobs require high levels of math. There can be no doubt that mathematics has economic utility. It often serves as a filter, or a base requirement, for jobs. Thus, those who do not succeed at a certain level of mathematics course work can be blocked from consideration for some jobs. However, the mathematics that most people actually use at work is probably taught by the middle of ninth grade. And the percentage of people who actually use calculus on the job is fantastically small. So, contrary to the myth, much of school mathematics has little actual vocational utility.[1]

The answer that 'mathematics improves general mental strength' is predicated on the belief that doing mathematics improves formal logical reasoning and problem-solving in a broad way. That is, mental processes are improved by math practice, and then become available for use in other situations — whether mathematical or otherwise. While mathematics training may or may not strengthen the mind generally,[2] it has been shown time and again that standard mathematics curricula are not good mathematics training. Most students lack facility with even the simplest real-life problem solving.[3] So here again what is being claimed has a questionable relationship to reality.

Now, I support math education that is vocationally useful, and that might make students stronger thinkers. However, the reality is that the path to calculus that we attempt to lead students down does neither of these well. There is one thing that this path does very well, however. It puts students into hierarchical lists that universities and employers use to simplify their choices of student and employee candidates.[4] When teachers respond to 'Why?' with the name of a test, it is because they understand the importance of a student's position in that queue. It is a very real and valid concern. In the bigger sense, however, if most students get little vocational or logical power from the ways we currently teach mathematics, then the way that mathematics matters most now is as a major part of our society's publicly-funded human sorting service. And even though some do benefit from the system, all students are cheated of the potential critical power that a strong mathematics education might help them develop.

I do not mean this exploration of answers to 'Why?' as an attack on teachers. The assumptions we educators call upon to justify our practices partly mirror those of school systems themselves, of which we are all products. I started realizing the power of these assumptions while muddling through attempts to develop curricula that I felt might matter to my students in ways other than as a sorting device. This led me to consider my own assumptions about why mathematics is important for students to study, and to imagine how my practice might be brought more in line with those assumptions.

WHY DOES MATHEMATICS MATTER?

My beliefs about why math is important are informed by three basic ideas. The first is that mathematics can give students a powerful way to relate to the world, not the mythical world of future jobs where they will utilize calculus, but rather their immediate world — the world that they actually inhabit during the time they are students, and that they will continue to inhabit after graduation. Students deal with situations, concerns, and activities every day that are rich with mathematics. They are bombarded with numbers from jobs, stores, ad agencies, the government, etc. Mathematical knowledge can be used to help students analyze and raise questions about such numbers and their implications, as well as to use numbers to understand the world in different ways.

The second reason — simply an extension of the first — is that mathematics knowledge is necessary for full participation in our democracy. The one way that mathematics knowledge (or lack thereof) will bear directly on the life of every student is in her/his role as citizen. Numerical data and mathematical models are integral parts of our reality.[5] They are used every day to decide such things as how many Americans need to be kept unemployed to ensure a "healthy" economy and how many dollars a human life is worth to an insurance company. Those who make such decisions wield great power to shape the reality that we all experience. In highly technical societies such as ours, mathematical competence is a major portion of democratic competence.[6] Math is increasingly used as a means of developing technology and directing public policy.[7] Even though most people will not use advanced math on their own jobs, all people need to be prepared to evaluate the work of those who make such decisions and to engage with the mathematical aspects of important social issues.

The third reason — a further extension on the theme — involves the relationship between mathematics and rational thought. Thanks in large part to Descartes, rational thought is widely accepted as the only worthy mode of cognition in western societies.[8] As a result, rational argument is, at least theoretically, the only accepted mode communication in public debates. Being able to rationally justify positions is a skill needed for individuals to gain public validity for their ideas. Thus, it is critical for citizenship. There is a clear tie between rational argument and the logical justification of mathematical results. Indeed, deductive mathematical proof is considered the purest type of rational argument. While the two are not equivalent, there are similarities that could be capitalized on by honing students' understandings of the specific structures of deductive logic. Thus, mathematics education might further enhance students' power as citizens by helping them make and critique rational arguments.

These overlapping justifications for my job, which will be expanded on in the following sections, have brought me to believe that we should teach mathematics that matters in two senses: it should matter to students and their immediate lives, and it should matter to the imperative of democratic citizenship.

MATHEMATICS THAT MATTERS TO STUDENTS

I believe that we need to help students learn to engage mathematically with their immediate worlds. Traditional mathematics curricula have not been successful at doing this because math is traditionally taught in a largely formal way. That is, it is taught without reference to the objects of people's real experiences. Such teaching stems in part from the beliefs that math is, by it very nature, abstract, and that it is math's abstract nature that allows it to transfer — or to be used — across a variety of concrete, real-life situations. This conception of transfer has been called into question,[9] and there has been some movement away from strictly formal learning in current reform trends.[10] However, most mathematics is still taught in ways that are artificial to students. Even curricula that claim to be 'realistic' are not sufficient. There is a gulf between teaching 'realistic' mathematics -which is word-problem- and situation-based — and 'real' mathematics, which actually involves the lives and interests of the students in the classroom.[11] It is through 'real' mathematics that I believe teaching should take place.

Mathematics education based on a real problem curriculum would directly involve students in exploring their worlds with math. We know that students have interests, but they normally do not become part of mathematics classes. Bringing students' lives into class can help on a very basic level: learning can happen far more easily when students see the direct relevance of what is being learned.[12] My experience has been that, when students' contexts were being studied, students involved themselves more actively, and I could concentrate on their intellectual development rather than on behavioral manipulation.

Since I believe that school mathematics should be geared toward helping people interact with each other and the world, this* is *a call to teach entirely applicable mathematics.

Teaching mathematics based on real contexts and situations familiar to students serves another important purpose. It allows curriculum to respect and capitalize on the rich collections of personal and cultural knowledge that students come to class already possessing. Schooling generally disregards and devalues much of students' personal and cultural experiential knowledge.[13] And, because of differences in their relationships to the dominant school culture, students from non-dominant cultures are especially mis-served by schools.[14] Opening the starting points of mathematical explorations to the concerns and interests of students can allow math teachers to become part of the remedy to this situation by broadening the bases of curriculum to include students' lives.[15] Multiple interests, concerns, and viewpoints could then be allowed a place in classroom discussion. Of course, all students do not share interests, concerns, and viewpoints, and opening the classroom to a multiplicity of voices invites in 'negative' along with 'positive' influences. This can greatly complicate classroom interaction.[16] However, since such complications are part of the reality with which I would like to help students engage, I prefer to incorporate them into, rather than exclude them from, the classroom discourse.

Teaching such mathematics would involve starting with particular real situations of interest to the students, and mathematizing them. Mathematizing involves gaining understandings about real situations by using mathematics.[17] Pedagogically, I like to think of it both as using math to uncover patterned relationships, and as imposing mathematical order on unordered realities. So to mathematize means analyzing a real situation either by mathematically modeling its components, or by quantifying its characteristics with statistics. By mathematizing the life contexts of particular students, and by using such mathematizations as the bases for learning, it becomes possible to inform the mathematics with the ideas and cultural constructs that students already possess.[18] This is not a call for a curricular add-on, but rather for a deep shift in our thinking about the relationship between mathematics and people's lives. Such a shift might happen if we take the lives and world views of all students seriously.[19]

I can hear the formalist questions arising: "What kinds of mathematics can be taught this way? That is not mathematics at all, but mathematics applications."[20] My answer to that charge is: well...yes and no. Since I believe that school mathematics should be geared toward helping people interact with each other and the world, this *is* a call to teach entirely applicable mathematics. However, that does not mean that mathematics need never be addressed at the formal level. There is a great variety of mathematics that can be soaked from and used to analyze even the simplest real situations. And there would no doubt be times, as multiple real contexts are mathematized, that formal mathematical issues would need to be addressed.[21] It is, after all, the exploitation of similarities of pattern across situations that gives mathematics its power. So, in attempts to help students comprehend that power, there would need to be explorations of the similarities between the patterned aspects of different real situations. That is exactly how much of mathematics was historically developed in the first place.[22] So such abstraction and pattern seeking should certainly continue as one goal of mathematics education. My point is that mathematical abstraction should not be viewed as the only goal of mathematics education. Its importance needs to be reconsidered.

What I am arguing for is that mathematics be taught through mathematizing (or 'making mathematical') rather than through concretizing (or 'making concrete').[23] Traditional school mathematics, especially from the onset of algebra, introduces concepts and objects at the abstract level, and then gives concrete examples of them. That is, it starts with abstractions and concretizes them. What we need to do is reverse that priority, put concrete examples in the foreground, and build abstractions from those through mathematization.[24] Now, as contexts are mathematized, and as teachers and students work to formalize some of the mathematics that arise from those mathematizations, certainly some concepts that are currently taught would fail to show up. In those cases, I suggest we would need to rethink their curricular importance. That is not to suggest that concepts that don't arise from students' situations and interests should never be taught in schools. However, with a curriculum that is driven by calculus — a math that few people would ever have reason to use in real life — it is obvious that we need to rethink our curricular priorities. The dilemma posed by mathematics that are absent from mathematizations of reality would make a fine starting point.

By starting with the concrete and then moving to the abstract, we side-stepped a tension that arose earlier when students that I first taught abstractly tried to 'apply' the mathematics.

I learned the value of using students' contexts as the basis for teaching through two experiences teaching the graphing of points, lines, and functional relationships in the x-y plane to pre-algebra students. The first time through was with a group of middle track students. I started with traditional methods, including a few graphing games, to familiarize the students with plotting in the abstract x-y plane. Then we used tables and simple calculations to graph patterns and lines. We finished up with activities that were a bit more realistic, using function-like calculations to answer questions based on local bus travel and other familiar situations. What surprised me is that the concept of plotting points never seemed to make sense to most of the students, and the simplest graphing was a haphazard undertaking even at the end of the nearly two-week-long unit. And since they didn't understand how to graph in the x-y plane, graphing on different types of axes (for instance graphing *time worked* and *dollars earned*) was very difficult for most to grasp.

The next time I taught this was with students in a special program called School Within a School (SWAS) for the school's repeating ninth-graders. We happened to be in the middle of hurricane season and hurricane Opal had just done a dance around our end of the state. To introduce graphing, we started with hurricane maps. The coordinates to plot the location of the hurricane du jour were published daily, along with information about wind speed, velocity, and direction of travel. We first learned how to find locations by plotting Opal, by plotting and reading the plots of several fictitious hurricanes, and by exploring the daily changes in direction and speed and how those appeared on the graphs.

From there, we tied the ideas of latitude, longitude, and compass direction into the structure of Miami's grid-like street map. In Miami, all east-west roads are called streets and north-south roads are called avenues, and each is sequentially numbered starting with zero downtown. This means that the street address of any building includes all the information needed to go there. Now, all of my students knew exactly how to find places by their addresses. We used their knowledge of the address system, along with the similar situation of hurricane mapping, to develop an intuitive understanding of the abstract idea of plotting points in the x-y plane. With that solid base, we continued on to explore tables of values and graphs of functional situations. This time the students, who for the most part had long records of poor math performance, didn't flinch when we switched from plotting (x , y) points to forms such as (hours worked, dollars earned). By that time they were familiar with several types of points, e.g. (latitude, longitude) and (street name, street number). By starting with the concrete and then moving to the abstract, we side-stepped a tension that arose earlier when students that I first taught abstractly tried to 'apply' the mathematics.

Another important benefit of the fact that we were exploring their real world was that several discussions arose about differences between different areas on Miami's map. Although I didn't capitalize as much as I might have on discussions of disparity of wealth in different areas, we did touch a bit on that issue as well as others. Had I been more experienced, I'm sure I would have been able to weave such issues more tightly into the content. On the other hand, had I not used the students' real context as a base for teaching the topic, such issues would never have surfaced at all.

MATHEMATICS THAT MATTERS TO THE DEMOCRATIC IMPERATIVE

Teaching mathematics that matters to students' immediate lives is important. However, it also needs to be taught in ways that matter to the democratic imperative. I believe that all education should help improve students' qualifications as citizens, and mathematics is no exception. Many mathematics reformers, including the NCTM, acknowledge that mathematics knowledge is important for an informed citizenry.[25] What a functioning democracy needs, however, is not simply informed citizens. Because democracy is, in theory, about self-government, it requires active involvement to function correctly. What we need, then, are both informed and engaged citizens, who can engage intelligently with societal issues and debates once they become informed of them. Traditionally, the teaching of school subjects does not encourage such engagement.[26] Mathematics, with its ethos of abstract detachment, tends to be the worst offender of all on this measure. And, as mentioned above, there are numerous ways in which mathematical competence is necessary for intelligent engagement with today's important social issues.

If one accepts that we should teach mathematics by mathematizing students' contexts, the next question is: how can we make such mathematics democratically important? Doing that requires, whenever possible, mathematizing situations that involve socially relevant issues that students can engage with.[27] It also requires using the process of mathematical engagement as the basis for making judgments and taking actions based on those judgments. This can help prepare students to become the confident question posers, problem solvers, and mathematical/rational communicators that our democracy requires.[28] In this sense, mathematics classrooms can serve as places where students actually enact democratic principles through the practice of democratic citizenship. An example of this is another unit I taught to my SWAS ninth-graders. This unit was designed to introduce both descriptive and inferential statistics, and was a first attempt at fully enacting my beliefs about the need to teach both personally and socially relevant mathematics.[29]

Inferential statistics is the statistics through which inferences are made about entire populations from a few representatives of the population; unfortunately, it is not generally taught in K-12 curricula. This unit was designed to teach it because inference is what gives statistics nearly all of both its strengths and weaknesses. It is a critically valuable mathematics for citizens, but is usually never seen by students, especially like those in SWAS who were not bound for any mathematics beyond geometry or Algebra II.[30]

We started with reading statistical graphics from newspapers and discussing the information they represented, the questions that could be asked about them, and the means by which the information may have been gathered. We then created a survey to be taken anonymously by members of the school's student population. The choice of survey questions was left to the students. This helped gain great interest, and resulted in some of the most engaging discussions we'd had all year. The data we gathered, along with the data from several smaller surveys the students administered, were analyzed as we explored the ideas of populations, random selection, and inference; created graphic representations of the data; and discussed how confident we could be about our inferences.

As a culminating unit project, each student had to create and administer her/his own one-question survey on a topic s/he felt was socially important. This involved deciding on a target population, determining how to obtain a random sample of it, and then writing up a short report with mathematical justifications for the inference made. The students were also to share that report with someone in a position of authority who they thought should be familiar with the knowledge they had created.

We experienced all the problems attendant with teaching something the first time, the end-of-the-year jitters, and working with students that the school system had miserably failed. Even so, the unit was as mathematically successful as anything else we had done. This itself was a victory given that the mathematics we were dealing with was of a much higher level than usual.

Regarding teaching mathematics that matters to students, we worked with topics that were of immediate concern to them. We used the mathematics we were learning as a means of gaining deeper understandings of their immediate surroundings. This was not a "teach them now so they'll know how to use it later" approach. Rather, it was "learn as you do." The real-life basis for what we learned allowed us to have very thoughtful discussions about student interests and concerns and gave us a place to ground the more abstract mathematics we were exploring.

In terms of informed and engaged citizenship, simply mulling over the data we collected made the students aware of things they'd not known before about their environment. We used the data to engage in discussions about how statistics might be used to deceive and what the requirements for making valid inferences are. Much to my dismay, bad planning meant that students did not have time to report their findings to a figure of authority. However, the requirement to do so did actively engage many of them, including several who otherwise had shown little interest in the class all year. Many designed their surveys to address questions of specific relevance to certain authorities. As examples: one surveyed the student body so she could let the new principal know how students felt about his first year's performance; another surveyed pregnant teens in her housing complex about reasons for getting pregnant so she could report it to the school's health clinic counselors to help them better counsel girls about pregnancy; and a third surveyed male students to find out how they felt about teen fathers' role as parent so he could inform the guidance counselors of males' thoughts on the topic.

This unit offered the students a small experience with creating knowledge about something that concerned them, and putting it in an 'officially sanctioned' form that allowed them to participate in the discourse of authority. All students deserve to have such opportunities, and the imperatives of our democratic technological society demand that they do. Given that mathematics is a major part of our society's official mode of discourse, students must have experiences where they learn to be comfortable utilizing mathematics to communicate within it.

RATIONAL ARGUMENT AND MATHEMATICS

In our highly rational and scientific world, mathematical logic serves as the prototypical means of finding and proving truth. It is also very closely related to the rational language of the public sphere. Helping students master the art of logical argument has long played a role in mathematics education. Geometric proofs have usually served as a means of introducing students to this art. However, this is one area of mathematics that is absolutely never related to the real world. Proofs are the ultimate source of mathematics' assumed power of abstraction and generalization, but they never refer to things that actually exist, only to abstract mathematical objects.

Being able to rationally justify one's positions is an important part of gaining validity for one's thoughts in our rationalistic society. It was with this in mind that I made the decision not to teach formal proofs when I taught geometry. Instead, we focused on learning how to write coherent and rigorous paragraph justifications for solutions to specific problems. My feeling was that if students could learn the structures behind written justifications for specific solutions, they would be a step ahead in the work of learning to rationally justify other thoughts as well. In my classes many students excelled at this, even some of whom were struggling in other areas. Such justification should not wait until geometry, however. It should start in the earliest years of grade school mathematics.[31] In the statistics unit with my SWAS students, we did focus on what knowledge was necessary to be able to confidently make an inference. This was a small but necessary step in learning the need to justify statements.

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One mistake I made while teaching paragraph justifications was that we never explored the real-world implications of rational thought in reference to objects other than mathematical objects. A key to mastering both mathematical deduction and rational justification is understanding that what you can conclude depends entirely on what you assume. Mathematical work is based on logical movement from a set of assumptions to a set of conclusions. Because of the strict logical structure of mathematical argument, the assumptions entirely dictate what can be concluded.

In everyday rational argument, similar logical rules apply, at least theoretically. However, in everyday argument assumptions are not usually the focus of scrutiny. Often, when someone does or says something that doesn't make sense to us, we take for granted that the speaker/actor is not making sense at all. This presumption is very often wrong. Often, when someone's reasoning doesn't seem rational or reasonable, it is simply because s/he is working from a different set of assumptions. As with mathematics, two people can make perfectly sensible arguments that result in opposite conclusions if the assumptions they make are different.

Much misunderstanding between different individuals and groups of people in our society results from inattention to assumptions. Learning to question the assumptions that allow people, including ourselves, to arrive at the conclusions we do is an important step in being able to take a critical role in rational discourse. Mathematics is, in part, the art understanding this relationship between assumptions and conclusions. Thus, the mathematics classroom seems like a fine place to involve students in learning to question the assumptions they and others make about issues and problems that matter.

QUALIFICATIONS ON MATHEMATIZATION

So far I've argued that we need to teach mathematics through the mathematization of real, socially relevant situations. There are certainly benefits for citizens of our highly technical democracy to learning through and about the processes of mathematizing. However, mathematization should be not be uncritically adopted as an all-encompassing mode of analysis or understanding. Students need to be exposed to these modes of analysis not only so that they can use and learn from them, but also so that they can take an active part in critiquing them.

Mathematics is only one of many ways of making sense of the world, and it is probably our most morally vacant. Much has been written on the negative effects of the detached, positivist, and essentialist modes of interpreting the world that have developed along with our ability to quantify and categorize.[32] The authority of numbers is pervasive in our society, and they are often used to gain authority for misleading, and even untruthful, analyses.[33] Earlier I described mathematizing as "imposing mathematical order on unordered realities." There are situations where such order helps us understand things we might not have otherwise. However, mathematizations can just as easily be used to bad ends as good. The determining factor is the set of assumptions made in the process of mathematizing.

As already discussed, assumptions largely determine conclusions, and with the complexities involved with mathematizing, there is certainly room for disagreement about which assumptions should be made and which should not. A prime example is the already mentioned SAT. Because I assume that achievement is not illustrated by one's ability to answer multiple-choice questions, I would not attempt to mathematize it by means of such a test (and perhaps not at all). Those who publish the test obviously make a different set of assumptions to arrive at the conclusion that the test is valid — a conclusion which they use statistics to 'prove.'

Students need to be made aware of the ways that math can harm as well as how it can help. In accepting mathematizing as a mode of mathematics instruction, it would be critically important to also accept the critiquing of mathematizing as part of that mode. Where would such issues be addressed if not in mathematics classrooms?[34]

CONCLUSION

Many people, including my former students, feel that school mathematics is irrelevant to their lives. This is not the result of their inability to comprehend reality. It seems to be simply a common sense recognition of the fact that, as it is currently taught, mathematics does not matter for most people except in its role as a sorting mechanism. We lead students to calculus when what they are exposed to in real life is statistics. This fact alone gives credence to people's questioning mathematics' relevance. If we want people to think that school mathematics is Important in their lives, we need to teach mathematics that actually is important.

These ideas are a preliminary sketch of ways we might reconfigure mathematics education so that it actually does matter in real people's real lives. The units described are some of my first attempts to do so. Obviously such approaches require a very different orientation toward mathematics than most of us math teachers, as products of schools ourselves, are familiar with. Hans Freudenthal, who pioneered work in the area of 'realistic' mathematics education, wrote that "Mathematics is an activity, a behavior, a state of mind... an attitude, [and] a way of attacking problems."[35] In order to take Freudenthal seriously, we need a more open-ended approach to mathematics education that requires deep involvement with real problems rather than simply the acquisition of skills that are never applied to real problems. As mentioned, however, realistic mathematics is not sufficient. Even if we approach mathematics as "an attitude, [and] a way of attacking problems," the question remains, whose problems are worthy of consideration?[36] Mathematics that is based on either abstractions or pre-defined general types of situations fosters classroom atmospheres that lock out students' experiences, concerns, and cultural backgrounds. It also locks out discussion of the types of issues that matter to our democratic society. We need to help students understand how to critically engage with the world mathematically, as well as how to engage critically with mathematics.

Mathematics educators need to have a good and honest answer to the question: Why do we need to learn this? If we want school mathematics to matter, we need to teach it as active engagement with problems that matter — both to students and to our democracy. If we don't care that school mathematics matter, we could continue teaching it based on the imperatives of calculus. Then we could rest assured that, in the future as now, much of the mathematics taught will not matter, and much of the critical mathematical power that could be developed in our society will lay dormant.

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[6] Ole Skovsmose, "Mathematical Education and Democracy." *Educational Studies in Mathematics* 21 (1990), 109-128.

[7] See Apple, "Do the Standards Go Far Enough", where Apple describes mathematical knowledge used in this manner as "technical/administrative knowledge," the control of which he posits is concentrated in the hands of a limited group of people. Making the distribution of that control more equitable is, in large part, the goal of the type of teaching described here.

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[14] Lisa Delpit, *Other People's Children: Cultural Conflict in the Classroom*. (New York: The New Press, 1995).

[15] See Gloria Ladson-Billings, *Dream Keepers*. (San Francisco: Jossey-Bass, 1994). Here Ladson-Billings discusses the importance of including students' lives in the classroom.

[16] Deborah Loewenberg Ball, "Transforming Pedagogy: Classrooms as Mathematical Communities. A Response to Timothy

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[20] Robert Thomas, "Proto-mathematics and/or real mathematics." *For the Learning of Mathematics* 16 (Number 2, 1996), 11-18.

[21] See Robert P. Moses, Mieko Kamii, Susan McAllister Swap & Jeffrey Howard, "The Algebra Project: Organizing in the Spirit of Ella." *Harvard Educational Review* 59 (Number 4, 1989), 423-443. The authors offer a fine example of using the experience of riding the subway to help students develop deep understandings of the abstract concept of negative numbers.

[22] See Anna Sfard, "On the Dual Nature of Mathematical Conceptions: Reflections on Processes and Objects as Different Sides of the Same Coin." *Educational Studies in Mathematics* 22 (1991), 1-36, Here Sfard gives short descriptions of the development of both the number and function concepts.

[23] Skovsmose, "Mathematical Education and Democracy," and Bill Rosenthal, "No More Sadistics, No More Sadists, No More Victims." *UMAP Journal* 13 (Number 4, 1990), 281-290.

[24] Christine Keitel, "What Are the Goals of Mathematics for All?" *Journal of Curriculum Studies* 19 (Number 5, 1987), 393-407.

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[26] Henry A. Giroux & Roger Simon, "Popular Culture and Critical Pedagogy: Everyday Life as a Basis for Curricular Knowledge." In Henry A. Giroux & Peter L McClaren (Eds.) *Critical Pedagogy, the State, and Cultural Struggle*. (Albany NY: State University of New York Press, 1989), 236-252.

[27] In "Mathematical Education and Democracy," Skovsmose discusses the seeming contradiction between teaching based on students' interests and teaching based on the democratic imperative.

[28] Tate, "Mathematizing and the Democracy."

[29] The statistics unit described was designed in collaboration with Bill Rosenthal. The collaboration was part of our ongoing efforts to understand how to best bridge the divide between critically-oriented university academics and progressive classroom teachers.

[30] Bill Rosenthal, "No More Sadistics, No More Sadists, No More Victims." *UMAP Journal* 13 (Number 4, 1990), 281-290.

[31] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*.

[32] See, for example, P. J. Davis & R. Hersh, *Descartes' Dream: The World According to Mathematics*. (New York: Harcourt, Brace Jovanovich, 1987), cited in Putnam, Lampert, & Peterson, "Alternative Perspectives on Knowing Mathematics in Elementary Schools."

[33] Stephen J. Gould, *The Mismeasure of Man*. (New York: W. W. Norton, 198 1), cited in Tate, "Mathematizing and the Democracy."

[34] Terezinha Nunes, Analucia Dias Schliemann, & David William Carraher, *Street Mathematics and School Mathematics*. (Cambridge: Cambridge University Press, 1993).

[35] Hans Freudenthal, IOWO- Mathematik Fur alle und Jedermann. *Neue Sammlung* 20 (Number 6, 1980) 634-635, cited in Keitel, "What Are the Goals of Mathematics for All?"

[36] Michael W. Apple, "Do the Standards Go Far Enough? Power, Policy, and Practice in Mathematics Education." *Journal for Research in Mathematics Education* 23 (Number 5, 1992), 412-431.

Platonism and All That...

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own research: yes, proofs I have invented, but the patterns which the proofs legitimate seem to have been there, waiting to be found. I have no idea what absolute reality is like, but I can tell you what it felt like to find these things.

And, so, back to Plato and his cave; the firelight casting shadows on the wall. We face the wall, and guess, if we will, what makes the shadows. Sometimes mathematics seems firm, unshadowlike. But sometimes the shadows waver. In *Proofs and Refutations*, Lakatos (1976) documents the wavering which may take place. He says we never know whether our proofs are right, but he believes we can be sure of their improvement. And what of Gödel? Undecidability promises that we will never come to the end of our search, because the choice amongst the undecidables will remain, and the absence of a consistency proof is the guarantee that shadows, not ultimates, are what we see. I think I am a Platonist at night.