Humanistic Mathematics Network Journal

Issue 18 Article 11

11-1-1998

Changing Ways of Thinking About Mathematics by Teaching Game Theory

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> "It saddens me that educated people don't even know that my subject exists."
> Paul R. Halmos

Halmos' words (1968), which served as the motto of the article, "Mathematics Today" (Steen, 1978), are equally relevant to this present project. It originated with the recognition of the narrow and limited conception of mathematics prevalent among the general public. Very few have any idea of what mathematics really is.

One of the main purposes of teaching mathematics in schools is to contribute to the enrichment of the mathematical world-view of the students. As in any scientific discipline, a mathematical world-view is formed by means of personal experience. The broader and richer the individual's experience, the more enriched and profound will be the corresponding world-view. Accordingly, the more diversified the encounter with mathematics, the richer the mathematical world-view.

> ***When the subject is art or "the arts"...teachers make every effort to ensure that students experience art in the broadest sense...in the teaching of mathematics, however, no real concern is shown for what mathematics really is.***

Schools act as the crucible in which the student's world-view of scientific disciplines is formed: they are responsible for the impoverished mathematical world-view common among students and graduates. When the subject is art or "the arts" (music, painting, sculpture, literature), teachers make every effort to ensure that students experience art in the broadest sense. They try to introduce them to as many kinds of art as possible; in the teaching of mathematics, however, no real concern is shown for what mathematics really is.

We agree with Aumann (1985) that "the case for thinking of mathematics itself as an art form is quite clear" and "if one thinks of mathematics as art, then one can think of pure mathematics as abstract art, like a Bach fugue or a Pollock canvas . . . ; whereas game theory and mathematical economics would be expressive art, like a cubist painting or Tolstoy's *War and Peace*."

In order to help students to sense the spirit of mathematics, effort must be made to introduce them to as many kinds of mathematics as possible. This may be accomplished by means of new curricula and new approaches to instruction. In Israel, a mathematics curriculum for high school upper grades composed of a combination of compulsory courses and 90 hours of elective studies was approved in 1975. The change in curriculum structure gave rise to the idea of creating an elective in game theory. Game theory both satisfies the criteria of the elective mathematics curriculum and exemplifies a branch of the discipline which may contribute to a change in attitudes and approaches to mathematics.

A course in game theory was created such that it is constructed of four topics dissimilar in character and bearing little mathematical relation to each other. The four topics were elected on the basis of their being of special interest beyond their mathematical content, not demanding specific prerequisite knowledge in mathematics, and providing general knowledge about game theory and its concerns (see Gura, 1995).

The first chapter of the course is called "Mathematical Matchmaking" and deals with the stable marriage problem that was raised by Gale and Shapley (1962). They concluded their paper with the following comment:

Finally, we call attention to one additional aspect of the preceding analysis which may be of interest to teachers of mathematics. This is the fact that our result provides a handy counterexample to some of the stereotypes which non-mathematicfi believe mathematics to be concerned with. Most mathematicians at one time or another have probably found themselves in the position of trying to refute the notion that they are people with 'a head for figures' or that they 'know a lot of formulae.' At such time it will be convenient to have an illustration at hand to show that mathematics need not be concerned with figures, either numerical or geometrical .... What, then, to raise the old question once more, is mathematics? The answer, it appears, is that any argument which is carried out with sufficient precision is mathematics.

Following Gale and Shapley, we strove to create a course which would employ a predominantly verbal mode of presentation and which, at the same time, would be sufficiently varied in content and level of explication.

RESEARCH

Game theory as a high school course is new to the existing curricula; therefore, the research requires the methodology of a case study. We carefully examined whether game theory can be taught at the high school or equivalent level and, if so, at what level of explication? At what depth? In order to answer these questions, we focused on topic selection and the teaching of these topics. We planned an intensive study which would investigate the teaching of game theory in its natural environment, the classroom. There were no special control conditions, only the tools of the classroom framework — exams, written work and questionnaires. In order to obtain convincing results and conclusions, the research was conducted on three different types of classes — one high-level mathematics high school class, two pre-academic preparatory classes studying equivalent low-level high school mathematics and one teachers' college class majoring in arithmetic instruction for elementary school classes studying at high school level mathematics. The structure of the research corresponded to Yin's (1984) definition of a case study. It was an empirical study investigating the teaching of game theory within the framework of the real context, the classroom, in which the boundaries between content, quality of teaching, approaches to teaching, the class situation and teacher-student interaction are quite vague.

Data were gathered via exams, questionnaires, and detailed journals of what went on during the lessons. Our primary interest was qualitative information, although we also compiled quantitative results. There were only four small classes and therefore the statistical analysis is limited; emphasis was placed on the qualitative analysis. From teaching in the classroom, we learned that several topics in game theory can be taught at both levels of mathematics. Although there were difficulties, students at all levels were successful in dealing with the course material. In the two

Class	N	Initial Grade $\bar{x}$	Final Grade $\bar{x}$
C1	22	50.0	66.1
C2*	19	52.9	78.3
C3	22	52.9	71.8
C4	20	56.1	74.8

Table 1 : Average grade (in percentages) of the c level (the lowest) classes in the pre-academic courses, 1897
*studied game theory

Class	N	Initial Grade $\bar{x}$	Final Grade $\bar{x}$
C1	20	70.2	80.9
C2	19	76.7	74.4
C3	18	72.4	81.9
C4*	16	75.3	88.7

Table 2 : Average grade (in percentages) of the c level classes in the pre-academic courses, 1988
*studied game theory

	Class a pos.	Class a neg.	Class b. pos.	Class b. neg.	Class c pos.	Class c neg.	Class d pos.	Class d neg.	Total
Before the Course	3 21.4	11 78.6	13 65.0	7 35.0	12 61.1	7 68.9	2 25.0	6 75.0	60
After the Course	1 10.0	9 90.0	6 30.0	14 70.0	5 31.2	11 68.8	0 0.0	8 100.0	54
Total	4	20	19	21	16	18	2	14	114

Table 3 : The distribution of positive and negative answers to question 1 (in absolute numbers and percentages) before and after the course

	Class a pos.	Class a neg.	Class b. pos.	Class b. neg.	Class c pos.	Class c neg.	Class d pos.	Class d neg.	Total
Before the Course	0 0.0	14 100.0	13 65.0	7 35.0	16 66.7	6 62.5	5 62.5	3 37.5	60
After the Course	0 0.0	10 100.0	7 35.0	13 65.0	6 37.5	10 62.5	2 25.0	6 75.0	54
Total	0	24	20	20	18	16	7	9	114

Table 4 : The distrubution of positive and negative answers to question 2 (in absolute numbers and percentages) before and after the course

preparatory classes we also were able to compare the average grade in mathematics of four parallel classes when three of the classes did not study game theory.

Note that the class which studied game theory did not have a higher grade average from the start but had the highest average final grade. Variance analysis shows:

Table 1: F = 10.660 with significance level of 0.001
Table 2: F = 16.706 with significance level of 0.001

The game theory courses indirectly contributed to raising the level of grades in mathematics in general. The object of my discussion here is to offer some of the results which point out the effect of this course in game theory on changing ways of thinking about mathematics. The results are gathered from a questionnaire designed to ascertain the subject's world-view of mathematics and from an attitude survey.

First, two questions from the questionnaire which was filled out by students before and after the course will be discussed:

1. What is your opinion about the statement that is often heard that the activities of the mathematician are based on computations?
2. What is your opinion about the statement that mathematics is nothing but a game of symbols and formulas that were invented by the human mind?

The table indicates that there is a connection between the type of answer (positive or negative) and the time it was answered (before or after the course). Calculating χ^2 for the whole population shows $\chi^2 = 7.274$ with significance at level of 0.01. We may say, therefore, that there is a significant relation between the type of answer and the time it was given. It would appear that the course in game theory contributes to reducing the number of students who relate to mathematics as mainly technical.

Question 2 explores whether the approach to mathematics is a formalistic one. A positive answer would attest to a formalistic, at least to some degree, approach to mathematics. The table shows that after the course in game theory the number of answers representing a formalistic approach to mathematics decreased. A χ^2 test calculated for the whole population shows $\chi^2 = 4.954$ with the significance at a level of 0.05. Clearly, there is a significant relationship between the rejection of the formalistic approach to mathematics and the studying of the course in game theory.

A real change in students' attitudes and approaches to learning mathematics was also observed and is best illustrated in the following questions from the attitude survey:

1. Was the course in game theory very interesting, interesting or not interesting?
2. Did the introduction of a new mathematical subject change anything in your overall approach to mathematics? If so specify what sort of change.
3. Do you think that the course in game theory can

help you in areas which require abstract thinking and are not connected to mathematics? Explain.

Answers to question 1 indicate the student's degree of interest in the course and those aspects of the course which stimulated this interest. In their answers the students relate to the technical aspect of mathematics, its relevance to life, to the different way of thinking — the very components that are significant in the overall mathematical world-view.

Examples of answers to question 1:

1. "Very interesting — a different approach from the usual one at school; the emphasis is on mathematical principles in addition to techniques."
2. "The course was very interesting because it was a new subject and because it deals with aspects of life which are concrete and more attractive to me."
3. "The course was interesting, more than any other subject I have studied in mathematics so far."
4. "Very interesting. This course enabled me to become acquainted with a 'different' mathematics and think in a different way."
5. "Interesting, different from other mathematics courses, theoretical and requires logical thinking."

The answers to question 2 revealed that more than 60 percent of the students changed their approach to mathematics following the course in game theory.

Examples of answers to question 2:

1. "The introduction to a new mathematical subject changed my attitude towards mathematics because I saw that mathematics is not just numbers and arithmetic operations, but there is something deeper and it is much more developed than I ever thought."
2. "Yes, by understanding the idea of proofs as a basis for mathematics."
3. "Yes, it changed my attitude because I understood that mathematics may be found in almost every area; mathematics is not just plain drill."
4. "To some extent yes, because the idea of mathematics being a useful instrument to the social sciences is new to me. I feel the same way about the use of verbal explanations alongside numbers. "
5. "Yes, I don't like mathematics because it says nothing to me, but game theory interests me because in game theory there are real life subiects and therefore it is easier for me and it is more interesting...suddenly..."
6. "Yes. This subject was more interesting than any other subject I've studied and has given me a strong desire to study more subjects of this kind in mathematics."
7. "I did not change. I've always loved mathematics."
8. "I did not change but my interest in the subject was strengthened."
9. "The introduction of a new subject based on mathematics did not change my attitude but familiarized me with mathematics as a whole which was much broader than I'd realized."
10. "No change. In fact it is hard for me to accept game theory as mathematics."

Answers 7-9 were given primarily by students studying high-level mathematics, who brought with them an interest in mathematics from the outset. The change in attitude was more pronounced in those students studying low-level mathematics.

Examples of answers to question 3:

1. "Yes. The course helped me see that every problem has several relevant aspects. I also understood that there are several approaches to a solution and that sometimes one has to choose a specific one. I think that the course developed abstract thinking."
2. "Of course. Game theory requires abstract and analytic thinking in order to discover proofs and corresponding processes to prestated principles."
3. "Yes. Game theory enables me to look at a subject from a wide perspective, that is from above, from different angles and various possibilities."
4. "The course helped me realize that mathematics can be brought closer to the social sciences with the help of mathematical thinking which gives precise results."
5. "The course is based on logic and analysis of real situations and can therefore help in areas of abstract thinking."
6. "Of course. This material, with its deep proofs, develops abstract thinking."

One could qualify our conclusions by saying that the students' answers do not guarantee an improvement in abstract thinking following the course in game theory and therefore this is not factual evidence. However, we must take into account the feelings expressed

by the students.

The range of answers included those ideas which were anticipated as a direct result of studying game theory. Among the answers: a feeling that mathematics is everywhere, an understanding of the usefulness of mathematics, its relevance to life, its depth, the proof as a basis for mathematics — in general, a change in the modification of the mathematical world-view. Answer number 10 to question number 2 is atypical, but exists nonetheless; the perception of mathematics as technical may be so strongly rooted in some people that even this course could not change it. The answers to the attitude survey seem to validate our choice of game theory to enrich and enlarge the student's conception of mathematics.

In conclusion, our hopes for the course were realized. As a result of the course, the number of students with an open-minded attitude to mathematics increased; the students were able to see mathematics as not only technical and computational, but also as an expanding and developing world of its own. Students discovered that the world of mathematics is much richer than they had previously thought. Indeed, it appears that the very encounter with a new sphere of mathematics in and of itself creates a new receptivity in the students to the assimilation of new concepts and values.

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Nine and One Third Circles of Rejoicing

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Alleluia!
Noble word
We greet you with joyful
Jubilations
You and your three thousand three hundred sixty
Permutations.
Let each one inscribèd be
Round a circle on each degree
Then with clarity will be heard
The sound of nine circles and a third.