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Badiou's *Logics*: Math, Metaphor, and (Almost) Everything

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Abstract

Mathematics plays a central role in the philosophical system of Alain Badiou. The aim of this essay is to situate this appeal to mathematics in the broader context of his work, including its literary and political elements.

Alain Badiou's appeal to mathematics is a process, a system that has been developing for close to half a century. Although there is an overarching mathematical theme, the system is too expansive and variable to admit a simple description. I focus here on a few questions that relate to the role of mathematics in his later work, as presented in the *Logics of Worlds* [9] and in some of the relevant but less formal initial meditations. I felt I had to let the text unfold or digress, following the multiple logics of its subject, toward extra-mathematical considerations—literary, artistic, political—from which Badiou's work cannot be fully separated.

In the first part [29] of this incomplete record of my attempts to understand the position and significance of mathematics in Badiou's philosophy, questions arose about the choice of the formalization that acts as the mathematical framework of *Being and Event* [5]. At that stage of the evolution of Badiou's work, ontology is identified with set theory, which in turn is presented in the ZFC axiomatization, although other choices are possible.

The question of principles guiding this and other selections—of the axiomatic system and "foundational" backdrop or framework, mathematical and non-mathematical examples, terminology and style—is not the only one that could be discussed. Highlighting this particular set of problems also represents a choice, a decision (this time on my part); other questions necessarily arise in a philosophical system of Hegelian scope and ambition. The question of the choice of formal framework, however, seems to me central to any discussion of the role of mathematics in Badiou's *Logics of Worlds*, also known as *Being and Event 2*.

I found it best to approach the problem indirectly and let it emerge from other questions that could (or should) be of interest to mathematicians. A possible starting point is an important and unusual invariant of Badiou's system: despite its transformations and development over the years, it features extensive technical discussions of mathematics—formal deductive systems in *The Concept of Model* [6], set theory and the theory of forcing in *Being and Event*, topos theory (or a fragment thereof) in *Logics of Worlds*. Whatever else might be said of this, it is clear that Badiou's love of mathematics, or of certain aspects of mathematics, has generated genuine interest in the subject, enthusiasm even, in surprisingly diverse academic locales. I recently saw him give a talk on (basically) the construction of reals to students and professors of comparative literature at New York University.

The mathematical community did not rush to reciprocate with a comparable level of attention; it is a safe guess that most of Badiou's readers are not mathematicians. Indeed, Badiou occasionally deflects critique by questioning the mathematical competence of his opponents. Thus, referring to a critique of his references to mathematics, he remarks on "the blatant insufficiency, to my great regret, of the purely mathematical competence of my critics", and notes that often his "adversaries—who are very numerous—totally ignore the mathematical and logical apparatus used in my philosophical constructions" [13, pages 362-363].[1]

While there are still only a few detailed mathematical analyses of Badiou's major works, several reviews have appeared in recent years. I assume that this relative scarcity of commentary—relative, that is, to the volume of Badiou's philosophical output—is at least in part due to the familiar reaction: references to mathematics by a contemporary Parisian philosopher tend to be mechanically filed in the nebulous category of "nonsense". On the other hand, caution is understandable, and cannot be entirely attributed

[1]The statements are part of a polemic that took place in *Critical Inquiry* [13, 17, 27, 28]. Regrettably, the exchange culminated in allegations regarding the opposing teams' understanding of the definition of a one-element set, a topic almost certain to produce the opposite of excitement in a generic mathematician.

to the proverbial philosophical reluctance of the "working mathematician". For example, some of the main concepts of *Being and Event* invoke the theory of forcing, which, although a major achievement of twentieth century mathematics, belongs in a relatively narrow field of specialization that many (myself included) would rather leave to the experts.

Two questions immediately arise in this context. First, there is the question of Badiou's grasp of mathematical methods and the correctness of his retelling of well-established arguments (he does not prove new theorems). The second question is that of Badiou's philosophical rendering—an additional layer of interpretation—of the technical results he adduces; that is, the question of consistency of the philosophical interpretation, internally and with the "raw" mathematics cited. These aspects of *Being and Event* were studied by the logician Maciej Malicki, who expressed reservations about the key concepts Badiou adopted from mathematics (or mathematical logic):

> [their] mathematical structure and [their] implications for Badiou's philosophy turn out not to meet expectations. They are ill-designed, and this leads to mathematical inconsistencies as well as philosophical consequences that contradict Badiou's intentions. Far from rejecting the philosophical substance of these categories, I claim that *their mathematical content may serve as an inspiring metaphor or analogy, but* [this metaphor] *has clearly defined bounds of meaningful interpretation.* ([23, pages 2-3], emphasis added)[2]

For example, Malicki concludes after a thorough analysis that two of the key "mathemes" of Badiou's *Being and Event*, the unnamable and the indiscernible, "are built out of mathematical concepts that do not fit together; moreover, their properties are not as Badiou claims." Corrections are required, possibly major ones, at least in the system presented in *Being and Event*. In subsequent private correspondence with Malicki, Badiou acknowledged these critiques and claimed he would be able to provide appropriate corrections. Until this happens, if it does, one can only suspend judgment on the philosophical claims built upon these mathematical concepts and supported by an appeal to the mathematical criteria of formal rigor.

[2]Page numbers refer to the arXiv version of [23].

There is, however, an additional complication. Even if Badiou is taken at his word that the serious omissions noted by Malicki can be corrected, the corrections may have broader implications, in this case philosophical, which could affect the entire system. Thus, the question of interpretation and selection of the mathematical background remains, and in a sense becomes more acute. Malicki's conclusion makes this crucial point:

> Mathematics—forced to accept compromises going definitely too far—responds with outcomes which are hostile to fundamental philosophical motivations of Badiou's doctrine. Despite some points of convergence, his generic theory of truth and his philosophy of event can coexist only at a price of *selective and instrumental interpretation* of the mathematical component. ([23, page 14], emphasis added)

This is a question to which I found myself returning while reading *Logics of Worlds*, although the status of the formal correctness of Badiou's mathematical arguments seems to be different in this case. Badiou's preparatory notes and studies of topos theory have been published separately as *Mathematics of the Transcendental* [14], and have been reviewed by mathematicians. A review by the category theorist Colin McLarty [25] is unequivocal: "Much in the present notes, though, is mathematically proffered theorems, proofs, and exercises in category theory. Bellicose critics will be sorry to learn that these are correct."

Since this essay is primarily concerned with the system presented in the *Logics of Worlds*, the question of correctness will not be central in my discussion. However, the question of choices, selections, persists. "Badiou looks at a limited and logic-oriented part of the subject", McLarty observes, and adds [25]: "This raises a puzzle. Why treat only the localic case? It is much narrower than the general case and not much simpler. [...] The restriction has been sharply and knowledgably criticized by Veilahti."

An analogous puzzle is raised by Andrej Bauer: "Is it really necessary for Badiou's philosophy to use precisely toposes and not some other kind of categories? It looks like most of the philosophical analogies would still hold in a more general setting [...]. Similarly, why should the study of the 'transcendental' be limited to frames?" [18, page 1071]

Antti Veilahti's article, to which McLarty refers, is too technical to describe here, but reveals similar difficulties that arise from the choice of the mathematical framework of *Logics of Worlds*. The key point for the present purposes—a point to which I return— is that, despite references to topos theory, Badiou selects a framework that does not represent a radical departure from set theory: "Badiou's own topos-theoretic formalism, however, turns out to be confined only to a limited, set theoretically bounded branch." [30, abstract]

Before I attempt to speculate on the possible reasons for choosing a more restrictive mathematical setting, let me quote the conclusion of Bauer's review:

> I shall not criticize a philosopher for not knowing everything when he expended an amazing amount of energy to build not one, but two bridges from his land to mine. I am impressed by the lucidity of Badiou's remarks on the philosophical significance of category theory, especially in relation to set theory, and I invite philosophically minded mathematicians to be so too. [18, page 1071]

Such a sentiment has fueled my attempts to understand the role of mathematics in Badiou's work. But the puzzle noted by McLarty and Bauer (and by Veilahti in more technical terms) remains unresolved: Why restrict the mathematical framework? Although it sounds plausible that he (like most "working mathematicians") could not master everything that category theory has to offer, there is also evidence that he did in fact consider a more general framework—and decided to not use it. The remainder of this article will therefore be concerned with attempting to unravel this puzzle of "why".

It is useful to recall part of Malicki's assessment of Badiou's appeal to set theory and forcing in *Being and Event*: "selective and instrumental interpretation of the mathematical component". A discussion of Badiou's interpretive operations, and his extraordinary glossing of (selected fragments of) mathematics, seems inevitable.

The method consists of adding a rich philosophical layer to set theory and, later, topos theory (or a special case of it). Interpretive and metaphorical elements of this procedure are, in theory, not entirely uncontrolled. But even if this is taken for granted, the limits of such an "application" of mathematics are not clear. If mathematical selections and their metaphorical overlay are somehow instrumental, that presumes a goal or at least a motivation.

My thesis is that Badiou's motivation—to the extent that I am able to divine it—does not come from mathematics alone; it is entangled with other "conditions" of philosophy.

Badiou views philosophy as the space of virtually unobstructed circulation between its conditions: mathematics, art, politics, and love. He supports this demand for freedom of philosophical play by arguing that attempts to limit it would be, and have been, detrimental: favoring one condition over the others leads to familiar difficulties. Roughly speaking, Plato (and much of "analytic" philosophy) favored mathematics over poetry; Heidegger (and much of "continental" philosophy) favored poetry over mathematics. Contemporary philosophy should dialectically overcome these traditional positions if it is to formulate truths of the era in which the word "truth" tends to appear in scare quotes.

That may well be the best of all possible philosophical worlds. But there is a sort of spontaneous symmetry breaking at the heart of the Badiousian circulation, notably between the mathematical and the poetic conditions. Badiou's philosophical texts are imprinted with a stylistic, rhetorical, writerly component (also a personal component, as explained in "Philosophy as Biography" [8]). These "effects of the text", that is to say literary effects, are not under the jurisdiction of the mathematical condition. Mathematics, however, is subjected to a number of poetic operations, some of which override the conditions set by formalization.

It is an interesting question whether mathematics already includes an "obscure" poetic element, a set of interpretive principles or guiding imagery, which sometimes goes by the name of "philosophy"—as in "Langlands philosophy" or "Grothendieck philosophy". To this question Badiou responds in the negative: mathematicians do whatever it is they do, but what they produce (namely: mathematics) is absolutely transparent.

Mathematics is a thought that "faces the void", which I suppose means that it clears out all particularities and thinks only forms of being as such (in *Being and Event*). But to pass to the description of any particular existent from pure being—a "dumb generality", as Marx indelicately put it—something more is required: something that makes it appear in a world and relate to other beings appearing in that world. To solve this and other problems, Badiou changes the mathematical framework in *Logics of Worlds*.

The solution he proposes is fascinating, and its mathematical form follows a certain shift in mathematics. However, the literary element remains and apparently continues to be necessary. (It is not deployed in the same way in *Being and Event* and *Logics of Worlds*, but it is present in both texts.) Mathematics may be transparent, but its philosophical import has to be explained and elucidated. This requires a supplement, an image, that is: metaphor.

Sometimes, though not always, Badiou explicitly marks some of his disquisitions as metaphors or based on "metaphorical affinity". These literary supplements—according to Badiou, to the extent that I could discern a consistent position—are not to be viewed as contingent on personal preference: some of the metaphors originate from mathematics, so they are not arbitrary fictions (not under the jurisdiction of the literary condition alone). Hence the range of literary operations is, in theory at least, somehow limited.

This is a point which has to be addressed with due care. On the one hand, some limits are indeed imposed on Badiou's philosophy by its mathematical condition. For example, if ZF set theory is fixed as the ontological framework, it is not possible to assert the existence of an absolute One (the "set" of all sets); nor is it possible to claim that language magically conjures beings. On the other hand, these are straightforward consequences of the axiom that ZF set theory is ontology; the example is therefore not representative of the depth and range of literary operations to which mathematics is exposed in Badiou's work.

There are many analogies and metaphorical affinities that Badiou invokes to construct textual connections between the state (as a political entity) and the power set (the set of subsets of a set), nature and the collection of ordinals, love and partially ordered sets, the subject and (infinite) groups, independence of the Continuum Hypothesis and the triumph of politics over the realism of the unions—to list a few random items from the hefty catalog. The boundaries of paralogical operations are in these cases not so easy to detect.

For instance, Badiou outlines ethical reasons why philosophy should think Man not as a finite being, being-for-death, but as the Immortal he is capable of becoming [2, pages 11-12]. The metaphor of the Immortal is part of a larger objection to the "hateful spirit of finitude" [10, page 118]. In this broader context Badiou has opined that all situations (basically: nonempty sets) are better thought as infinite. This is also a metaphor, but one that transgresses the limits imposed by the ontology of *Being and Event*: there are finite sets in ZF. Of course Badiou at once points out that he does not actually deny the existence of finite sets on the level of "pure objectivity", so there is no contradiction. Rather, what happens in this case is that the political condition (of the war on finitude) joins forces with the poetic condition to override the mathematical condition.

That is only an example, but it appears to belong to a category of similar examples that cause difficulties with determining the precise conditions that are invoked. In the framework of *Logics of Worlds*, there are many worlds, as the title itself announces. Indeed, "man is the animal that appears in a very great number of worlds" [9, page 114] and, moreover, it is "the essence of the world that there are several worlds" [9, page 102]. Yet in the non-technical book *The Meaning of Sarkozy* [7], published soon after *Logics*, Badiou argues for the principle that there is only one world: "that of living men and women". That there is only one world ("that of living men and women") is an axiom of collective action ("in this single world") and a political principle so important that Badiou dedicates an entire chapter to it [7, "Only One World", pages 53–70]. I can only assume that the meaning of "world" is not the same in the two texts. In that case, however, the political and poetic conditions of philosophy trumped the mathematical one—again.

A clearly stated goal of Badiou's project is to undo philosophy's linguistic turn, including the antiphilosophical sophistry of Wittgenstein as well as Heidegger's antimathematical poetic ontology. The objective is to cure a certain condition—the postmodern condition, so called—described by a medical metaphor in *Infinite Thought* [3, page 39]: "Truth is suffering from two illnesses. In my opinion, it is suffering from linguistic relativism, that is, its entanglement in the problematic of the disparity of meanings; and it is also suffering from historical pessimism, including about itself."

In the same lecture he notes that something has to be done—"the world is asking something of philosophy"—although it may well be a hazardous intervention [3, page 39]: "the problem is knowing whether this illness is mortal or not, knowing what the diagnostic is, and knowing whether the proposed remedy is not in fact, as is often the case, exactly what will finish off the patient."

For this to work as I think Badiou thinks it should, namely by having mathematics, as the science of being as such, set limits to the effects of philosophy's preoccupation with language and interpretation, mathematics has to be absolutely transparent. Ontology (that is, mathematics) is where the play of interpretations stops [4, page 56]: "Mathematics has the virtue of not presenting any interpretation. The real does not present itself in mathematics as if upon a relief of disparate interpretations."

But if mathematics must be supplemented by interpretive operations coming from the literary and political conditions of philosophy (or from Badiou), it is not at all transparent how mathematics can simultaneously guard against the excesses of interpretation. That is possible only if some higher principle ensures that transparency is not, as it were, lost in translation. Formalization by itself offers no comfort, especially when the levels of discourse are deliberately conflated. This is one of the difficulties of *Being and Event*.

Naming a mathematical concept by an otherwise laden term (such as "the state") is a decision, though not one that "binds your thought to being", as Badiou states. He is not deciding upon what exists in such cases; he is deciding on how to rename certain forms of thought (or certain forms of being), some of which already have mathematical names. Whether these are absolute decisions, made by Badiou after the fashion of a "new Carl Schmitt", is an important and disturbing question. In the case of events and their naming, an analogous objection forced him to abandon the terminology of naming, as explained in an interview published in *Infinite Thought*:

> Lyotard said that I was an absolute decisionist, a sort of new Carl Schmitt. But I think there's some confusion here because, after all, the crucial question is the event and the event is not the result of a decision. The difficulty is that in *L'Etre et l'evénement*, I say that the name of the event is the matter of a pure decision and I have to change that point. It's not very good terminology, the terminology of the nomination. I now think that the event has consequences, objective consequences and logical consequences. The effect of the event is a profound transformation of the logic of the situation—and that is not an effect of decision. The decision is uniquely to be faithful to the transformation. [3, pages 129–130]

This is an interesting solution (or idea for a solution), although it addresses only the special case of naming events, which in the system of *Being and Event* are by definition not sets. A remark similar to Lyotard's, however, applies to language games in which Badiou engages within set theory. He does provide motivation and makes intricate rhetorical appeals to explain his renaming of mathematical concepts, but ultimately it is his decision whether the collection of ordinals should be named "nature" rather than, say, "divinely instituted hierarchy" or (why not) "transcendental totalitarianism". At the point of this nomothetic decision the entire politics of the linguistic turn could erupt into Badiou's meditations. But it seems to be the only way in which set theory could be made to appear in less intangible worlds.

Resolving this difficulty in the system of *Being and Event* required a return to, and serious reconsideration of, the idea that logic is a species of mathematics and not the other way round. Badiou describes a complex dialectics of the relation between mathematics and logic in his *Second Manifesto for Philosophy* [11]:

> what Heidegger calls ontological difference can be said to be the immanent gap between mathematics and logic. It would be proper then, in order to continue to follow him, to call 'metaphysical' any orientation of thought confusing mathematics and logic under the same Idea. Now, there are two ways of perpetrating this confusion. Either one reduces mathematics to a simple logical theory, which is what Frege, Russell and Wittgenstein all do in their own manner. Or one considers logic to be but a specialized branch of mathematics [...]. As such two metaphysics can be said to exist. [11, page 40]

To overcome these two metaphysics, Badiou writes, "I am also going to borrow something from Hegel—namely that existence must be thought as the movement that goes from pure being to being-there, or from essence to the phenomenon or appearing, as explained in two profound and obscure chapters of [Hegel's] *Logic*." [11, pages 45-6]

Constructing a mathematical framework that elucidates how pure being "goes" to being-there, how it appears in a world, is one of the reasons why in *Logics of Worlds* Badiou seeks to incorporate the topological outlook of Poincaré and Brouwer, among others. Sets-as-such were of less interest to them than spatial structure revealed by mappings among spaces; as topologists, they looked at how curves and surfaces appear in a space. Poincaré, in particular, advanced the view that mathematicians are not concerned with the internal composition of objects but with their structural properties, or their relations to objects and structures of a given class. Ultimately, this leads to the point of view of category theory.

In the framework of *Logics of Worlds*, which (to a certain extent) follows the movement of the "history of ontology", much of this insight is swept away by the complexities of categorical logic—a later and in a sense independent development. The outlook described above could just as well lead to choosing category theory as the mathematical framework of Badiou's system, but he is interested specifically in topos theory rather than category theory in general. In fact, although it has become standard to speak of topos theory in reference to *Logics of Worlds*, Badiou's choice of mathematical framework is more specific.

Let me recall McLarty's comments cited in the introduction: "Badiou looks at a limited and logic-oriented part of the subject", which "raises a puzzle. Why treat only the localic case? It is much narrower than the general case and not much simpler. [...] The restriction has been sharply and knowledgably criticized by Veilahti" [25]. Veilahti in turn notes: "Badiou's own topos-theoretic formalism, however, turns out to be confined only to a limited, set theoretically bounded branch" [30, abstract]. This is an important point.

To attempt to explain or speculate on the possible motivation for Badiou's choice, it is illustrative to recall a metaphor from John L. Bell's *Toposes and Local Set Theories* [19] (one of Badiou's sources, though apparently not a favorite): "the topos-theoretical viewpoint suggests that the absolute universe of sets be replaced by a plurality of 'toposes of discourse', each of which may be regarded as a possible 'world' in which mathematical activity may (figuratively) take place." [19, page 245]

In Bell's description, the topos-theoretical viewpoint suggests a kind of pluralism of possible worlds, where presumably no world is a priori privileged over any other. But a proliferation of toposes of discourse, all equally legitimate as worldviews, resembles the very illness of philosophy that Badiou proposes to cure by the bitter mathematical pill. This does not fit well with the political condition of Badiou's project: "*First Thesis:* We have to break with the linguistic turn that has seized philosophy. [...] *Third Thesis:* At the heart of the conditions of the linguistic turn, there is the formal identification of logic and mathematics [...]. *Fifth Thesis:* Mathematics is posited as the science of Being qua Being, or ontology strictly speaking." [4, page 111]

The role of mathematics is, in part, to reassert the primacy of ontology over language—or at least to find a balance between the two—and thus to arrest, albeit on a highly "Platonic" level, the proliferation of disparate discourses. It is reasonable to assume that this is why only a fragment of topos theory is presented in *Logics of Worlds*. Within this narrower framework, Badiou can argue that languages and logics are mathematical (ontological) entities: ultimately, they are built up from sets. The realm of sets is by definition not a world, but it shadows every possible world as pure being that appears in the given world in a certain way. Since worlds are appearances of pure being, set theory—"ontology in the strict sense"—underpins all possible worlds and serves as their ontological common ground.

In the topos theoretic outlook, each world has its local language. These worlds and their languages are mathematical constructs: a "world" is a collection of mappings from the universe of sets to a certain structure, and these mappings govern the logic of how being-as-such (the universe of sets) appears. A "world" defined in this manner is a topos (a special case of topos, as noted by McLarty and Veilahti), but the framework is in essence set-theoretical. Ontology "in the strict sense"—the ghostly realm of pure sets—is now prior to each possible world and the corresponding local language of that world. Being in this sense independent of each local language, perhaps even in some sense "prelinguistic", ontology is beyond the reach of linguistic relativism. The linguistic turn has been overturned.

It is surprising that such a radical consequence for philosophy—a mathematical escape from the Nietzschean "prison house of language"—should follow from the material compressible into a few chapters of, say, MacLane and Moerdijk [22]. The result would be spectacular, but it does not quite follow from purely mathematical considerations. Topos theory can be formulated axiomatically in an appropriate language, without mentioning set theory, which remains for Badiou the ontology-proper, the science of being as such, as it was in *Being and Event*.

Thus it could be argued that Badiou's ontology-proper (namely: set theory) does not have the special status of the science of being-as-such that floats behind all appearances and worldly discourses, but is only an interpretation of a different type of language. It is Badiou's choice to not present it that way. "Insofar as his [mathematical framework] is constituent of his theory of the worlds," Veilahti writes, "this 'constitution' isn't mathematically necessary but relies only on *Badiou's own decision*" [30, page 7, emphasis in the original]. The decision involves suppressing the broader context of category theory, which does not privilege sets. Keeping a set-theoretic backdrop is not an unusual choice, and it is not inconsistent with mainstream mathematical practice, but it is not the only possible choice.

A key feature of Badiou's "materialist dialectics" is that he argues against the universe as an absolute, yet somehow keeps set theory around as an umbral absolute limit. On the one hand, "there is no possible uniformity among the determinations of the thinkability of multiples [pure sets], nor a place of the Other in which they could all be situated. The identifications and relations of multiples are always local" [9, page 112]. The universe of sets is, by definition, not a world. It is not even a universe. On the other hand, this no-place of pure sets is inscribed into the very mathematical framework chosen by Badiou.

This dialectic requires strenuous explanations that obscure the clarity of the mathematical setting. To illustrate: "We will say that a multiple, related to a localization of its identity and of its relations with other multiples, is a being (to distinguish it from its pure multiple-being, which is the being of its being)" [9, pages 112–113]. Thus, while laboring to explain that sets are not there, Badiou in effect states that each localized being has its pure set, "the being of its being". Pure sets are somehow written into the system, though we are not supposed to see these specters until they appear.

The system of *Logics of Worlds* is calibrated so that it is quite simply the nature of being to appear in a world: each world (by definition) comes equipped with an appropriate logic of appearing. The function that Kant attributed to the transcendental subject ("the transcendental unity of apperception") is a feature of the system itself: a world of phenomena, by definition, includes a mapping of pure sets into an algebra that constitutes and structures the logic of appearing. The algebra is also an order structure. This is what Badiou calls "the transcendental" of a world.

Instead of the Kantian transcendental subject, each world has its own "transcendental", a logic, which inherits the function of establishing an order and logic of appearance. Mathematically, it takes the form of a Heyting algebra: it specifies the intensity of appearing in that world of each being (and the intensity of relations among beings). In an amusing twist on the Quinean tenet—"To be is to be the value of a variable"—in the system of *Logics of Worlds*, to be-there is to have a value in a Heyting algebra; more precisely, to be-there is to be assigned a degree of intensity of appearance relative to other beings.

Although this maneuver allows Badiou to reinvent a sort of "Kantism without the transcendental subject" and find a place for ontology in an elaborate revision of Kantian analytics, the topos-theoretical pluralism of worlds and their logics of appearing re-opens the question of interpretation. For Kant, we can know only phenomena, the ways in which the being-as-such appears, and the transcendental subject guarantees that these perceptions have a universal form fixed by a priori intuitions. Intuition of space is Euclidean, according to Kant, and hence our world appears to all of us as Euclidean.

In *Logics of Worlds*, on the other hand, examples are given of various worlds, but the question of which specific logic of appearance obtains in these example-worlds does not seem to arise. The concrete difference between the logic of the world of a painting and that of the world of a battlefield is not made explicit on the mathematical level but only on the less rigorous narrative level, where it seems obvious in the first place.

The role of Badiou's passage to this rendition of topos theory is to illuminate the general form of how pure sets appear in worlds. His system offers a "generic" theory of worlds and their logics. Unlike Kant, who committed to a particular geometry and thus to a particular logic of appearing, Badiou is vague about questions of the type: Which Heyting algebra, specifically, is the transcendental of this (or that) particular world? The poetic condition of philosophy is called upon at this critical moment. In the place of a concrete Heyting algebra, one finds rhetoric: red ivy, autumnal shadows. But dissolving a rigorous mathematical framework in literary supplements seems inconsistent with the political program of arresting the play of interpretation and undoing the effects of the linguistic turn.

The thesis of the prelinguistic character of mathematics, which Badiou comes close to asserting despite sophisticated dialectics of the relation of mathematics and logic, is not exactly new. It acquires a new and far more delicate form in Badiou's work, but traces of its history appear in *Logics of Worlds*. Sounding almost like Brouwer—an adversary of formalism who argued that mathematical thought is prelinguistic and that logic is a sort of sociology, a record of how folks organize their thoughts (see [20])—Badiou writes of "pre-linguistic operators" and states that "logic, formal logic included, not to mention rhetoric, all appear for what they are: derivative constructions, whose detailed study is a matter for anthropology" [9, page 174].

There are, however, fundamental differences. By "pre-linguistic", Brouwer has meant that mathematics is a linguistically irreducible activity of a founding absolute subject, an indivisible flow of acts of free choice unfolding in the mind of the "ideal mathematician". Such mysticism (or, less harshly, such romantic idealism) is of no philosophical interest to Badiou. The days of a founding subject are behind us. Yet the Brouwer-event left a trace in the form of intuitionistic logic and Heyting algebras, which (albeit only in retrospect) turned out to be related to the Grothendieck-event and to the work of a few other people mentioned parenthetically (MacLane, Eilenberg, Lawvere, Tierney); which in turn enabled Badiou to select from this complex history of ontology the fragment that appears in his later work.

The carefully chosen mathematical framework creates an opportunity to address questions that the system of *Being and Event* could not, such as a more nuanced theory of change, which can now be gradual and does not always take the radical form of an event. Lawvere's idea of topos theory as a theory of dynamic or variable sets was probably motivated by his interest in the foundations of physics, and there is a sort of physics in *Logics of Worlds*, something that Badiou's system could not quite address in its earlier stages. It does not, however, include a discussion of the fact that mathematical physics has developed an interest in topos theory and, more generally, category theory. That is certainly not a philosophically irrelevant subject, as noted by Yuri Manin, who describes the category-theoretic view as follows:

> there is no equality between mathematical objects, only equivalences. And since an equivalence is also a mathematical object, there is no equality between them, only the order equivalence, etc, ad infinitum. This vision, due initially to Grothendieck, extends

> the boundaries of classical mathematics, especially algebraic geometry, and exactly in those developments where it interacts with modern theoretical physics. [24, pages 51–52]

Badiou's decision to retain an essentially set-theoretic outlook thus misses an important possibility of establishing a firmer connection to an important part of contemporary science. Instead, mystical effects are introduced. According to Badiou's analysis of the logic of death, what happens is that an "exterior cause" changes the intensity of a being's appearing in a world (by mapping the departing being to zero in a transcendental algebra); death is merely a change in the logic of appearing. It is a possible viewpoint on death, and not an especially novel one, apart from the reference to mathematics: a candle is blown out. But with set-theoretic soul-forms shadowing all worlds as ontology-proper—akin to Berkeley's "ghosts of departed quantities"—there seems to be nothing that would prevent an apparently dead being from rising and reappearing: logics of worlds, Badiou writes, are subject to "sudden lifting of an axiomatic prohibition, through which the possibility of the impossible comes to be" [9, page 391].

A different species of "materialist" reading of the topos-theoretic metaphor of many worlds might be that there is no privileged viewpoint from which events could be seen as changing the logic of appearing of pure beings. Beyond worlds there might be no unlocalized form, but only an absolutely contingent vanishing of one world and popping up of another—something like Quentin Meillassoux's "hyperchaotic Time".[3] Or, to stay closer to the path of Badiou, there might be a mathematical structure that is not included in the specific framework he chose. Despite arguments to the contrary, the formal element of *Logics of Worlds* indicates that Badiou is theorizing the ways of appearing of the being of *Being and Event*: the universe of sets. Some of the old questions reappear, therefore, in the new setup.

I noted earlier that naming-games were problematic in *Being and Event*, leading to the question of how the choices of names are made. It does not follow that by doing this Badiou was not formulating a truth. It follows only

[3] "Things, people, events, physical laws, correlation itself: to be is to be determinate—to be this or that—and thus to be able to change without any reason whatsoever, in perfectly contingent fashion, within a Time capable of destroying every entity, whatever its mode of being." [26]

that he introduced a new meta-mathematical "axiom of truth" every time he renamed a set-theoretic concept by an otherwise laden term. That, however, adds up to a large number of new axioms and consequences, and decisions have to be made on what to present. These are meta-mathematical decisions, occurring outside of a rigorous formalization, which therefore have something to do with ordinary language and its rhetorical function in philosophy. Mathematics produces an astonishing number of ontological assertions, only some of which, and with utmost care, may be transcribed into ordinary language; others lead to perplexing results.

Suppose, as a simple thought experiment, that I am watching a football game. Being a mathematician, I consider it in the mathematical (ontological) mode: there are humanoid forms traversing paths around a rectangular form in three-dimensional Euclidean space, apparently in relation to a spherical form which seems to be the center of the action and the common cause of their desires. I wonder, therefore, what ontology tells me about spherical forms.

There is, for instance, the Banach-Tarski Paradox, which is not at all a paradox but a beautiful theorem. It states that any ball in three-dimensional Euclidean space can be cut into a few pieces, which can be moved around without deformation and reassembled into two balls of exactly the same volume as the original. In this case ontology tells me something that is in some sense inadequate to the situation under consideration. Certainly the theorem shows that, as Badiou might say, it is possible to think of such a duplication of a ball, no matter how scandalous it may appear to our notions about the physical world. That is why the theorem acquired the name of a paradox.

Furthermore, this theorem is valid in certain worlds (those whose "local set theories" support the Axiom of Choice), not only in the spectral non-world of sets as such. One might claim that "the world of living men and women" does not support the Axiom of Choice, and that therefore magical duplications do not happen on football pitches in this world. But in that case one is simply asserting that the Axiom of Choice is in some sense inadequate as part of the logic of appearing in this world.

The very notion of a more or less adequate "correspondence" between the formal and the empirical is proclaimed already in *The Concept of Model* to be part of "bourgeois epistemology"—an ideological question, dismissed by

Badiou as such; the argument is repeated in various shapes throughout his work. Nevertheless, a related problem reappears on the level of discourse. The paradox occurs precisely on the meta-mathematical level: what is paradoxical about it is, so to speak, only an effect of the text, an effect created by the polysemy of the word "cut", which in ordinary language suggests an action very different from the mathematical "cut" involved in proving the Banach-Tarski theorem.

Hence it seems that *some* transcriptions of ontological and logical statements (in Badiou's sense) into ordinary language produce results that are in *some* way inadequate. I am not suggesting that this happens in Badiou's system. On the contrary, its overall coherence is an astonishing philosophical feat. Precisely because of that, it is difficult to imagine that such an achievement is possible without any guiding idea of adequacy.

Badiou selects his examples with care, understandably so: he aims to explain, demonstrate, illustrate, convince. "Using natural language in studying and teaching the sciences", writes Yuri Manin, "we bring with it our values and prejudices, poetical imagery, passion for power and trickster's skills [...]" [24, page 80]. The difficulty is compounded when one also rejects any notion of adequacy, as Badiou does: "Modern philosophy is a criticism of truth as adequation. Truth is not *adequation rei et intellectus*" [3, page 45].

Thus the question resurfaces about the criteria involved in Badiou's decisions on which parts of ontology to cite and transcribe into examples, and which to "pass over in silence". For instance, ontological duplication of a ball during a World Cup final does not, to the best of my knowledge, occur in the long catalog of examples that illustrate the relevance of Badiou's formalism to "real life" or "historical" situations. Admittedly, it would look bizarre. But a meditation on the battle of Gaugamela—an example that "reveals the extent to which military genius is really the genius of the transcendental functor" [9, page 288]—is apparently not too far out. What are the criteria that guide the choice involved in giving one example and not the other?

I can only speculate that the difference has to do with the fact that Banach-Tarski theorem is not "realistic"; it would not serve the rhetorical function of convincing the reader of the relevance of ontology to the world of football, never mind the military genius of Alexander. A battle, on the other hand, lends itself to "realistic" literary treatment; as do discussions of the ontology of the state, where [5, page 110] we see Lenin in a moment of despair ("ready to die"), as well as a dejected Mao ("more phlegmatic and more adventurous"). The process of incorporation of "amorous truth" is elsewhere exemplified by a Hallmark image of a stroll on the beach.

The level of detail and the intensity of "realistic effects" vary, but Badiou's examples are constructed much as one does in a realist novel (with a mathematical clef). Defoe is probably not as impressive a literary reference as Mallarmé, but might work equally well: from the scattered traces of a shipwreck-event that subtracted itself from being—leaving only shoes that are not fellows, some hats and caps—Robinson Crusoe formulates an axiom of truth about his new situation. It could be called the generic procedure of literary realism.

There seems to be something like a mimetic principle involved in Badiou's choices. He mentions only "resemblances" and "metaphorical affinities", so the principle of selection is not likely to be that of bourgeois epistemology, namely the notion of there being a correspondence between the formal and the empirical. But the related ideological criterion of utility appears to be in force: fragments of ontology (mathematics) are suppressed because they are not adequate to the production of "realistic effects" of the text. Thus a kind of realism silently operates on ontology, not only as a metaphorical supplement but as a higher authority that decides what, among all that mathematics can express about being qua being and being-in-a-world, is or is not expressed.

Admittedly there is a notable difference in the intensity of "realistic effects" in *Being and Event* and *Logics of Worlds*. In the later book they are cranked up to a purpler hue of prose "in which the golden tint of the stone is prisoner of the vague-blue". Perhaps this is because the operator of appearing in a world, with its fuzzier, more nuanced semantics, allows Badiou to let loose the literary genies of lyricism, inspirational rhetoric, and realism all at once. The outcome has been euphemistically described, by a sympathetic reviewer, as verging on neo-baroque excess.

There is, nonetheless, a continuity between the two systems when it comes to mathematics. Indeed, the fundamental problem remains exactly the same: Badiou's decisions on which fragments of mathematics to present, and which to set aside. Mathematics can say more, and has said more, than he allows it to say.

One could take what Badiou would call an "aristocratic" view of this predicament: "Perhaps this is more an accident of Badiou's studies at the time he wrote the notes than a decision" [25]. Or, "I shall not criticize a philosopher for not knowing everything" [18, page 1071]. Or one may take a less forgiving, more rigorous stance: "the evidence supporting the accident hypothesis is only circumstantial. [...] The philosophical implications, in contrast, support the claim that such an 'accident' occurs philosophically on purpose" [30, page 37].

This is not an undecidable problem. Textual evidence suggests that the choice was not accidental, and that some time between *Briefings on Existence*[4] and *Logics of Worlds*, Badiou decided that full-fledged category theory is too much:

> Unfortunately, and contrary to *my chosen angle*, the majority of presentations remain prisoner to an often heavy-going examination of the entire arsenal of category theory. Accordingly, *after having located the philosophical fragment that interests me* ([9, page 538], emphasis mine; prisoner is Badiou's.)

It seems somehow disingenuous to conscript mathematical thought as an ally only to dismiss parts of what this thought thinks as though it were an annoying technicality. But it is not a question of "sincerity"; it is, rather, a philosophical question of fidelity. The full arsenal of category theory—written off in the end-note cited above—could conceivably offer a substantially different philosophical insight than the fragment presented in *Logics of Worlds*. Meditations on the more general framework of category theory appear briefly in Badiou's *Briefings on Existence*, then disappear. The mathematical material did not fit the philosopher's chosen angle.

I may well be too sensitive to the ideology of academic and bureaucratic *decideurs* who are interested only in what, at a given moment and from their chosen angle, appears to be useful. Certainly I am not comparing Badiou to them. One of the many reasons his work should be of interest to mathematicians is that he wrote against the "economic enslavement" of mathematics

[4]The French original was published in 1998 (*Courte traité d'ontologie transitoire*, Seuil). The book includes what appear to be Badiou's preliminary considerations of the shift toward category theory.

almost half a century ago, in 1968. He spent the intervening decades developing a system of thought in which formal innovation (in arts and sciences alike) is an ethical and political task. Of Badiou's love of mathematics there can be no doubt: his recent *In Praise of Mathematics* [16] confirms it in a conveniently concise form. But love can have a darker side, described by the Lacanian motto (as cited by Žižek): "I love you, but there is something more in you than you [...], so I mutilate you."[5]

There is a surplus in mathematics that is at once its great mystery and the source of philosophy's "horror". Sometimes those annoyingly technical and apparently useless parts of mathematics—those proverbial games in the void, which receive disparaging nicknames such as "pointless topology", for example, or "abstract nonsense" in the case of category theory—become relevant to the world, and nobody seems to know exactly why, how, or when such an event might occur.

I am not saying anything here that Badiou has not said in his dispute with Hamlet: "there is nothing in our philosophical capacity which could not come to be in the reality of the world" [1, page 189]. I take this to mean that there is only one world, and that our mathematical capacity relates to it in ways that no a priori relation can describe. If mathematics is thought's sin (as Badiou remarks on Wittgenstein's dislike of set theory [12, page 143]), then Badiou has sinned in his thoughts more than most philosophers. But he is still too moderate in this special vice. He takes what he can use, even when he knows there is more.

Despite a number of novel and philosophically radical claims, Badiou appears to have hesitated before the implications of the thesis that mathematics—all of it, as-such, regardless of which fragment preoccupies him at any given time—is ontology. Hence one can only say, with Sade: One more effort! Let there be duplicable spheres, fields of one element, noncommutative spaces, Grothendieck universes, categories and higher categories and homotopy type theories. Let a hundred flowers blossom. There is always, there could be, *Being and Event 3*.

[5] At a lecture I attended, Badiou took a sanguine view of the question of his mathematical selections: he makes choices and decisions on what parts of mathematics to cite, just as when one speaks of poetry one does not need to quote every poem. The problem of criteria involved in his mathematical choices is not made easier by such a reply.

Badiou's *Métaphysique du bonheur réel* [15]—where mathematics is related not only to happiness but to beatitude—suggests that the next stage of his system may call upon the mathematical framework of paraconsistent logics. One can only hope that this framework will help further illuminate the fascinating interplay of philosophy's "conditions", particularly of mathematics and metaphor.

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