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Adam Glesser
California State University, Fullerton

Martin Bonsangue
California State University, Fullerton

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Cover Page Footnote

The first author would like to thank California State University, Fullerton and its students for their support in the making of the videos related to this article.

Jay Leno and Abstract Algebra

Adam Glesser[1]

Department of Mathematics, California State University, Fullerton, USA
aglesser@fullerton.edu

Martin Bonsangue[2]

Department of Mathematics, California State University, Fullerton, USA
mbonsangue@fullerton.edu

Synopsis

The Jay Leno skit *Jaywalking*, showing ordinary people struggling to answer basic questions, is both entertaining and applicable to teaching. This article describes how an instructor can strengthen students' conceptual understanding by creating an element of confusion, or "cognitive dissonance," in the students' minds using Jaywalking-style interactions in the classroom.

Keywords: *abstract algebra; Jay Leno; mathematics videos; teacher-student communication; cognitive dissonance*

1. Jaywalking on *The Tonight Show*

One of our favorite late-night skits was Jay Leno's "Jaywalking" on *The Tonight Show*. This consisted of Jay Leno walking around the Burbank area and asking people questions that many would find fairly basic. Examples included, "In what galaxy do we live?", "How many commandments are there in the Bible?", and "Who was the first president of the United States?".

[1]Adam Glesser has been teaching mathematics since people started teaching him mathematics. He enjoys the state of being perplexed as much as the state of resolution.

[2]Marty Bonsangue has taught all levels of mathematics from grade school to grad school. He remains fascinated with the study of mathematics as a very human endeavor.

The comedy came when the person asked—who usually thinks of herself or himself as being reasonably well-educated—either has no idea or confidently gives an incorrect answer. Here is an exchange from an actual clip.

Leno: In what country would you find the Panama Canal?

Person on Street: I have no idea.

Leno: Well, in what country would you find the Great Wall of China?

Person on Street: China?

Leno: Yes! So, in what country would you find the Panama Canal?

Person on Street: China?

Part of the attraction is, of course, the wonderful feeling of superiority one enjoys while watching people fail at something one considers simple. However, the reactions of people for whom the answers are not obvious can range from a self-aware chuckle to visible embarrassment and discomfort. While these may not be the ideal emotional states for getting people to open up, they can be exceptionally useful for learning.

2. Jaywalking in Educational Theory

Pointing out the failures of others is not new in educational theory. Derek Muller's 2008 doctoral dissertation [4] reports on using videos to teach difficult concepts from physics. Groups of students were pre-tested and then given either a video providing a straightforward explanation of physical laws, or a video in which incorrect explanations were given but then followed up with a dialogue between a tutor and a student in which these errors were discussed in greater depth. While students preferred the expository style of the lectures in the first video, students watching the latter video performed significantly better on a post-test than those who only had watched the lecture. Muller explained this by observing that the approach in the second video necessitated a higher cognitive load than that used in the first video.[3]

[3] Reducing the cognitive load may increase student satisfaction, but also lead to less robust long-term understanding. This is supported by the 2014 Ph.D. thesis of Roy [7], who experimented with e-Proofs in teaching proof comprehension.

Muller expanded on the videos in the study and now produces more such videos on his YouTube channel *Veritasium* [5].

A somewhat related finding involved the long-running Comedy Central satirical news show, *The Colbert Report*. The show's host, Stephen Colbert, mocks the conservative Fox News talk show hosts by becoming a caricature, amplifying their personalities and taking their arguments to their logical conclusions. Baumgartner and Morris [1] studied the effects of watching *The Colbert Report*. They observed that, while humor did change people's attitudes towards the topics, because he presented his opinions as truth, Colbert was actually convincing conservative viewers to become even more conservative in their beliefs. This is an example of what LaMarre, Landreville, and Beam [3] called "motivated message processing." The Baumgartner and Morris study was in stark contrast to their previous 2006 study on the effect of Jon Stewart's *The Daily Show* on the beliefs of the audience: Stewart's approach helped to point out the absurdity of certain political situations, while Colbert's approach tended to clarify people's political views rather than reinforcing them.

These findings support classic education theory. More than forty years ago, Jean Piaget and Barbel Inhelder [6] wrote about the importance of *cognitive dissonance* to develop deeper and more abstract levels of thought. Robert Karplus, a Berkeley physicist-turned-educational-theorist, described the Learning Cycle in which initially incorrect thinking can lead to meaningful correct thinking, as well as increased self-efficacy, or self-confidence, in the subject [2]. Specifically, Karplus posited that active exploration and invention can lead to higher-level processes including discovery, expansion, application, and extension (see Figure 1).

The phenomenon suggested by and described by motivated message processing, cognitive dissonance, or the learning cycle is evident in the mathematics classroom as well. It is always discouraging to walk through the classroom and see that a student has incorrectly written down what the instructor just painstakingly finished explaining. In fairness, students will often say that what they wrote is what they heard the instructor say. Thus, while the error was one of reception rather than transmission, correcting the error remains a shared responsibility.

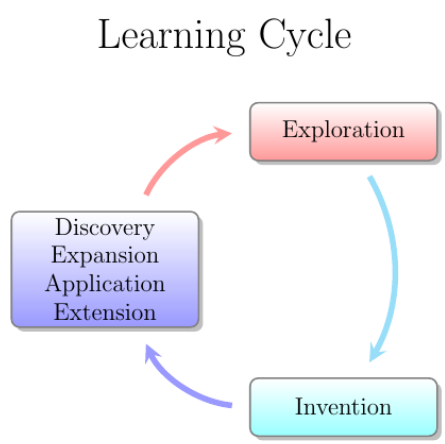

Figure 1: The Learning Cycle (Karplus, 1968 [2]).

3. Jaywalking with Symmetric Groups

One useful approach is to have students present their work at the board, allowing for other students to see the successes and failures—the latter possibly being more useful than the former. We have tried to make this process more intentional by producing videos for abstract algebra that show students making common mistakes and then having those mistakes corrected. For instance, one video shows several students mistakenly using the converse of Lagrange's theorem to conclude the existence of subgroups of a certain order. Another video that aims to clarify the cycle notation used in symmetric groups, shows three students incorrectly computing the composition (1 2)(1 2 3), each using a different, but frequently used, method, as described below.

Student 1: Working right-to-left, we see that 1 goes to 2 goes to 3, and then 3 is fixed—so 1 goes to 3. Then 3 goes to 1 goes to 2, and 2 goes to 1, so 3 goes back to 1. So, (1 2)(1 2 3) = (1 3).

Student 2: 1 goes to 2 in the first cycle and then 2 goes to 3, so 1 goes to 3. Now 3 is fixed in the first cycle and then goes back to 1, so 3 goes back to 1. Thus, (1 2)(1 2 3) = (1 3).

Student 3: The 1 2 in the first and second cycles cancel, so we just get (3).

However, after giving the students a geometric representation, they all deduced that the correct answer is (2 3) as seen in Figure 2.

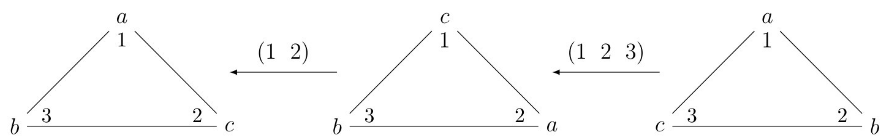

Figure 2: Composition of Cycles.

A more general explanation for how to compose cycles follows. The video concludes with a more complicated example, and then a student presenting the solution. The video, hosted on YouTube, has a link in the show notes to a supplementary worksheet that gives explanations as well as problems and solutions.

4. Jaywalking for Beginners

The *Primary Teacher's Fallacy* is that we believe our students understand our words in the same way in which we mean them. This is not to say that they cannot or will not listen effectively, only that the barrier to reception—which includes distractive social media, the coffee about to spill off their neighbor's desk, the homework due in their next class, or, simply, their inner monologue—is often exceptionally high. This means that effective transmission attempts should be engaging, brief, and memorable.

The use of videos is a natural way to avoid attention-deficit difficulties. Students can watch the videos at their leisure, easily repeating the viewing if a distraction arises. However, creating videos is an expensive proposition requiring more time, equipment, and expertise than is reasonable to expect from a typical faculty member. What, then, is possible outside of the video realm?

Faculty frequently encourage student engagement by having students present at the board. The mistakes students make in this setting are often instructive to the student presenter, attentive audience members, and even the instructor. A variation of this activity is to make these presentations performances. Consider stacking the deck by meeting with students ahead of time and prompting them with incorrect responses! Students that are hesitant to participate because of the fear of making an honest mistake in class are often happy to join in when they are a conspirator. Have, say, three or four students answering questions in front of the class. Some of the answers should be common mistakes, while others should be believably unbelievable. The former answers will address some—perhaps most—of the problems the students have, while the latter will help the activity be memorable.

After showing the incorrect student answers, increase the cognitive load by giving a simple example that contradicts the answers. Let the students discuss the situation long enough to let the incongruity percolate, but not so long that their frustration becomes a distraction. At this point, give the students an explanation as simply and as plainly as possible. Again, we are aiming for brevity because long-winded explanations and complicated but revealing examples are products of the Primary Teacher's Fallacy. Follow up by giving the students examples to work that build slowly in complexity and which allow them to internalize the deeper concepts.

5. The Moral of the Story

The difficulty in transmitting information to our students is that there are many obstructions to reception and very few error-corrections done on their end. One way of subverting this dilemma is to make the errors themselves the transmission. This may have the effect of increasing student engagement and decreasing the conceptual misunderstandings that prevent students from a deeper level of comprehension.

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