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## A NOVEL METHOD OF CONSTRUCTING SORTING NETWORKS

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Summary

The construction of sorting networks has been a topic of much recent discussion [1] - [5]. In view of the apparent difficulty of verifying whether a reasonably large proposed sorting network actually does sort, the most useful approach for constructing large networks seems to be to devise a recursive scheme which constructs a network which is guaranteed to sort, obviating the verification phase. Examples of this approach are presented in [1],[5]. In this note, another such approach is presented.

The most economical 16-line sorter known has been constructed by Green [3], [4]. His approach is to successively sort lines whose indices differ in one component of the binary expansion. This yields a partial ordering of the lines which is isomorphic to a Boolean "n-cube" configuration. This configuration is then further sorted to yield a linear order. The network for accomplishing this is constructed in a clever, but ad hoc manner, and no techniques for extending this approach to larger numbers of lines have appeared.

In this note such a technique is presented. However, it suffers from the fact that it produces networks which are no more economical than the odd-even merge networks of Batcher [1]. Nevertheless, some insight may result from a knowledge of this technique.

The approach is to reduce an n-cube configuration to an n-m cube in which the vertices represent linear orders of m components. A recursive rule is given which applies this technique to obtain a complete sorting network and the correctness of the rule is proved. It is then shown that the number of comparisons for an n-line network are the same as Batcher's construction, although the networks are definitely not isomorphic to Batcher's. For certain numbers of lines, this method yields net-

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works which are related to Batcher's by a kind of "flipping" operation described in [2]. Precisely what relation holds between these two constructions has not yet been discovered.

A complete presentation of these results appears in [6]. The construction is derived for the more general k-ary n-cube, but upper bounds are only shown for k=2 (the "Boolean" case). Whether other values of k yield better results has not been thoroughly investigated. Proofs of correctness are done in terms of partial orders, using a useful and general lemma about "cross products" of partial orders and the technique of Liu [7].

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