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What We Say, What Our Students Hear: A Case for Active Listening

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I want us to think about what our students hear, which is often not what we are trying to convey. I think we can all believe that things go on in our students' heads that we don't understand and that we need to pay more attention to. I suspect that more than one of you has put a problem on a test that lots of students have answered incorrectly and you've said, "How could they mess that up?" I hope that this paper will give you some clues about why they "messed that up."

More importantly, I want to encourage you (and me) to listen more carefully to what our students do say. The "active listening" in this paper involves listening on OUR parts.

MESSAGES OUR STUDENTS HEAR

What are some messages that our students hear?

We say, "This won't be on the exam."
They hear, "This is not important."

We say, "You will need this concept next year."
They hear, "I don't need to learn this concept this year."

We say, "We want you to use algorithms quickly and automatically."
They hear, "Mathematics does not require thought."

We give timed tests.
They hear, "Mathematics must be done quickly." They, therefore, will not struggle with problems that they cannot complete quickly.

We give them lots of exercises with no words.
They hear, "Mathematics is not a language of communication, only computation."

We don't give partial credit.
They hear, "The mathematics is the final result, not the process. Mathematics is either all right or all wrong; there is no middle ground."

ONE MODE OF REASONING

To begin to understand more deeply what our students hear, I want to think about mathematics, and about one suggested style of reasoning that people might use in mathematics.

- Gets right to solution in a structured, algorithrmic way, stripping away any context.
- Uses a mode of thinking that is abstract and formal.
- Geared to arriving at an objectively fair or just solution upon which all rational persons can agree.
- Employs a legal elaboration of rules and fair procedures.
- Confident to judge.
- Is analytic.

How does this reasoning style relate to mathematics? Think for a moment about this reasoning style. Does it describe mathematics for you? Do you think it describes mathematics for your students? Is it like reasoning in mathematics? What are its limitations?

Is this what we would like our students to be able to do with the mathematics we teach them? Let's think about this reasoning mode.

Gets right to solution in a structured, algorithmic way, stripping away any context.
We want them to be efficient. We want them to be able to apply the mathematics to varied situations. We want them to use the precise, formal, logical processes that we have taught them.

Uses a mode of thinking that is abstract and formal.
Of course!

Geared to arriving at an objectively fair or just solution upon which all rational persons can agree.

Isn't this the goal of mathematics? Mathematics is well defined. When you pick up a mathematical object you know immediately whether or not it fits the definition at hand.

Employs a legal elaboration of rules and fair procedures.
Mathematics is a formal system. The definitions, axioms, theorems, algorithms, give us a common basis for discourse.

Confident to judge.
Because it is well defined, we can work within it with certainty.

Is analytic.
Mathematics is an analytic system, which we develop and use analytically.

But when we read this list we probably do not all hear the same things that our students hear. A mathematician may read "abstract and formal" and see an abstract system that is pretty well laid out. If there were any ambiguities, we have removed them by our choice of definitions and by staying in a small domain. It is complete with underpinnings of human exploration that we know are present.

But many of our students hear "abstract and formal" as "coming from outside without meaning." Many believe that they need to give up their own ways of thinking and memorize these algorithms, definitions and proofs that are meaningless to them. They do not see a system that is applicable to many contexts. They see isolated sets of instructions that are highly compartmentalized.

I have shared this reasoning style with many in mathematics and mathematics education over the years. Most have agreed that this list illustrates the way that mathematics is conveyed in the classroom, in traditional textbooks, and in our professional writing. We present elegant, well-polished proofs, carefully devised sets of examples, collections of theorems and corollaries, and sets of applications. Mathematics is polished and complete. (See Buerk 1985, pp. 63-64.)

But what we present is like the part of an iceberg that we see above the water. WE know what the tip of the iceberg is sitting on. We may know exactly what the underpinnings of a theory are. If not, we do know that what we see has underpinnings that we could study if we wanted to.

Many of our students see just the tip of the iceberg, which we present to them. They have no idea what is under the surface of the water. They may describe what is under the water in a very different way than we would. Some even believe that the theory or concept has no underpinnings, that it has no human or mathematical connections.

> "
> ***We say, "This won't be on the exam." They hear, "This is not important."***

We want our students to know that mathematics is grounded in human thought, human exploration, and human questions. Will this happen if we only present them with the tip of the iceberg? Will this happen if we only share with them the public image of mathematics as a completed formal system?

STUDENT VOICES
Let's listen to several articulate students who have seen or heard only the public image of mathematics-only the tip of the iceberg.

> Peg writes:
> On the eighth day, God created mathematics. He took stainless steel, and he rolled it out thin, and he made it into a fence, forty cubits high, and infinite cubits long. And on this fence, in fair capitals, he did print rules, theorems, axioms, and pointed reminders. "Invert and multiply." "The square on the hypotenuse is three decibels louder than one hand clapping." "Always do what's in parentheses first." And when he was finished, he said, "On one side of this fence will reside those who are good at math. And on the other will remain those who are bad at math, and woe unto them, for they shall weep and gnash their teeth."
>
> Math does make me think of a stainless steel wall - hard, cold, smooth, offering no handhold, all it does is glint back at me. Edge up to it, put your nose against it, it doesn't take your shape, it doesn't have any smell, all it

> does is make your nose cold. I like the shine of it—it does look smart, in an icy way. But I resent its cold impenetrability, its supercilious glare. (Buerk 1982, p. 19)

Here is a creative, insightful woman who presents a view of mathematics as created by God, not by human thought. Mathematics is a fence separating people—for Peg it separates those that are good at mathematics from those that are bad at mathematics. That division is absolute, for the fence is too high and slick to climb over, and it is too long to go around. The fence presents to us all the rules of mathematics. Mathematics and this stainless steel wall have no human warmth, no smell, no flexibility—just a "cold impenetrability" and a "supercilious glare." This view of mathematics as an absolute, closed system with no human connections is clear and well defined for Peg. She would like a way to connect with mathematics, but finds none.

> Jackie, a second student, writes:
> I was exposed only to the public image of mathematics. To me, there seemed no room for interaction with the content, no possibility of connection with the ideas. Mine was the role of tourist who merely looks out at the sights that surround [her] as they travel past in a blurred rush. (Buerk & Szablewski 1993, p. 151)

Jackie, like Peg, wants to find a way to connect with mathematics. She feels like a tourist on a whirlwind tour with no time to catch her breath or appreciate the sights. She elaborates, in an assignment to write a letter to her next mathematics teacher:

> I realize that in order to help us realize all that already exists in the world, in order to guide us through all the worlds of mathematics, you must keep to a strict itinerary. If you didn't, we would not be exposed to all we must be exposed to in order to reach the destination of "mathematician," "chemist," "well-rounded person." But don't you see that in your well-intended efforts to show us all the "landmarks" of those worlds, you are not allowing us to touch? How can we come to say that we believe in a thing, a concept, an idea, if we ourselves do not know it is real? (Buerk & Szablewski 1993, p. 152)

Jackie lets us know how frustrating it is to not be allowed to touch and experience mathematics. She also shared her discomfort in the mathematics classroom in the following:

> Unlike English class, math was not a place for ideas in process. You could not say or share something you were thinking about. You could only share with the class completed perfected thoughts, and I simply had no such thoughts concerning math. (Buerk & Szablewski 1993, p. 152)

For Jackie, mathematics requires a kind of thinking different from her own, because her thoughts in mathematics do not come out as completed thoughts. She needs to slowly develop her thoughts, but she believes that her own thoughts are not allowed in the mathematics classroom. Since she believes that she cannot think in mathematics in a way that works for her, she becomes silent. Seeing only the tip of the mathematical iceberg reinforces these beliefs in many of our students, even when we tell them that mathematics is more than what they see on the surface.

Jackie tells her story in an article we wrote together in MAA Notes #32, *Essays in Humanistic Mathematics*. The essay is entitled, "Getting Beneath the Mask, Moving Out of Silence."

> A third student, Lee, wrote,
> Doing math can result in a precise answer or an estimate but it is not a thinking process. Rather it is a process of identifying, comparing, and doing a problem in relationship to that identification and comparison. (Buerk 1990, p. 80)

Be careful as you read this. Lee means that he does not think when doing mathematics. His process is mechanical. For him "identifying and comparing" mean that he tries to find the correct algorithm or procedure.

> Listen to Lee explain:
> The math process is one in which all attention is focused on a narrow subject. My mind is not allowed to create, or wander, or to think

> about doing the problem. My mind says, "Compare this problem to others that are like it and base your answer on the way you found your answer to that other problem." (Buerk 1990, p. 81)

If there is no algorithm in his collection then he cannot solve the problem, he has no recourse, but to give up and wait for someone else to solve the problem for him.

Peg, Jackie, and Lee are showing us a conception of mathematics that is quite different from our own. For them mathematics is a complete, closed system. While it is important, it is beyond their grasp. It seems very cold; it is not something they can relate to or touch. They do not believe that mathematics was created by people. They believe that learning mathematics requires following exact patterns, algorithms, and rules that someone else has given to them. They view mathematics as rote. For many students holding this rote conception of mathematics, doing mathematics means putting aside their own thinking, and instead, memorizing algorithms that have no meaning for them.

STUDENT INTERACTIONS

Let's look at some examples of interactions with students who have rote conceptions of mathematical knowledge.

First let me tell you about my encounter with Jake, a college freshman. Jake accepted a rote conception of mathematics without concern.

> In a class discussion about exponents, [he] told me that exponents were added when multiplying factors with the same base. I asked him why. He said, "That's the rule." I asked him why the rule said that. "It just does," he replied. "It is the rule I was taught." "But why?" I persisted. He looked at me very seriously and asked, "You mean there's a reason?" Jake was very surprised to hear that there might be a rationale for this mechanical manipulation. (Buerk 1985, p. 60-61)

Second, let me share with you a situation that is not atypical. I'm sure that we all work to justify as many algorithms, extension of rules, and definitions as possible. In that spirit, consider the following:

Said at the blackboard:
Question: If we decide to rewrite $\sqrt{x}$ in exponential form, x^r, why does the value of r have to be 1/2?
Suppose we assume that there is a real number r such that $x^r=\sqrt{x}$
We know that: $\sqrt{x}\,\sqrt{x}=x$.
Since $x^r=\sqrt{x}$, we can replace $\sqrt{x}$ by x^r, so $x^rx^r=x$.
Now we want to preserve the rule (pattern) $x^ax^b=x^{a+b}$, that we know works for integers, so we can write $x^rx^r=x^{r+r}$.
Then $x^{r+r}=x$.
Therefore, $x^{2r}=x$.
Which tells us that $2r=1$.
Hence, $r=1/2$and, therefore, $\sqrt{x}=x^{1/2}$.

We go through an explanation something like the one above, and we write the steps on the board as we go along.

Written on the blackboard:
Question: If we decide to rewrite $\sqrt{x}$ in exponential form, x^r, why does the value of r have to be 1/2?
Suppose that $x^r=\sqrt{x}$.
$\sqrt{x}\,\sqrt{x}=x$.
$x^rx^r=x$.
$x^{2r}=x$.
$2r=1$.
$r=1/2$

My colleague, Ann Oaks, the Chair of the Mathematics and Computer Science Department at Hobart and William Smith Colleges in Geneva, New York, saw a student in her office the day after she gave an explanation like the one above. She happened to notice his notes. He had written the following:

Student Response:
Step 1: First you set $x^r=\sqrt{x}$.
Step 2: Then you set $\sqrt{x}\,\sqrt{x}=x$.
Step 3: Then you set $x^rx^r=x$.
Step 4: Next you write $x^{2r}=x$.
Step 5: Then you say $2r=1$.
Step 6: $r=1/2$

Not only had the student turned the explanation into a set of steps, he did not even include the question his steps were responding to. Ann checked with her class and found that many others had the same kind of notes. Her explanation had become another procedure to be memorized.

Many of our students are not listening to our explanations as a way to help them make meaning of the concept for themselves—to really understand the concept. They do not see that we are trying to place the definition in a context for them. No, the students are seeing our explanation as another procedure for them to learn.

> *We say, "You will need this concept next year." They hear, "I don't need to learn this concept this year."*

Third, in a recent research project at a small liberal arts college, two good Calculus II students were given two problems to solve, and their process was audio-taped. The first problem was a rather easy integral, but the process to solve it was not obvious. Students had to stick with it, but they had the skills to solve it. The second problem was a mathematical puzzle problem that was much harder to solve than the integral problem. The students gave up on the integral fairly quickly, because their instructor had not shown them how to do that type of problem. Their rote conception of mathematics took hold. But, they stayed with the puzzle problem until they got the solution. They kept saying, "We can keep trying." They solved it. They saw the integral as a classroom problem. They gave up because they had not yet been shown the algorithm to use. Like Lee, they believed that they needed an algorithm, or they could not approach the problem. The other problem was not a "school" problem, so they used their ingenuity to solve it. For other problems, non-school problems, they could explore. Their rote conception of mathematics applied only to "school" problems.

While many students' view of "school" mathematics problems is disturbing, it is exciting to notice that our students still retain their innate intellectual curiosity. We can, and must, build on this curiosity. We must couch more of our problems in forms that prevent our students from looking for past models and, instead, encourage their creative thought. We can ask questions differently. Many students have a mechanical method to approach "solve." We could try "verify," "explain," "explore," "discuss," or "describe to someone not in this class." With these options many begin to realize that they are being given permission to think.

We must be careful when we listen to students. They may use words that give us hope that they are really understanding the mathematics that we are teaching them, when they are not. For them "math is important" may refer only to the kinds of questions we ask on tests and quizzes. For them, "math is useful" might refer only to the kinds of mathematics problems that they can solve. When they focus on "understanding" they often mean that they know which algorithm to use. We must listen carefully, or we will not really hear how our students view mathematics. And sometimes their views are ones that we really do not want to hear.

The reasoning mode we have considered conveys to many only the public image of mathematics, only the tip of the mathematical iceberg. But if our students have a rote conception of mathematics, they do not use our first reasoning style. In fact, they do not reason at all.

A SECOND REASONING MODE

Let's look at a second suggested style of reasoning.

- Tries to experience the problem, relate it to personal world, clarify language, create context, remove ambiguity.
- Uses mode of thinking that is contextual and narrative.
- Geared to looking at limitations of any particular solution and describing the conflicts that remain.
- Tolerant in attitude toward rules and more willing to make exception.
- Reluctant to judge.
- Is intuitive.

How does this reasoning style relate to mathematics? Think about this reasoning style for a moment. Does it describe mathematics for you? Does it describe mathematics for your students? Is it like reasoning in mathematics? What are its limitations?

My colleagues with whom I have shared this reasoning style find that it relates to mathematics as well. Their consensus is that this reasoning style represents the way that mathematicians do mathematics. "Mathematics is intuitive," they say. They stress the creative side: attention to the limitations and exceptions to theories, the connections between ideas, and the search for differences among theories and patterns that appear similar. (See Buerk 1985, pp. 63-64.)

Think about this. Mathematicians use a reflective, contextual, groping strategy to develop mathematics, but they share with the world ONLY the polished, finished product, which gives no clue to the process used to create it. Mathematics has a public image of an elegant, polished finished product that hides its human roots. It has a private life of human joy, struggle, challenge, puzzlement, and excitement. This works well for us, who know both the private and public worlds of mathematics. It does not work well for the student who sees only the tip of the mathematical iceberg and does not know something is below the water's surface.

For me, personally, mathematics is creative, dynamic, and evolving. I value its personal, intuitive, logical, and reflective dimensions. I enjoy the process through which mathematics is created. This process, which I see as involving conscious work, unconscious work, intuition, conjecture, reflection, redefining the question, asking new questions, and finally a degree of certainty, is a very human one resulting in a formal, logical, and consistent presentation of a complete idea. As you finish one problem or proof you are often left with many new questions to pursue. This process is used by mathematicians, educators, students, me, and in fact, by any inquisitive person approaching ANY question that is new to them. I want my students to know this side of mathematics as well.

> **"**
> ***We say, "We want you to use algorithms quickly and automatically."***
> ***They hear, "Mathematics does not require thought."***

By accepting the public image of mathematics, many thoughtful people find our discipline easy to reject, for it seems not to offer the opportunity for their own thought. Others find this image intimidating; they struggle to model someone else's thought process without truly understanding that process. Others reject mathematics because they find no way to include within it their own intuitive understanding of quantitative concepts or to use it as an opportunity to further their own quantitative intuition. Our presentation of mathematics in traditional ways gives a distorted picture of mathematics to many who are not in our discipline. Peg, Jackie, and Lee have made this clear.

STUDENT VIEWS OF MATHEMATICS

Our students form views of mathematics based on what they hear about mathematics in our culture, but more importantly, what they hear and experience in their mathematics classes. Let's listen to some more views of mathematics held by some mathematics students. Think about how each of the following students views mathematics as you read his or her words.

> First,
> Math is most like an earthquake. If an earthquake was to hit, even just a tremor, it could knock down and ruin a lot of things. Just like in math, if you make one error in a problem, even a small one, it can ruin or tear down all of your work. (Gibson 1994, p. 8)

This student is saying math is something over which he has no control. In fact, mathematics is like a natural disaster over which no one has control. Also only the correct answer matters. In his classroom he gets no credit for the process or for the ideas that would lead to the correct answer. He does not have confidence in his ability to work with mathematics. He is not in control.

> Second,
> Doing math is like driving through a city that you used to know but that has grown more complicated in your absence. You start down certain streets that seem familiar, but then you realize that these are dead ends. You can see the place you're trying to get to, but the way is full of detours and traffic lights that seem to be stuck on red...When you finally get to your destination, it feels good that you've traversed this dangerous and confusing city, but sometimes the thrill isn't worth the effort put forth

> to get there...Math is scary, like a big city. (Buerk 1996, p. 28)

For this student doing mathematics feels like being subjected to unnecessary roadblocks that often make the effort to maneuver through them feel like time wasted. While this student may be successful in mathematics, she finds much of what she learns to be a waste of time. She knows that she will not be able to retrace her steps the next time she comes to that city. She will probably take only the courses that are required for her major.

> A third,
> For me math is like a toolbox. The tools in the toolbox represent concepts, formulas, and techniques needed to solve problems. However, I could always use the wrong tool, or maybe my toolbox doesn't have a tool I need. The tools can be used to construct something, or they can be used to strip down a complicated machine so that all the parts can be analyzed. Some tools can become obsolete if I acquire new ones. When working with the tools of mathematics, I could just as easily use them to fulfil my needs by solving problems. (Buerk 1988)

This student, unlike the first two, feels empowered and reasonably confident in his ability and skills. He does the mathematics asked of him. We must hope that his tools are not just algorithms and procedures, but that he has other tools that are reasoning skills and problem solving strategies. The use of a tool analogy could be very limiting. Does the student see various uses for each tool? Can the student create his own, new uses for a tool in his toolbox. We would like this student to have the tools to work with mathematics in an integrated, not compartmentalized way.

> And, a fourth,
> To me, math is like a used car that you get for a good price: sometimes it runs smoothly, but on certain days things go wrong. It's frustrating, like a car can be, when it won't go right. You have to sweat, yell and curse, and sometimes pay a price to get the car going, but once it does go everything's great. With math, things don't always work out right. I don't know how many times I've screamed and pulled my hair out trying to "fix" a math problem, but when I finally figure it out, I feel fantastic; like I've accomplished something. Sometimes you break down, in a car or during a math problem, but if you work with it, you'll get to where you've going! (Gibson 1994, p. 9)

This student finds much in mathematics to be a struggle, but she stays with it. She doesn't give up easily; she has to work for her successes. She knows she is travelling in an old car, lacking many of the advantages of a newer one. Therefore, she knows she must compensate for having the old car, by developing a determination to fix things that go wrong. The determination we hear from this student is missing from many of our students who give up much too easily and wait for someone else to give them the answer.

I hope that you are beginning to hear how your students view mathematics. Their rote conception is deeply imbedded in their beliefs, and, therefore, very difficult to change. While students may not like mathematics as they view it, it is the only mathematics they know. Remember the example I gave about the square root of two? The instructor was trying to help her students understand that their definition of fractional exponents was consistent with the system that they were using. The students saw this explanation only as another procedure to follow. As we try to help our students see mathematics as a human endeavor, we will face resistance.

Ann Oaks, Hobart and William Smith Colleges, teaches a course called "Discovering in Mathematics," in which students actively create mathematical ideas and have lively discussions about them. They do become excited and do approach mathematics in very different ways. However, her students consistently call the course, "Discoveries in Mathematics." They do not see themselves discovering mathematics. They do not see anyone discovering mathematics. They still see themselves as learning about the discoveries that are mathematics. While they can change their behavior, it will take more experiences to fully change their rote conception of mathematics.

OUR TWO REASONING MODES

I want to come back to the two suggested reasoning styles that we have discussed. They may sound a bit awkward to you. They were not originally written with mathematics in mind. I drew them from another source and asked my colleagues to help me to understand how they might relate to mathematics. It was my colleagues who helped me see the connection to the public and private worlds of mathematics. It is clear that mathematicians need a blend of these two styles. One to create and do mathematics, and the other to present to the world the mathematics that we develop and use. We could not do mathematics if we did not integrate these two methods of reasoning.

I developed these lists back in 1983 while reading the work of Carol Gilligan, especially her book, *In a Different Voice*, and the related work of Nona Lyons. In dealing with hypothetical and real moral dilemmas they found that men tended to prefer the first reasoning mode, which is often called "separate" and that women tended to prefer the second reasoning mode, which is often called "connected."

"

We give timed tests.
They hear, "Mathematics must be done quickly."

By sharing only the tip of the mathematical iceberg, the public image of mathematics we are encouraging a rote conception of mathematics. We are encouraging students to memorize symbol strings that they find meaningless. But, by sharing only the tip of the mathematical iceberg, the public image of mathematics, we are also reinforcing to many females the cultural stereotype that mathematics is a male domain.

It is exciting today that because of the reform movement in mathematics education, we have a wealth of pedagogical and curricular resources available to help us show students more than just the tip of mathematical iceberg. Materials from the various reform projects encourage students to use both of these reasoning modes. Students are given experiences in the second mode to explore, to experience, to follow their own thoughts in process, to listen to the ideas of others, to touch, to feel, to work collaboratively, and to write. These experiences are necessary to help break down rote conceptions of mathematics and to see what is supporting the tip of the iceberg. The students can then see, and even help define, the finished product, and see it with meaning.

It is my personal hope that as we incorporate more of the ideas of the reform movement at all educational levels, we will see fewer rote conceptions of mathematics and less fear of mathematics in our students in the future.

I am of the generation that saw and participated in remarkable changes in opportunities for females. In the fifties my interest in mathematics was encouraged, but my only career option was teacher. While I never regretted that career choice for me, I have worked very hard in the ensuing years on issues of gender equity in mathematics. I have also worked hard to have the strategies of the reform movement included in mathematics classrooms. Those of us using these strategies in the seventies thought of them as feminist pedagogy, though we avoided using those words as much as possible. It has been exciting for us to see them come into the mainstream in the 1989 Curriculum and Evaluation Standards for School Mathematics and in the reform movements in mathematics education.

A MATHEMATICIAN'S STRUGGLE TO HEAR HIS STUDENTS

I have tried to help you understand what our students hear and to some extent why they hear it. I hope that I have helped you to see why the strategies of the reform curriculum are important for our teaching. But the fact remains that listening to our students can be difficult. We really have to practice hearing what they are really saying to us. To reinforce this idea I have chosen the words of David Henderson, a research mathematician at Cornell University, who describes his experience eloquently in "Mathematics and Liberation," which appeared in *For the Learning of Mathematics* in 1981. He says:

> Let me relate what happened to me when I started teaching calculus for the first time (after I was already an established mathematician).
>
> I tried to listen to the people in the class. I tried to understand what their questions were. I found that some people were not thinking clearly because of emotional problems or because of rigid reactions that came from previ-

> ous conditionings. But other people were obviously thinking clearly, and I tried to understand what they were saying. In many cases I found this terribly difficult—my gut reaction was that it couldn't possibly be right—it felt like nonsense. I felt threatened—here was something which I couldn't see in an area I felt certain about.
>
> Gradually, after much persistence and with the help of friends, I began to sense that I had blinders on—that my ways of understanding calculus had blinded me to other ways of perceiving. I saw that many of the people in the class had real questions about the meaning of limits and derivatives—questions which I could not answer or questions which I then started to explore for the first time. I lost a certain narrow feeling of certainty but gained a broader perspective. Now I perceived calculus in a different way. (Henderson 1981, p. 12)

Henderson documents his struggle to learn to really hear his students. In his essay he then reflects on what his lack of hearing, lack of listening, might have meant to his students.

> What was happening to the people in my class who were asking a real question I couldn't understand? Some correctly sized up the situation and blamed my blinders, but this was rare. Most blamed themselves.
>
> It is a hurtful experience to have someone whom you see as an authority not understand a real question of yours. When this and other distressful mathematics experiences happen to people enough times over the years, they feel stupid, they feel they can't think about mathematics. They then react to mathematics through fear or in rigid, rote ways. Their reactions are reinforced by the cultural view that mathematics can only be understood by a select few. (Henderson 1981, p. 12)

Henderson understands how a rote conception of mathematics might develop and might be reinforced. He also understands how this conception of mathematics is linked to the fear and anxiety that many feel in the face of mathematics. Henderson continues,

> Recently, I was thinking back over the times that my perception of mathematics had been changed by the insights or questioning of a person in my class. Suddenly, I realized that in almost all of these cases the other person was a woman or from a different culture from by own. I don't think that this is just a coincidence. (Henderson 1981, p. 13)

Focusing only on the public image of mathematics excludes many voices from the mathematics dialogue and limits our own understanding of our discipline.

A WORD OF CAUTION

Because of their rote conceptions of mathematics, our students often do not hear what we want them to hear. They often DO hear what we leave unsaid and we do not intend for them to hear. We need to reflect on how we view mathematics and be clear about it for ourselves, because we will convey our views to our students, even when we do not intend to. Do we believe that mathematics must be learned by rote? Do we believe that most students are not capable of doing the mathematics we are teaching them? If we believe it, our students will know it. Do we believe that mathematics is a wall separating people by gender or by cultural heritage or by socio-economic background? If so, our students will know it. Are we insecure about the mathematics that we teach? If so, our students will know it. Are we afraid of the creative ideas, insights, or real questions that our students have or might have if we give them the opportunity? If we are, they will know it. We need to be aware of our beliefs. Our students will hear our unspoken beliefs. They are very perceptive.

I have been reading a good bit about death and dying lately. My close friend and professional colleague, Janet, is dying of cancer. Elisabeth Kubler-Ross talks about helping health care professional learn to listen to their dying patients in *To Live until We Say Goodbye*. I found the following passage kept bringing my thoughts back to my students, our students, and to the things our students hear, in spite of what we say, or leave unsaid. Dr. Kubler-Ross writes:

> The purpose of my seminars was to teach young students in the helping professions to take a good hard look at their own fears, their own unfinished business, their own repressed

pains which they often unwillingly projected onto their patients.

Those physicians who were most afraid of the issue of death and dying never revealed the truth to their patients, rationalizing that the patients were not willing to talk about it. These professionals were not able to see the projection of their own fears, their hidden anxiety, yet the patients were able to pick up these feelings and, therefore, never shared their own knowledge with their physicians. This situation left many dying patients in a vacuum, unattended and lonely.

It is also true in our mathematics classes that what we leave unsaid is conveyed to our students. If we believe, but would never say, that mathematics is really hard, or elitist, or something to feel anxious about, they will hear that mathematics is really hard, or elitist, or something to feel anxious about. Many will believe they cannot do the mathematics we are teaching them. Many will believe that they are not the right gender or from the right cultural background to do mathematics. We may ask our students for their questions, but we may have trouble taking their questions seriously, as David Henderson did. They will hear that we do not want to hear their questions or that their questions are not appropriate, or that they must be really dumb to have asked their questions. They will stop asking their questions, but they may be left intimidated, fearful, and wanting to avoid mathematics at all cost. They, too, may be left "in a vacuum, unattended and lonely."

> ***We give them lots of exercises with no words. They hear, "Mathematics is not a language of communication, only computation."***

OUR OWN CONCEPTIONS OF MATHEMATICS

I hope that you are thinking about ways you can listen to your students and help them see both the private and public worlds of mathematics. Let me suggest that you begin by thinking about how you see mathematics. For you, is mathematics most like a maze, a puzzle, a set of tools, or a quilt of intricate design and artistic delight? What is your metaphor for mathematics? How do you fill out this image once you have chosen it? Thinking about your own metaphor for mathematics may help you think about your own conception of our discipline. Then you can continue devising ways to help your students gain a complimentary conception and thinking about the pedagogical and curricular strategies you will use to help them achieve this new conception.

I have for many years asked my students to write their metaphors for mathematics. I have shared a few with you today. [Note: A protocol to gather metaphors in a classroom setting appears in Gibson 1994, pages 11 and 12.] I was delighted to hear what my students thought about mathematics and how frank and honest they were in the metaphors they so freely shared with me. But before their metaphors could be really helpful to me I needed to clarify my own conception of mathematics. I was finally able to express my conception in the form of a metaphor as I ask my students to do. Let me conclude this paper by sharing with you my metaphor for mathematics. I wrote it following my visit to the ACEER Laboratory in the Peruvian Amazon rainforest and my walk on the canopy walkway that allowed me to experience all levels of the rainforest and to climb to the top of the canopy for a perspective from above.

> For me mathematics is like the Amazon rainforest, vast and filled with much that I know and much more that is new or unknown to me. The plants and animals of the rainforest have developed intricate interdependent relationships and adaptations for their mutual survival. By using the canopy walkway and talking with those I meet who point out new things to me and answer my questions, I come to appreciate the variety and differences at all levels of the rainforest and finally stand above the canopy to gain perspective. I read, observe and question; I experience and I touch; I share my knowledge with others always listening for their perspective and understanding. While I have much to learn about the plants, birds, and insects, and their interdependence on each other, I can appreciate the whole that they have collectively created. The rainforest is vast, sometimes thick and dark, other times

quite open; it is empowering, not frightening. On each visit I see more, understand more, and feel more connected with nature, knowledge, and myself. (Buerk 1996, p. 27)

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Math Induction

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Adapted from Camp, Dane R. "Math Induction" in Math Song Sing-a-long, edited by John A. Carter and Dane R. Camp. Booklet presented at Illinois Council of Teachers of Mathematics Annual Conference, Springfield, Illinois, 1998. May be sung to the tune of "Blowing in the Wind" by Bob Dylan.

How can you prove that a statement is true
For any counting number n?
Cause there's no way you could try them all—
Why you could barely begin!
Is there a tool that can free us
From this quand'ry we're in?
The answer, my friend, is math induction,
The answer is math induction!

First you must find an initial case
For which the statement is true
Then you must show that if it's true for K,
Then K+1 must work, too!
then all statements fall like dominoes
Tell me, how did we score this coup?
The answer, my friend, is math induction,
The answer is math induction!