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Cover Page Footnote

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Some Thoughts on the Epicurean Critique of Mathematics

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Abstract

In this paper, we give a comprehensive summary of the discussion on the Epicurean critique of mathematics and in particular of Euclid's geometry. We examine the methodological critique of the Epicureans on mathematics and we assess whether a "mathematical atomism" was proposed, and its implications. Finally, we examine the Epicurean philosophical stance on mathematics and evaluate whether it was on target or not.

1. Introduction

The mathematical and philosophical status, as well as the pedagogical relevance, of Euclidean geometry[2] has been extensively discussed in modern times. But this discussion is not new. As Proclus recorded, since the ancient times the Epicureans expressed skepticism towards the principles, results, and completeness of Euclidean geometry. Some researchers even suggest that the Epicureans proposed a "mathematical atomism", both as a critique and as a theory of geometry, to counter the principle of infinite divisibility, which is at the core of geometry.

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[2] As Stahl says in [28, pages 2–3], some mathematicians distinguish between Euclid's geometry and Euclidean geometry, i.e. Hilbert's axiomatization. In this paper, I use both terms interchangeably, to mean Euclid's geometry.

In this paper, we present the Epicurean critique, explain where it stemmed from, and examine whether enough evidence can be gathered to substantiate an Epicurean mathematical atomism. We see what the implications of such atomism could be, and evaluate whether the Epicurean methodological critique was on target or not. Lastly, we evaluate whether the Epicurean philosophical stance on mathematics was successful or not. We pose the following questions, as the basis for the discussion that follows:

(i) Why did the Epicureans criticize mathematics and what was the Epicurean critique? Where did it focus?
(ii) Was the Epicurean critique justified? Did the Epicureans suggest some different kind of mathematics?
(iii) Was the Epicurean philosophical stance on mathematics on target or not?

2. Incommensurability and Infinite Divisibility

The applicability and importance of mathematics in science and other human activities, given that the subject itself is based on abstract concepts such as infinity, empty set, and so on, is something that has raised many eyebrows. The modern reader may be familiar with Eugene Wigner's expression "the unreasonable effectiveness of mathematics", but the relevant philosophical concerns go a long way back in the history of thought.

For example, when someone proves that the number $\sqrt{2}$ is irrational, i.e., that it is not a fraction, this particular piece of knowledge, as well as the methods that produced it, lead to two important conclusions, one general and one specific: (1) Mathematics does not in principle depend on sensory experience. (2) Incommensurability gives support to the idea of infinite divisibility of line segments. Regarding (1), Alexander says:

> "In this connection, philosophers often distinguish between a-priori and a-posteriori knowledge. The latter [...] requires sensory experience for its justification, whereas the former does not. Mathematical truths, unlike, say, truths about the natural world, are known a-priori. You now know that is irrational, not on the basis of any measurements or observations, but rather on the basis of pure reflection." [1, page 5]

Regarding (2), Proclus says:

> "For when geometers demonstrate that there is incommensurability among magnitudes and that not all magnitudes are commensurable with one another, what else could we say they are demonstrating than that every magnitude is divisible indefinitely and that we can never reach an indivisible part which is the least common measure of magnitudes¿' [23, page 217]

Proclus' claim deserves a bit more analysis. There are two ways that magnitudes relate to infinite divisibility: (i) the one that relates to the continuum aspect of magnitudes, but does not involve incommensurability, and (ii) the one that relates to the dense aspect of magnitudes, and involves incommensurability. Aristotle basically discussed (i) and used it to argue against Democritus' atomism in *Physics* (B.VI) and in *On the Heavens* (B.III) [4], as well as against the atomism associated with Xenocrates in *Metaphysics* (B.I) and in *On the Heavens* (B.III) [4]. Proclus, as we read in the passage above, hinted at (ii) but without any clear explanation of how incommensurability and infinite divisibility actually relate. According to Karasmanis, the roots of the connection between the latter two concepts are found in Plato's *Philebus* in his notion of "apeiron" ("indeterminate"):

> "Plato's approach to apeiron is rather mathematical. He approaches magnitudes via incommensurability and not via infinite divisibility. Of course, all magnitudes are infinitely divisible in the Zenonian sense, and therefore continuous. But magnitudes are also dense, in the sense that they include incommensurable cuts. Is there any relation between infinite divisibility and incommensurability? Let me continue my reasoning beyond the text of the *Philebus*." [19, page 394]

And then Karasmanis continues to say that:

> "We have, therefore, two kinds of infinite divisibility: (1) Zenonian infinite divisibility which results in continuity; and (2) anthuphairetic infinite divisibility which produces the incommensurables and makes the continuum dense, thus generating magnitude. So supplementing the Aristotelean continuum — which is

> characterized by Zenonian infinite divisibility — with the incommensurables — which are found with the anthuphairetic infinite divisibility — we get magnitudes." [19, page 394]

So, how do incommensurability and infinite divisibility then relate? They relate via "anthyphairesis" ("alternated subtraction"), which is a process that successively subtracts the smaller of two magnitudes from the greater until the process results in the common measure of the two magnitudes. In that case, the process ends when the common measure is reached and the magnitudes are called commensurable. If the process does not end, then the magnitudes are called incommensurable and, hence, the magnitudes have no common measure. As Euclid states in Proposition X.2:

> "If, when the less of two unequal magnitudes is continually subtracted in turn from the greater, that which is left never measures the one before it, the magnitudes will be incommensurable." [12, page 17]

The discrete version of the process is also used in arithmetic in order to find the greatest common divisor of two numbers a and b, and is called the Euclidean Algorithm.[3]

[3] More precisely, if $a > b$, then successive divisions give:

$$\begin{aligned} a &= q_0 b + r_0, \\ b &= q_1 r_0 + r_1, \\ r_0 &= q_2 r_1 + r_2, \\ r_1 &= q_3 r_2 + r_3, \\ \vdots &= \vdots \\ r_n &= q_{n+2} r_{n+1} + r_{n+2}, \\ r_{n+1} &= q_{n+3} r_{n+2} + 0, \end{aligned}$$

where $0 \leq \cdots < r_n < r_{n-1} < \cdots < r_2 < r_1 < r_0 < b < a$. Then, the last non-zero remainder, namely r_{n+2}, is called the greatest common divisor of a and b. We write $gcd(a, b) = r_{n+2}$. For example, $gcd(33, 27) = 3$ because:

$$\begin{aligned} 33 &= 1 \times 27 + 6, \\ 27 &= 4 \times 6 + 3, \\ 6 &= 2 \times 3 + 0. \end{aligned}$$

Now, how does this relate to the Epicureans? As we will see in the next section, the Epicureans distanced themselves from Leucippus' and Democritus' notion of the "partless" atom, due to Aristotle's critique of the latter. Yet, the modified Epicurean notion of a theoretical "minimum" in an atom (understood as an indivisible common measure for atoms) does not seem to come to terms with the incommensurability of line segments, which implies infinite divisibility.

3. The Epicurean Position

For the Epicureans, both issues (1) and (2) in Section 2 were objectionable. Issue (1) was perhaps the reason why mathematics was not part of the Epicurean curriculum, as it did not accord with Epicurean empiricism. Yet, some Epicureans such as Polyaenus, Demetrius of Laconia, and Zeno of Sidon were mathematically knowledgeable, even though their interest in

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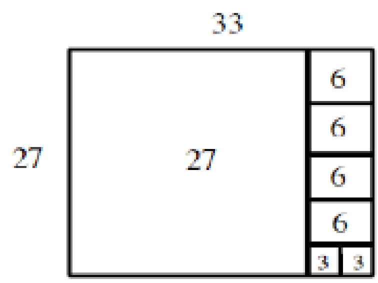
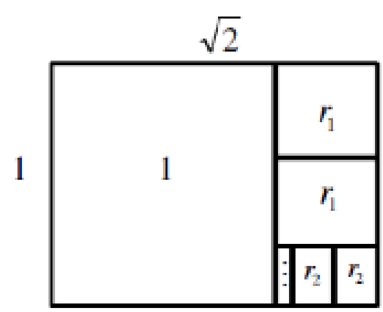
Observe, that this process relates to continued fractions as well. Indeed:

$$\frac{33}{27} = 1 + \frac{1}{4 + \frac{1}{2}}$$



Here notice that the numbers on the "diagonal" [1;4;2] in the continued fraction are the coefficients in the algorithm above. And, geometrically, they represent the subdivision of the rectangle into squares as in the figure above. But, now notice that in the case of $\sqrt{2}$, we have:

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2+\cdots}}$$



Hence an infinite fraction which means that the algorithm continues indefinitely. And, the subdivision of the rectangle above will never be complete.

mathematics as some claim had a philosophical "agenda."[4] We will focus more on issue (2), namely that incommensurability provided justification for the idea of infinite divisibility of line segments. Or, as Proclus[5] put it above (see Section 2), that every magnitude is divisible indefinitely and that we shall never reach the indivisible.

In the "Letter to Herodotus", Epicurus states that:

> "Furthermore, among bodies some are compounds, and others those of which compounds are formed. And these latter are indivisible and unalterable (if, that is, all things are not to be destroyed into the non-existent, but something permanent is to remain behind at the dissolution of compounds): they are completely solid in nature, and can by no means be dissolved in any part. So it must needs be that the first beginnings are indivisible corporeal existences." [9, page 2].

Also he says:

> "Besides this we must not suppose that in a limited body there can be infinite parts or parts of every degree of smallness. Therefore, we must not only do away with division into smaller and

[4] Sedley says that "Polyaenus wrote a work entitled Ἀπορίαι in which he set out the theoretical objections to geometry". He then states that at "least some of Polyaenus' ἀπορίαι about geometry concerned its incompatibility with the theory of minimal parts." [26, page 24]. White also says that "while there were some Epicureans who were — or previously had been — mathematicians, it seems that acceptance of the Epicurean world-view generally involved, for these individuals, a conversion from the practice of geometry." [30, page 297]. Finally, Furley says that due to the Epicurean doctrine one "should expect, therefore, a priori, that Epicurus would regarded geometry as irrelevant to the study of nature. . . " [15, page 156]. Regarding Epicurus, I am not sure how knowledgeable he was in mathematics. According to Sedley [26, page 23], he wrote an essay entitled "On the Angle of the Atom", so one could suggest that he must had been familiar at least with the basics of geometry, even though he did not have much appreciation of the subject as Cicero tells us [30, page 297]. Yet, as we are also told in Cicero [6, page 23], Epicurus was not even willing to learn geometry from Polyaenus and he was prone to geometrical errors, so perhaps he was not very well-versed on the subject.

[5] Proclus' argument against indivisible segments is essentially Aristotle's argument in *Physics* VI, 2 [10, page 268]. But, as we explained already, this type of infinite divisibility relates to the continuous aspect of line segments and not their dense aspect.

> smaller parts to infinity, in order that we may not make all things weak, and so in the composition of aggregate bodies be compelled to crush and squander the things that exist into the non-existent, but we must not either suppose that in limited bodies there is a possibility of continuing to infinity in passing even to smaller and smaller parts. For if once one says that there are infinite parts in a body or parts of any degree of smallness, it is not possible to conceive how this should be, and indeed how could the body any longer be limited in size? For it is obvious that these infinite particles must be of some size or other; and however small they may be, the size of the body too would be infinite." [9, page 4]

It is quite clear, I believe, that in the two passages provided, Epicurus intends this "atomism" for material objects, and not abstract objects. The argument from the two passages is quite clear: since material things do not vanish into non-existence, there must be a limit to infinite divisibility of material things. Furthermore, a finite body cannot consist of infinitely many parts, hence it cannot be divided into infinitely many parts.

But, there are some other passages from Epicurus and other writers (see evidence items (a)-(d) below) which prompt one to argue that Epicurus not only intended this "atomism" for physics but also for geometry.[6] The conclusion of that was the rejection of the notion of infinite divisibility of segments, and, perhaps, geometry overall. As Mau says:

> "As we see here, Epicurus explicitly rejects the so called '*εἰς ἄπειρον τομή*' ['infinite divisibility'].This seems to be a technical term for the mathematical axiom that any size can be bisected again and again ad infinitum." [22, page 422].

Nevertheless, as Vlastos claims (and most seem to agree), these passages do not provide enough evidence that point to some "special" Epicurean mathematics, in which the infinite divisibility of line segments was not considered.

Historically, according to Vlastos [29, pages 123–125], ascription of mathematical atomism to Epicurus follows the timeline shown below:

[6] Notice that in this case the arguments provided by Epicurus are not valid. For example, the interval $[0, 1]$ could be divided ad infinitum without vanishing. Also, an infinite sum of small terms could be finite. For example consider that $\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$.

- H. von Arnim, *Epikurs Lehre vom Minimun*, 1907 [2]. (first ascription).
- C. Bailey, *The Greek Atomists and Epicurus*, 1928 [3].
- S. Luria, *Die Infinitesimallehre der antiken Atomisten*, 1933 [20]. (most comprehensive)
- T. L. Heath, *Mathematics in Aristotle*, 1949 [18].[7]

Mau, Furley, Sedley and White [22, 15, 26, 30] base their views on prior authors mentioned in the timeline above and they draw their arguments, among others, from the following evidence:

Evidence (a): Epicurus' phrase "the least parts in the atom" [9, page 4] in the "Letter to Herodotus", which they understand as some sort of even smaller indivisible minima inside the atoms.[8] Specifically:

> "Now we must suppose that the least part in the atom too bears the same relation to the whole; for though in smallness it is obvious that it exceeds that which is seen by sensation, yet it has the same relations. For indeed we have already declared on the ground of its relation to sensible bodies that the atom has size, only we placed it far below them in smallness. Further, we must consider these least indivisible points as boundary-marks, providing in themselves as primary units the measure of size for the atoms, both for the smaller and the greater, in our contemplation of these unseen bodies by means of thought. For the affinity which the least parts of the atom have to the homogeneous parts of sensible things is sufficient to justify our conclusion to this extent: but that they should ever come together as bodies with motion is quite impossible". [9, page 4].

[7] As Vlastos says [29, page 125], Heath in [16] actually argued against atomism for Epicurus, but then he reversed his position in [18].

[8] In Greek "τὸ ἐν τῇ ἀτόμῳ ἐλάχιστον, τὰ ἐλάχιστα". Notice also that in Bailey's translation of the Letter (below, page 4) the "least indivisible parts" are usually referred to as "parts", but sometimes as "points". As a reviewer has pointed out, the latter term is quite misleading as the term "point" is not in the Greek text. The text mentions no noun, only two adjectives ("τὰ ἐλάχιστα καὶ ἀμιλῆ"), which better translates as "the smallest and indivisible parts" rather than "points" which have no size. (The Greek text can be found at http://www.epicuros.gr/keimena/k_Herodotos.htm, accessed on July 3, 2017).

This is a crucial passage and Mau argues that Epicurus claims that the atom, in spite of the fact that it cannot be mechanically cut up into parts, must in an abstract sense have parts. And, the question is whether this points to two different minima, namely physical and mathematical minima.[9] Also, Mau concludes that "it becomes most likely that in the 'Letter to Herodotus' the word '*ἐλάχιστον*' ('least part') also means a certain constant which is part of Epicurean physics and mathematics" [22, pages 425–427]. Finally, the statement from above [see also Footnote 8] that "these least indivisible points [are] primary units [for] the measure of size for the atoms" basically defines these minima to be the common measure of the size of atoms, something that relates to commensurability.[10]

Evidence (b): Simplicius' passage in his *Commentary on Aristotle's* Physics 6, which informs us that Epicurus had to abandon the term "partless" for atoms, after Aristotle's criticism to Democritus and Leucippus who considered atoms to be unchangeable because of their being partless. In particular:

> "Leucippus and Democritus, however, believed not only in impassivity (apatheia) as the reason why primary bodies are not divided, but also in smallness and partlessness, while Epicurus later did not hold that they were partless, but said that they were atomic [i.e. uncuttable (atoma)] by virtue of impassivity [alone]. Aristotle refuted the view of Leucippus and Democritus in many places, and it is because of those refutations in objection to partlessness, no doubt, that Epicurus, coming afterwards but sympathetic to the view of Leucippus and Democritus concerning

[9] Aristotle hinted at that earlier, in "On Generation and Corruption". He said characteristically, "And again, if the primary things are indivisible magnitudes, are these bodies, as Democritus and Leucippus maintain? Or are they planes, as it is asserted in the Timaeus?" [4, page 515].

[10] The term "unit" is actually not mentioned in the Greek text (see Footnote 8 for the Greek text), so one could argue that there is actually nothing to suggest that the minima are commensurable with one another or the atoms are commensurable with the minima. Those minima are simply to be understood quantitatively, as in the text we are told that where there are more of these minima the atoms are bigger, and where there are fewer minima the atoms are smaller [9, page 4]. Yet a few lines later in the text it also says that "these minima are the initial measure [*τῶν μηκῶν τὸ καταμέτρημα*] to determine all magnitudes, big or small". So, even though Bailey's translation is incorrect, those minima can still be understood as a common "unit" with which atoms are compared.

> primary bodies, kept them impassive but took away their partlessness, since it was on this account that they were challenged by Aristotle.” [27, page 17].

So, to avoid the Aristotelian critique, Epicurus requires his atoms to contain finite minimal parts that are conceptually indivisible, yet have extension. His dilemma, I suppose, is how to reconcile this with his empiricism. Are these minima composing the atom partless? Because, if they are, it is hard to see how they escape Aristotle’s critique. On the other hand, if they have extension then in what sense is the atom indivisible?

Evidence (c): Indivisible mathematical magnitudes were known when Epicurus wrote the “Letter to Herodotus”. According to Proclus, Xenocrates introduced a similar concept. As Proclus says:

> “That something is divisible and that it is divisible to infinity are not the same thing. One could use this problem also to refute the doctrine of Xenocrates that asserts indivisible lines.” [23, page 217].

Xenocrates’ atomism postulated two kinds of minima, namely mathematical and physical, that related to each other. As Dillon informs us:

> “His atomic theory is valid for the intelligible world, as well as for the physical, and the atomic units of the latter are essentially projections of the former.” [8, pages 117–118]

Interestingly, Dillon quotes Aetius saying that “Xenocrates and Diodorus defined the smallest elements [of things] as partless” [8, page 117]. So, it seems that the “minima” was a concept that had been already discussed. Epicurus apparently tried to modify his “minima” and tried to bring them to terms with his broader philosophy.

Evidence (d): Famous Epicureans such as Polyaenus, Demetrius of Laconia, and Zeno of Sidon who were all well versed in mathematics criticized Euclidean geometry in which the property of infinite indivisibility of magnitudes is essential ([30, page 297]; [26, page 24]; [7, page 77]). As Proclus writes:

> “Of those in this group whose arguments have become notorious some, such as the Sceptics, would do away with all knowledge [. . .] whereas others, like the Epicureans, propose only to discredit the principles of geometry.” [23, page 156].

And:

> "Since some persons have raised objections to the construction of the equilateral triangle with the thought that they were refuting the whole of geometry, we shall also briefly answer them. [23, page 168].

What was the motive behind the criticism and rejection of geometry? Sedley informs us that:

> "that Epicurus believed in a minimal unit of measure out of which not only atoms but also all larger lengths, areas, and volumes are composed, is nowadays widely accepted; and most would also agree that it is not merely a physical minimum, contingent upon the nature of matter, but a theoretical minimum, than which nothing smaller is conceivable. Others both before and since Epicurus have been seduced by similar theories without being led to reject conventional geometry. Yet this is precisely the penalty which a theory of minimal parts should carry with it, for one of its consequences is to make all lines integral multiples of a single length and therefore commensurable with each other, whereas the incommensurability of lines in geometrical figures had been recognized by Greek mathematicians since the 5th ce. Moreover the principle of infinite divisibility lay at the heart of the geometrical method commonly called the 'method of exhaustion', which was fruitfully developed by Eudoxus in the 4th ce." [26, page 23].

Clearly, Sedley's view is based on passage / evidence (a) above which he interprets as evidence suggesting mathematical atomism, and not just physical atomism. This interpretation might have some good grounds, as Epicurus is quite unclear on the nature of his suggested minima.

By "others", Sedley probably refers to the Platonists (maybe Xenocrates) or Democritus who suggested indivisible magnitudes and were criticized by Aristotle.[11] Epicurus seems to have suggested something similar and, perhaps, that was one of the reasons that prompted him to reject geometry, as

[11] Furley in [15, pages 4–5] considers Democritus as a mathematical atomist in the sense that he postulated theoretically indivisible units of matter. But, not in the sense of rejecting infinite divisibility of geometric magnitudes.

geometry was difficult to come to terms with his atomism. Unlike Epicurus, Xenocrates did not have to reject geometry as indivisible lines were part of his broader metaphysics. On the contrary, Epicureans had every reason to reject geometry because of their empiricism.[12] Finally, White quotes Cicero who said that, "Polyaenus, an eminent 'first-generation' Epicurean [...], (came) to believe that 'all geometry is false' after he had accepted the views of Epicurus" [30, page 297].[13] One of those "views", I suppose, was also the idea of indivisibility. Therefore, Mau, Furley, Sedley, and White believe that it is reasonable to believe that the Epicureans actually took a stand against the axioms and the principles of geometry and not only against its logical completeness ([22, page 425]; [15, page 156]; [26, page 24]; [30, page 297]).

4. Zeno and his Critique of Geometry

The case of Zeno at first sight seems to deviate a bit from the usual Epicurean stance on geometry. According to Proclus:

> "Up to this point we have been dealing with the principles [definitions, postulates, and common notions in Euclid's Elements], and it is against them that most critics of geometry have raised objections, endeavoring to show that these parts are not firmly

[12] Mau says that another reason the Epicureans rejected the main axioms of Greek mathematics was their theory of the "swerve" ("παρέγκλισις") of the atoms [22, pages 426–427]. Namely, the spontaneous and uncaused swerve from the atoms' straight motion. The swerve occurred only up to a minimum distance and it was a hypothesis incompatible with the geometric axioms.

[13] Polyaenus' view that "all geometry is false" could very well mean that geometry's conclusions do not apply to the physical world, as there are no physical perfect circles, etc. If so, geometry as an abstract deductive system could still be studied for its own sake, and contain truths such as the sum of the angles of a triangle equal two right angles. In that case, a more appropriate term would be "irrelevant", rather than "false". Indeed, the Epicureans might see no point in engaging in abstract deductive exercises, yet they would be wrong in considering geometry useless for such a reason. Geometry is useful in describing or studying the physical world, giving us intuitive insight, even if it is describing it approximately. Approximately true does not necessarily mean false. Our senses describe the world approximately, but I do not think the Epicureans would have considered our sense perceptions as false. If Epicurus wanted to measure the area of his Garden or build a cubic room in it, then he would have probably found geometry to be a useful tool and he could have applied it with high degree of accuracy.

> established. Of those in this group whose arguments have become notorious some, such as the Sceptics, would do away with all knowledge [...] whereas others, like the Epicureans, propose only to discredit the principles of geometry. Another group of critics, however, admit the principles but deny that the propositions coming after the principles can be demonstrated unless they grant something that is not contained in the principles. This method of controversy was followed by Zeno of Sidon, who belonged to the school of Epicurus and against whom Posidonius has written a whole book." [23, page 156].

Cuomo believes that Demetrius of Laconia might have not been an orthodox Epicurean, either. As she says:

> "Indivisibles conflicted with the assumption, common among geometers, that magnitudes could be divided indefinitely. Their existence in nature, on the other hand, was one of the main tenets of Epicurus, who put forth an atomic theory of matter inspired by Democritus. Indeed, [...] the Epicurean Demetrius set out to attack Euclid's *Elements*, or at least some of the contents of the first book. That, however, does not amount to evidence that the Epicureans spurned mathematics entirely. Demetrius seems to have been knowledgeable in the very subject he set out to criticize; two early 3rd ce Epicureans, Polyaenus and Pythocles, allegedly studied mathematics; the works of mathematicians like Apollonius and Hypsicles mention people (Philonides, Basilidees, and Protarchus, respectively), who can be identified by contemporary Epicureans by the same name." [7, page 77].

It is surprising that Cuomo does not mention Zeno in the passage above, whom Proclus, also above, specifically names as someone who at least accepted the principles of geometry. Indeed, Demetrius could have been knowledgeable on the subject, but could very well have rejected it too.[14]

[14] As Sedley says in [26, page 24], the Epicurean Demetrius wrote a book entitled *Πρὸς τὰς Πολυαίνου Ἀπορίας* to address some difficulties that Polyaenus, another Epicurean, faced in his book *Ἀπορίαι*, in which he discussed theoretical objections to geometry. Sedley believes that some of the explanations of Demetrius aimed to reconcile geometry with the Epicurean theory of minimal parts and address issues problems related to infinite divisibility.

Sedley and White cite Cicero ([26, page 24]; [30, page 297]) who informs us that Polyaenus, formerly a great mathematician, came to consider all geometry to be false, under the influence of Epicurus. Mau, Furley, Sedley, and White argue that rejecting geometry was mainstream Epicureanism ([22, page 429]; [15, page 156]; [25, page 24]; [30, page 297]). As White says:

> "Indeed, Epicurean aversion to the geometers' pulvis eruditus eventually became something of a rhetorical commonplace. Cicero (Nat.D. 2.47f) comments on this aversion and the consequent mathematical ignorance of Epicureans [...]. While there were some Epicureans who were — or previously had been — mathematicians, it seems that acceptance of the Epicurean world-view generally involved, for these individuals, a conversion from the practice of geometry. Polyaenus, an eminent 'first-generation' Epicurean, who was perhaps the most distinguished of the converted mathematicians, is described by Cicero (Acad. 2.106) as having come to believe that "all geometry is false" after he had accepted the views of Epicurus. Zeno of Sidon apparently was an exceptional figure who, according to the account of Proclus, seems to have continued his mathematical work as an Epicurean, arguing that (all? many? some?) theorems of geometry do not follow without some additions to the (normally accepted?) set of postulates. Despite a few problematic cases such as that of Zeno, it seems there is good reason to agree with David Sedley that "the wholesale rejection of geometry was still orthodox Epicureanism." [30, page 297].

Contrary to Vlastos' position, who views Zeno's critique simply as methodological and not as an overall dismissal of geometry, Mau and others think that Zeno too was part of the Epicurean "task" to undermine or reject mathematics in general, and his critique was intended to demonstrate that geometry was incomplete and inconsistent. As Mau says:

> "Professor Vlastos takes (3) [Proclus' statement above that Zeno admitted the principles but denied that the propositions coming after the principles can be demonstrated] as proof for the fact that Zeno of Sidon did not reject the classical mathematical system but was merely trying to fill in some logical gaps. This does not seems probable to me. For if, as Proclus states, the Epicureans

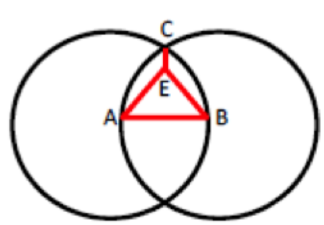
> were trying hard to overthrow mathematics generally, then they would be one step ahead if, taking as much testimony as possible from the enemy, they managed to prove that he is contradicting himself. That would hit the opponent much harder than just not accepting his principles." [22, pages 429–430].

One could also interpret Zeno's phrase "even if we accept [the premises]…" in [23, page 168] (see full quote below), as Zeno simply accepting the premises only for the sake of argument and not per se.

On the other hand, if White's assessment that Zeno seemed to have "continued his mathematical work as an Epicurean" derives only from Proclus, it would be simply a guess but not something with solid foundation. Nor is it very clear what White means by "mathematical work". It is possible that Zeno, as an Epicurean, who doubted the truth of geometry, was not interested in proving new theorems, adding new axioms, and so on. But, as a mathematician with higher standards of rigor than other Epicureans, he might have been interested in looking over earlier proofs and identifying their faults, without necessarily attempting to fix or complete those proofs. Since there is not enough evidence to prefer one or the other possibility, it might be reasonable to allow both possibilities until further evidence arises.

So, what did Zeno criticize exactly? According to Proclus:

> "Since some persons have raised objections to the construction of the equilateral triangle with the thought that they were refuting the whole of geometry, we shall also briefly answer them. The Zeno whom we mentioned above asserts that, even if we accept the principles of the geometers, the later consequences do not stand unless we allow that two straight lines cannot have a common segment. For if this is not granted, the construction of the equilateral triangle is not demonstrated.



> Let AB be the straight line, he says, on which we are to construct the equilateral triangle. Let the circles be drawn, and from their point of intersection draw the lines CEA and CEB having CE as a common segment. It then follows that, although the lines from the point of intersection are equal to the given line AB, the sides of the triangle are not equal, two of them being shorter than AB. But if their inequality is not established, neither are its consequences. Therefore, says Zeno, even if the principles are granted, the consequences do not follow unless we also presuppose that neither circumferences nor straight lines can have a common segment." [23, page 168].

So, Zeno is criticizing Euclid's Proposition I.1: "Given a finite straight line, construct an equilateral triangle." He is rejecting the assertion that this proposition can be deduced from the axioms, unless the further assumption is made, namely, that no two lines contain a common segment (perhaps an atom that is larger than a geometrical point, "mathematical atomism"?). Because, if they did, then it is possible that segments AC and BC could meet before at E, and hence we do not have an equilateral triangle (see Figure 1).

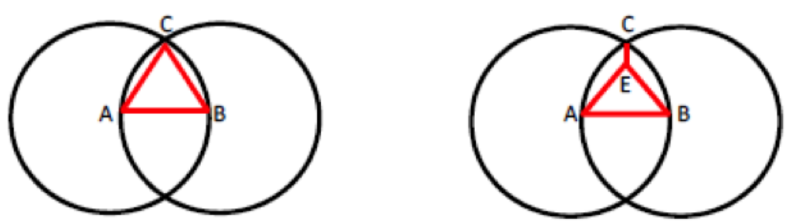


Figure 1: Perhaps Euclid's first proposition is false.

Proclus responds by saying that:

> "... in a sense it is presupposed in our first principles that two straight lines cannot have a common segment. For the definition of a straight line contains it, if a straight line is a line that lies evenly with all the points on itself. For the fact that the interval between two points is equal to the straight line between them makes the line which joins them one and the shortest; so if any line coincides with it in part, it also coincides with the remainder". [23, page 169].

Additionally, he says:

> "[...] furthermore, this principle is also evidently assumed in the postulates. For the postulate that a finite straight line may be extended in a straight line shows clearly that the extended line is one and that its extension results from a single motion." [23, page 169].

The latter is actually Postulate 2, in the *Elements*. In other words, according to Proclus there is a postulate that guarantees, as some believe, that the intersection of two lines is a point and not a line segment. What might have prompted Zeno to suggest that the intersection might be a line segment? It is not unreasonable here to suggest that the Epicurean minima were most likely the reason. The Epicureans believed in theoretically minimal parts in an atom, i.e., theoretically indivisible portions of matter. So, it is not farfetched to imagine theoretically indivisible portions of space as well, based on the Epicurean epistemic grounds of 'analogy'. On the other hand, and considering that Euclid did not exactly hold the highest standards of rigor in his geometry, Zeno might have had genuine concerns regarding that rigor and simply pointed to issues where geometry could be reformed, rather than trying to demolish it.

5. Some Remarks

In what follows, I make some critical remarks on the Epicurean position on mathematics and their critique of geometry, especially Zeno's critique.

Remark (a): It is a bit hard to make sense of a minimum part of an atom as the one suggested by the Epicureans. Namely, a minimum that is imperceptible, indivisible, with extension in space. And, one that is a common measure (a unit) of atoms, the latter being integral multiples of it. Firstly, what empirical evidence could Epicurus cite for the existence of such minima and their properties (unchangeable, etc)? Probably none. His argument is based primarily on analogy with ordinary visible bodies. Just as visible bodies consist of parts, so we could imagine (not observe) that atoms consist of parts too [9, page 4], which are indivisible. But, if we can imagine that the atoms have parts, why could we not imagine that these parts are divisible? We could perhaps imagine them as being some kind of geometric points, but Epicurus explicitly rejects that as his minima have magnitude. So, we could perhaps imagine them as some kind of pixels that compose the atom without being separable parts of it, similar to the pixels that compose a computer image. But, how do these pixel-like minima avoid the notion of infinite divisibility? Suppose for the sake of argument that these minima are small cubes with magnitude q, an integral number of which forms the atom below, see Figure 2.[15] Some magnitudes of this atom are subject to incommensurability and hence infinite divisibility in the "anthyphairetic" sense.

Figure 2: What are atoms made of? A speculation.

For example, if sides x and y of the atom in Figure 2 measure $3q$, then the diagonal d could not measure $3q$. As a matter of fact, d cannot be any integral multiple of q, and furthermore, d and x will be subject to infinite divisibility via anthyphairesis. Notice that the minima being cubes is beside the point here. We are not arguing about the incommensurability of the sides of the pixel cube, but about the incommensurability of certain lengths of the supposed atom above, given that it has minima in the shape of such pixel cubes. The argument still stands no matter the shape of the minima. As long as they are understood as common units of measure of some magnitude, they could be subject to incommensurability.

Purinton in [24], gives an alternative view on how to understand those minima, with what he calls the "relativist view". Namely:

> "the view that the minimal parts of atoms are only 'seen' as partless relative to a certain level of magnification in just the same sense that the minimal parts of a visible body are only seen as partless relative to the distance of observation." [24, page 119].

[15] Should we exclude such a shape? Epicurus does not give us any criteria on which shapes are acceptable. Furthermore, he stated that "and so in each shape the atoms are quite infinite in number, but their differences of shape are not quite infinite, but only incomprehensible in number" [9, page 2].

He then continues to say that:

> "on my view, minimal parts, being cubed-shaped, do have surfaces, though they are not 'seen' as having surfaces (or as cubed-shaped) at the level of magnification at which they are 'seen' as partless." [24, page 141].

But this last position can hardly avoid the argument we considered above, since the only thing that is relevant is that the minima are understood as units of measure. Even if they cannot be seen at the nth level of magnification, as Purinton claims, they still serve as the measure of an atom. And as such, their magnitudes are subject to incommensurability as our argument suggested. Furthermore, on the $(n+1)$th level of magnification, i.e. the level in which the minima could be seen, the sides of the minimal cube itself are now subject to incommensurability, as our argument also suggests.

Remark (b): Proclus goes to great lengths to show that even if the previous Epicurean concern (i.e. the intersection point) is not evident in the postulates, then it can be proved in a straightforward manner. He gives a proof by contradiction using circles, for which he considers also a second counterattack by Zeno ([23, page 169]; [10, pages 242–243]).[16] Zeno's complaint is basically that Proclus' proof begs the question in a similar manner, as it assumes that two circles cannot have a common segment.

> "To this demonstration Zeno would reply that the proof we gave that the diameter bisects its circle depends on our previous assumption that two circumferences cannot have a common segment." [23, page 169].

As Heath says:

> "That is, for anything we know, there may be any number of points C common to the two circumferences ACE, BCD. It is not until III.10 that it is proved that two circles cannot intersect in more points than two, so that we are not entitled to assume it here." [10, page 242].

[16] Zeno challenged Proclus' assumption that two circles intersect at a point. Something that was used in the proof but nowhere stated.

Proposition III.10 though, that Heath refers to, was not put forth by Euclid to address the circles segment problem. Unlike the straight lines segment which is presupposed in Postulate 2, as Proclus claims (see Section 4), Euclid probably took for granted that non-coinciding intersecting circles intersect at a point, and not a segment. It is also not asserted in any postulate, even though it is intuitively obvious. Yet, it was not obvious to Zeno. Was Zeno objecting to this intuitively obvious fact simply on geometrical grounds? As we said already, this fact is not asserted in any postulate indeed, and Zeno had legitimate grounds for complaint. Or, was Zeno objecting the fact because of the Epicurean preconceptions about indivisible units and their rejection of infinite divisibility? Just as in the intersection of lines, Zeno insisted that two circles intersect in a segment, and not a point. Zeno was a mathematician, but he was also an Epicurean. As such, he believed in minimal parts in an atom, namely in theoretically indivisible portions of matter. Hence, as we already claimed, it is not hard to imagine that Zeno might have considered theoretically indivisible portions of space as well. This is in accordance with the Epicurean epistemic notion of 'analogy', and it could explain Zeno's objection to the intersection point.

Perhaps, Proclus could have argued that the fact that intersecting circles do not have a common segment follows from the very definition (Definitions I.15 and I.16) of the circle. Namely:

> "15. A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another.
>
> 16. And the point is called the centre of the circle." [10, page 183].

In other words, a circle is a line all of whose points are equidistant from its center. So, assuming that the two intersecting circles have a common linear segment LM, instead of just a point P, it would force all points (except the endpoints) on the segment LM to violate the definition of the circle A, as they will not be equidistant from the center of A (see Figure 3). Similar reasoning could be applied if the circles intersected at a curved segment or at an area.

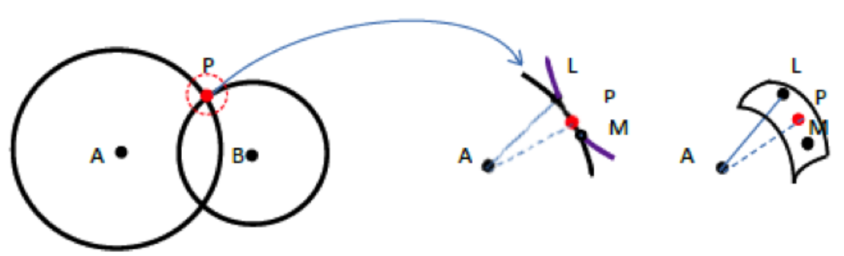


Figure 3: Two circles intersecting in interesting ways.

Hence, the suggestion that the intersection is a line segment, and not simply a point P, cannot stand because of the definitions of circle and line. From all points on LM one could reject all those points which violate the definition of the circle, such as all non-endpoints of LM, and one would end up with exactly one point P (essentially the point when L = M). And, therefore, one could still construct an equilateral triangle.

Remark (c): Vlastos in [29] says that there are no historical records of an Epicurean atomistic geometry, and, indeed, most experts seem to agree [15, 22, 24]. Yet, it is not unlikely that they entertained the thought or they experimented with such a geometric system, with perhaps no success. Some evidence pointing to this are: (i) Polyaenus' book in which, as Sedley says [26, page 24], he considered objections to geometry and its incompatibility with the theory of minimum segments; (ii) Demetrius' work addressing Polyaenus' concerns regarding that incompatibility; (iii) some fragments from the Herculanean papyri which appear to contain some mathematical work by Demetrius, which included "slight variants of Euclid's definition of circle" [5, pages 94–95]; (iv) Antiphon's view that a circle is a regular n-gon (where n is large) [29, page 130]. Vlastos presents the above as supporting evidence for an argument against ascription of mathematical atomism to Epicureans, claiming that had the Epicureans adopted an atomistic stance in geometry, they would have had constructed some special atomistic geometry; but the record shows no such geometry [29, pages 126–131]. Not necessarily though, as their atomistic stance in geometry is insinuated in their critique. Their objections and concerns about geometry might have stemmed thoughts of a geometric atomism, by analogy with their physical atomism, which they unsuccessfully tried to reconcile with geometry. Vlastos may just be reversing the logic of things. There is no atomistic geometry in the works of the Epicureans indeed, but there is some evidence that perhaps they experimented in that direction.

What was the point of re-defining the circle and re-addressing some of the geometric postulates? Surely, to use the latter for "polemical" reasons was one of the points. But, is it unreasonable to suggest that they might have explored where these new hypotheses lead, even though they did not lead anywhere? After all, there were some mathematicians in the Epicurean ranks. The fact that they did not produce such an atomistic geometry may simply mean that they probably could not. Vlastos basically answers this very same question, even though he draws the opposite conclusion. He says characteristically:

> "I find it hard to see what the finitistic geometry to which these scholars would commit, from Democritus down, would be like. It would be a system, constructed by mathematical techniques available at the time, whose elements would be exclusively discrete quantities, but would nonetheless retain sufficient contact with the mathematics of the special continuum to be recognizable as geometry." [29, pages 127–128].

Indeed, there were not too many available techniques to construct such a system in Epicurus' time, and such systems were defined more than 2000 years later. But why should the Epicureans bother with such a system anyway? Furley in [15, page 156] says that "Epicurus would have regarded geometry as irrelevant to the study of nature," and he also quotes Sextus from his *Adversus Mathematicos* saying that the Epicureans regarded geometry as "contributing nothing to the perfection of wisdom" [15, page 156]. He then concludes[17] that:

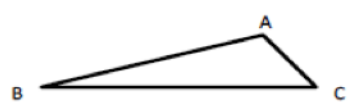
> "[Epicurus] made no attempt, apparently, to work out a fully systematic mathematical theory to support his physics. On his own premises, there was no reason why he should. His purpose was to reach peace of mind. However deplorable his rejection of philosophical inquiry may be, we may accept it as explaining why we can find no trace of a New Geometry." [15, page 157].

Therefore, it seems that the Epicureans neither could have constructed an atomistic geometry nor would they have been interested in doing so, which

[17] Mau also ends with "the information available does not allow the conclusion that there was some special kind of atomistic mathematics, or that there was none" [22, page 430].

is perhaps why we do not have any evidence of one. Yet, as we said earlier, their mathematical atomism was clear in their critiques. And the intention at least from some Epicureans in producing such geometry is not improbable.

Remark (d): Mathematically speaking, the Epicureans might have had some legitimate concerns about Proposition I.1, which refers to the construction of an equilateral triangle. But, philosophically speaking, one might observe an inconsistency in their critique. In Proclus' first defense of Proposition I.1 we recall that "the fact that the interval between two points is equal to the straight line between them makes the line which joins them one and the shortest" [23, page 169]. This actually is in accordance with Euclid's Proposition I.20, namely that "the sum of two sides of a triangle is greater than the third side", and it is something that the Epicureans accepted as obvious.[18] As a matter of fact, they ridiculed Euclid for it, by saying that "this proposition would be clear even to an ass" [23, pages Lii, 251], and that given the choice of routes for getting to his hay, the straight line BC or the broken line BAC (see Figure 4), the donkey will know enough to choose the former.



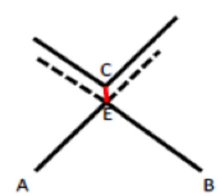


Figure 4: Even a donkey knows the triangle inequality.

Putting aside any concerns on the logical sequence of the two mentioned propositions for a moment, let us look at the issue a bit more generally. Let us look at the propositions per se. Epicureans complained that this statement is obvious and it does not require a proof. But, if this is something the Epicureans would accept as obvious, then why is it not obvious that two intersecting lines could meet at a point? Because, practically speaking, if the donkey knew enough to straight walk the line BC (and not the broken line BAC), then why would a particle not know? Or a point that moves on a line for that matter? Why is it common sense that the donkey or a particle in straight line motion will continue its straight motion, but a point on a line must somehow turn and produce a common section EC? (see Figure 4).

[18] It is known as the Triangle Inequality, namely BC > AB + AC.

Especially considering the fact that Postulate 2 says "To produce a finite straight line continuously in a straight line" [10, page 196], which as Proclus says implies that two lines have no common segment. Therefore, we probably have an inconsistency in the Epicurean critique. It seems that their "common sense" critique was applied selectively. And, it sounds a bit strange that the Epicureans might have held both that one should use one's spatial intuition as to what is obviously true instead of proofs, as Proclus claims they did regarding Proposition I.20, and at the same time object to Proposition I.1 that it assumes something not included in the axioms. It might be the case that common Epicureans did the former, where mathematically sophisticated Epicureans, such as Zeno, did the latter.

Remark (e): Perhaps the Epicureans should have insisted on a definition of the circle more along the lines of Antiphon's definition (see Remark (c) above). A circle is a bit more "fuzzy" and less exact. Perhaps they also imagined a circle that looks more like a polygon and with boundary that has a width and consists of minimum pixel points. In Antiphon's definition there is no mention of polygons having any "thick" sides, but for the Epicureans such an assumption is not incompatible with their views. Epicureans' theoretical minima were not geometric points, and they had spatial dimensions. Hence, they could have argued that the intersection of two such intersecting circles should be a segment (as in Figure 5). But, if we accept that circles are not exact and look more like polygons as below, then one must give up the old definition of the circle and provide a new one, say: "a circle is an n-gon, where n is large, which consists of pixel points".

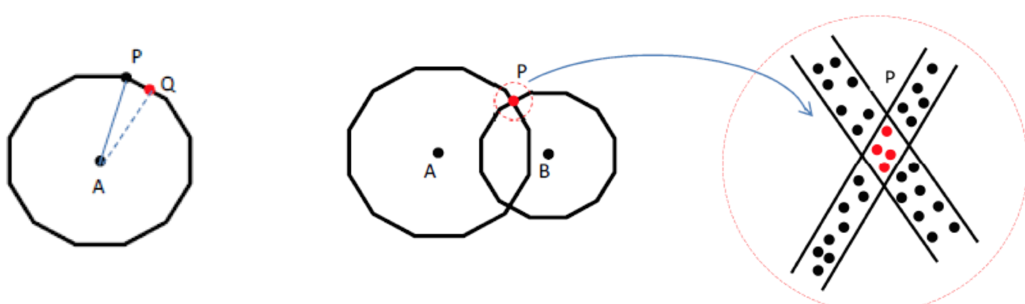


Figure 5: Other interesting ways of circles intersecting.

They would probably have to give up the triangle inequality too, so to avoid a criticism in the lines of Remark (b) above.[19] I doubt that the Epicureans had such concerns in mind.[20] Or maybe they did, but their experimenting with such definitions did not produce anything mathematically significant. Neither have they ever given criteria on what makes a circle more "realistic", and how to distinguish it from a polygon. So, they cannot maintain both the "classic" definition of a circle and a possible critique from experience. In geometry, the circle is defined as it is, and no claim in connection to reality is made. And, from the way it is defined certain things follow, such as the statement that two non-coinciding intersecting circles intersect at a point and not at a segment (or an area). The circle from experience is surely a different object, and it has different definition and properties. If sensory experience does not quite match the properties of a mathematical object, then this might not be too worrying to a mathematician. Hence, a critique from experience against geometry is not too relevant of a critique.

Remark (f): With the exception of Zeno, Polyaenus, and a few others, as some have suggested ([23, pages 275–276, 170–171]; [22, page 429]), perhaps most Epicureans did not have a clear understanding of the notion and meaning of "proof". As Proclus said, "granting the theorem is evident to sense-perception, it is still not clear for scientific thought" [23, pages 275–276]. In mathematics, even obvious things need to be proved from prior simpler propositions, all the way to the axioms.[21] Again, with the exception of Zeno, Polyaenus, and a handful of others, perhaps most Epicureans were

[19] In modern mathematics, for example in the geometry of metric spaces, the triangle inequality is one of the basic axioms. Yet, it is neither a totally obvious statement nor all geometries require it as an axiom. For example, any n-dimensional Lorentzian geometry does not require it as an axiom.

[20] In [5, pages 94–95] we are informed that some fragments from the Herculanean papyri appear to contain some mathematical work by Demetrius of Laconia, that include "slight variants of Euclid's definition of circle". In [7, page 73], we are also told that in those papyri "[Demetrius] reported, for polemical purposes, a definition of the circle [...]." Yet, in [13, page 210] we read, "Column 8 lines 9-17 [of the papyrus] cites *Elements* I Definition 15, the definition of a circle ...". From the related figures also cited in [13], and from what I understood from the evidence, it seems unlikely to me that Demetrius defined a new circle in the lines I hinted in Remark (e), and intended it as means to an experiential argument.

[21] Proclus was right that the triangle inequality, even though it states an obvious fact, must be derived from the other axioms of Euclid's geometry. (See also Footnote 19 above).

not well acquainted with how the axiomatic method worked, either, although its importance had previously been grasped by Aristotle, as is clear from his *Posterior Analytics*. After all, mathematics was not central in their curriculum and this could justify their relative "mathematical ignorance," as White informed us above.

Yet, aversion does not necessarily imply ignorance, as Cuomo informed us, and some Epicureans seemed to be knowledgeable in mathematics. So, what could justify their polemical stance against geometry? It could be that the Epicureans were a bit "dogmatic" and more interested in defending their own theories than actually generating mathematical work. According to Russell:

> "Epicurus has no interest in science on its own account [...]. Epicureans contributed practically nothing to natural knowledge [...] they remained, like their founder, dogmatic, limited, and without genuine interest in anything outside individual happiness." [25, page 236].

Cicero and Sextus may provide us with some evidence for Russell's assessment.[22] In *De Finibus*, Cicero says that Epicurus himself was not a particularly educated man, but one would expect that at least "he had not deterred others from study" [6, page 27]. Characteristically:

> "Again, it is unworthy of a natural philosopher to deny the infinite divisibility of matter; an error that assuredly Epicurus would have avoided, if he had been willing to let his friend Polyaenus teach him geometry instead of making Polyaenus himself unlearn it." [6, page 23].

Furthermore, Cicero informs us that Epicurus basically discarded formal logic altogether, and "he (did) away with definitions, and [...] he (gave) no rules for deduction or syllogistic inference" [6, page 25]. Finally, Sextus in his

[22] Regarding Epicurus' originality, Cicero says that:

> "where Epicurus alters the doctrines of Democritus, he alters them for the worse; while for those ideas which he adopts, the credit belongs entirely to Democritus — the atoms, the void, the images, or as they call them, eidola, [...] the very conceptions of infinite space, apeiria as they term it, is entirely derived from Democritus." [6, page 23].

Adversus Mathematicos says that the Epicureans regarded a group of subjects called *mathemata*, which included also geometry, as "contributing nothing to the perfection of wisdom" [15, page 156].

Being dogmatic or not, the Epicureans were certainly concerned with defending their doctrine. As Sedley suggested, "some of Polyaenus' ἀπορίαι about geometry concerned its incompatibility with the theory of minimal parts" [26, page 24]. Most likely, this was also the primary concern of most mathematically knowledgeable Epicureans, such as Zeno. Their mathematical interest most probably had the objective to address issues that challenged Epicurean atomism and empiricism. It might also be the case that Zeno had genuine mathematical interests, as he seemed familiar with the axiomatic method and his comments were legitimate. For most other Epicureans though, their difficulties with proof and the axiomatic method could also perhaps be explained by their underlying logic, namely inductive logic [21], which was essential to their philosophy and science, but not suitable to mathematics which is based on deductive logic.

6. Conclusion

Regarding the Epicurean position with respect to mathematics, and in particular their critique of Euclidean geometry, I think one can draw the following conclusions with a relative degree of confidence:

Conclusion (1): From the mathematical point of view, Zeno's critique was to some degree legitimate and to the point, especially regarding the foundations and logical structure of geometry. As far as "mathematical atomism" goes, even though there is no direct evidence that points to atomism and experts disagree on this matter, it seems only reasonable to ascribe mathematical atomism to the Epicureans. They postulated indivisible theoretical units and explicitly rejected infinite divisibility. Their atomism is insinuated in their critique, even though the sources do not show that they actually developed formally any atomistic geometry. At the time, probably such attempts lacked the necessary mathematical tools and what tools did exist could not take them mathematically too far. Euclidean geometry was the one that was developed and found useful applications.

Conclusion (2): From the philosophical point of view, if indeed the Epicureans not only intended to discredit mathematics but also rejected it as a whole, then the Epicureans were theoretically and practically incorrect on that matter. Mathematics, even if it is not based on experience, is necessary in science. It also contributes to our good living ("$\epsilon\hat{\upsilon}$ $\zeta\tilde{\eta}\nu$"), which was an important aspect of the Epicurean philosophy, with its many real life applications (flight simulation, magnetic tomography, video-games, etc.). Furthermore, mathematics is nowadays a core course in every country's curriculum with huge pedagogical value, especially for critical thinking. The Epicureans regarded mathematics useless and did not expect or encourage their members to do any mathematics beyond, perhaps, some very basic level.

The Epicureans, in general, did not have mathematics among their primary philosophical interests. That, of course, excludes the more mathematically inclined Epicureans such as Zeno. Yet, their belief that all knowledge is empirical and the inductive logic that guided their philosophy [21] do not seem to align with some of the most important aspects of mathematics, such as abstraction, axiomatic method and proof. Perhaps, the Epicureans could be justified in a way, as neither science nor mathematics were as advanced then as today, and they did not have a complete picture. It is possible that, today, they could have seen mathematics in a quite different way and recognized some of its special aspects.

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