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The Classroom Encounter

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As every newspaper reader knows, many people are trying hard to reform math education. The NSF, NCTM, MAA, and AMS are helping. Meetings are held. Grants are awarded. Textbooks are written and rewritten. "Technology" (use of calculators and computers) is introduced and expanded.

Has this activity made significant improvement?
"Too soon to tell."
When will be the right time to tell?

WHAT'S MISSING?

I hope this work succeeds, but I'm not optimistic. Why not? Because the reforms concentrate on *curriculum* and *teaching strategy*. The *encounter between teacher and student* is underestimated.

In the AMS *Notices* (1) Hyman Bass recently wrote: "Mathematical scientists typically address educational issues exclusively in terms of subject matter, content and technical skill, with the 'solution' taking the form of new curriculum materials. Curriculum is, indeed, a crucial aspect of the problem and one to which mathematically trained professionals have a great deal of value to offer. But, taken alone, it can and often does ignore issues of cognition and learning."

Of course the classroom is a place where information is transferred, but before that, it's a place where humans encounter each other—student with student, teacher with student. The successful teacher relishes that human encounter. He/she knows that teaching isn't just copying information from one abstract intelligence to another. "Covering the material" doesn't necessarily mean teaching the students. Human feelings and needs affect academic performance. In fact, research and experience have found a key ingredient in successful teaching—the relationship between teacher and students-sometimes called the "affective" aspect.

We don't talk about this very often. It's not stressed in our teaching literature, whether "conservative" or "reform."

Of course lectures should be correct, comprehensible, and interesting. But it also matters whether the student sees the teacher caring about her/him, as a human being.

What does it mean to care about the student? It means of course caring whether the student follows the lectures and does the problems. It also means caring why the student is in the class, and what his/her background, preparation, aspirations are. It even means caring when he/she has a crisis—in health, family, employment, or a significant relationship.

When a student has a crisis, does he/she have a reason to believe the professor would want to help?

More primitive: does the professor seek eye contact with the student, or avoid it? Does he/she talk to the students, or to the blackboard? When he loses the class, does he notice, and reestablish contact, or just go on obliviously? Does he let the students see him as a human being with feelings, needs, and weaknesses, or does he try to impersonate a talking, writing automaton?

Some mathematicians laugh such considerations out of court as "pedagogy." Some cry derisively, "Touchy feely!" I'm afraid that even the word "caring" is considered out of place in a mathematical publication. That very fact is a telling indication of the problem I'm talking about

I'll quote three sources. First, an MAA pamphlet of a quarter century ago. Second, a fantastically successful undergraduate math program in Potsdam, N.Y. Third, a study by two anthropologists on why undergraduates switch out of science, math and engineering.

MAA SUGGESTIONS

When I was young the *Monthly* had a section, "Classroom Notes," which accepted pedagogic contributions. It's no longer there. Maybe it was judged insufficiently mathematical. Paul Halmos wrote (12), regarding educational contributions to the *Monthly* when he was editor: "if the educational wisdom that an author had to offer made sense when the word 'mathematics' in it was replaced by 'geography,' say, throughout, then, I said, it should appear in a journal devoted to education, not mathematics. My aim, to make everything in the *Monthly* mathematical." But the readers of the *Monthly* don't read journals devoted to education. So, they're deprived of the wisdom in question.

I found an MAA publication of 1972 which had good advice for any math teacher: *Suggestions on the Teaching of College Mathematics*, credited to a committee chaired by D. W. Bushaw (2).

> "...the perceptive teacher who looks at students while he talks can hardly miss signs of puzzlement, boredom, or pleasure on their faces...if you sense that you have 'lost' a student you might pause and ask him if something needs further explanation. But a word of caution: impatience on your part with the nature of the student's question may result in an impassive and unreadable face on that student for the rest of the term....Another type of feedback that may be especially useful to the inexperienced teacher is obtained by spending the last few minutes of each period discussing what went wrong and what went right that day. LISTEN CAREFULLY to what the students have to say, even if it seems unreasonable...If the class was dead, say so. Make it clear that the students share responsibility if the class is a drag.
>
> "Every good teacher wants rapport with his class, but it is amazing how many instructors give their lucid explanation to the blackboard, the walls, a window, or a point about one foot over the students. LOOK THEM IN THE EYE!
>
> "Encourage conjectures and do not ridicule inept questions or wrong answers. Give the students the feeling that they are all on an equal footing in your esteem. You can learn from their mistakes. Above all, avoid sarcasm in any form. Nothing can damage your relationship with a class faster than sarcasm, however warranted it might seem.
>
> "LISTEN TO YOUR STUDENTS. When someone volunteers an answer to one of your questions, you may realize as he begins to talk that he is on the wrong track. Resist the urge to quiet him. Instead, try to understand what he is saying, acknowledge any merit in it, determine his misconceptions, and tactfully point them out to him. Then let him try again, or give someone else a chance. Many instructors misinterpret a question before it is completely formulated. After you have tried to answer a question, give the student who asked it a chance to say whether he is satisfied.
>
> "In general, strive for as much informality in the classroom as your own personality and the circumstances will allow. Don't be defensive when you make a mistake. No one is perfect, and an impression of integrity is more important than an impression of omniscience. Request help from the students and correct the error together."

In my opinion, asking every math instructor or professor to read this little book once a year would do more for math education than several committee meetings on calculus reform.

There have been many recent publications about college math teaching, mostly from the MAA. In particular, (4), (5), (7), (9-11), (15), (16), (19), (20), (22-29). These and others have excellent suggestions about curriculum and teaching strategy. Most of them don't strongly emphasize the relation between teacher and student.

A recent MAA publication that deals substantially with the teacher's interaction with students is *Keys to Improved Instruction by Teaching Assistants and Part-Time Instructors*, edited by Bettye Anne Case (3). It includes an anthology of guides for TA's and part-time and temporary instructors, collected from 10 universities. This is very good. Of course, TA's and part-timers aren't the only ones whose teaching can improve. But, unlike tenured faculty, TA's and part-timers do have

to listen when told to improve their teaching.

Is the teaching of tenured faculty above criticism? An anecdote from a prestigious East Coast school: A student complained that Professor X treated him unfairly in an oral exam. The chairman gave the student a second exam. The student did about as badly as the first time. So far so good. But then the chairman told Professor X what he had done! Professor X was indignant to the point of never again speaking in a civil manner to the chairman. Moreover, X's wife and the chairman's wife had been friends. End of friendship. Probably an extreme case.

"A MODERN FAIRY TALE"

My second evidence is a remarkable article from the *Monthly* March, 1987. It's titled, "A Modern Fairy Tale" (20). It describes an amazingly successful undergraduate mathematics program at a little known college, Potsdam College of the State University of New York.

Potsdam is a small town in far northern New York State. It's the home of Clarkson University, formerly Clarkson Institute of Technology. The author, John Poland, is in the Department of Mathematics and Statistics of Carleton University in Ottawa, Canada. He wrote:

> "Tucked away in a rural corner of North America lies a phenomenally successful undergraduate mathematics program...Picture a typical, publicly funded, Arts and Science undergraduate institute of about 5,000 students, with separate departments of Mathematics and Computer Science. While the total number of undergraduates has remained relatively fixed over the past 15 years, the number of mathematics majors has doubled and doubled again and again to over 400 now in third and fourth year. They don't offer a special curriculum...It is just a standard, traditional pure mathematics department.
>
> "More than half the freshman class elect calculus, because of the reputation of the mathematics department carried back to local high schools. And, of the less than 1000 Bachelor degrees awarded, almost 20% are in mathematics. In case you are unaware, 1% of Bachelor degrees granted in North America are in mathematics. These students graduate with a confidence in their ability that convinces prospective employers to hire them, at I.B.M., General Dynamics, Bell Laboratories and so on...
>
> "Do they just lower their standards? Mathematics teachers in the university across the street say, 'no.' They see no significant difference between their performance and that of their own students....
>
> "The students say the faculty members really care about them, care that each one can develop to the maximum possible level...*It is simply the transforming power of love, love through encouragement, caring and the fostering of a supportive environment*...By the time they enter the senior year, many can read and learn from mathematics texts and articles on their own...They graduate more women in mathematics than men. They redress a lack of confidence many women feel about mathematics. In the past ten years, almost every year the top graduating student at this institution, across all programs, has been a woman in mathematics.
>
> "What must a mathematics department do to attain this success? The faculty must love to teach, with all this means about communication, caring for students and for their development They would teach at a pace which allows students time to struggle with the problems and resolve them, rather than primarily to cover material...They would recognize that students need time to build the skills, understanding and self-confidence to handle more advanced mathematics. The faculty would encourage and reward the successes of the students, bringing all or most of them to a high level of achievement (and high grades), rather than using the grade to filter the brightest and quickest students into further mathematics studies. The recipe for success at Potsdam is very simple: instill self-confidence and a sense of achievement through an open, caring environment."

The atmosphere and attitude at Potsdam are largely the creation of Prof. Clarence Stephens, who was chairman of the math department and gradually remolded it according to his vision. Prof. V. C. Cateforis, the present chairman, says that changes in the incoming freshmen have diminished the number of math majors. But, the teaching philosophy is still the same. In every course, the expectation is still that all students will learn to write correct proofs, a goal some other departments would think hopelessly unrealistic.

When Poland's article appeared in 1987, 1 expected a sensation. Many other departments would seek to emulate Potsdam, I imagined. Alas, no. Even though Potsdam was held up as an example in Leonard Gillman's retiring presidential address (10) to the MAA, less than half a dozen math departments sent visitors or observers. I don't know of one that succeeded in following Potsdam's example.

Why?

TALKING ABOUT LEAVING

My third document is a book, *Talking About Leaving: Why Undergraduates Leave the Sciences*, by Elaine Seymour and Nancy M. Hewitt (23). Seymour directs ethnography and assessment research in the Bureau of Sociological Research, University of Colorado, Boulder.

They asked, why do 40 to 60 per cent of undergraduates leave science, mathematics and engineering (SME) majors? They studied seven four-year institutions of seven different types.

> "We discovered that the same set of problems lead both to switching and to serious discontent among those who persist.
>
> "...What distinguishes the survivors isn't the nature of their problems, but whether they're able to surmount them quickly enough to survive. The concerns of both switchers and nonswitchers are the same issues across all seven campuses, regardless of size, mission, funding, selectivity, or reputation. In contrast to the common assumption that most switching is caused by personal inadequacy in face of academic challenge, we find that a high proportion of switching arises from institutional sources, or from students' career concerns.
>
> "Ranked by their contribution to switching, these causes are:
> • loss of interest in science
> • belief that a non-SME major is more interesting or gives a better education
> • poor teaching by SME faculty
> • feeling overwhelmed by the pace and load of curriculum demands.
>
> "Criticisms of faculty teaching contribute to a third of all switching and were the third most common factor in switching decisions. Complaints about poor teaching were near universal by switchers (90.2 percent) and were the most common complaints by nonswitchers (73.7 percent)... Rejection of SME careers is partly rejection of the models which SME faculty and graduate students present to undergraduates. SME faculty are often seen as unapproachable or unavailable for help with academic or career planning concerns.
>
> "The curve-grading widely used by SME faculty is perceived to reflect disdain for the potential of most underclassmen. This grading is seen as intended to drive most students away, rather than to give students realistic feedback.
>
> "Harsh grading is part of the traditional competitive SME culture. It discourages collaborative learning, which many students view as critical to understanding the material...
>
> "Students [made] inferences from faculty teaching:
> • Faculty find the subject dull.
> • They have little understanding of how people learn.
> • They dislike teaching, don't care about students.
> • They don't see themselves as responsible for students learning.
>
> "Students didn't believe there was anything intrinsically dull about the SME class material. Same material, different professors, different outcomes."

Seymour and Hewitt checked the widespread notion that TA's with poor English drive students away. According to the students, TA's are not the problem.

There are bad teachers in all subjects, but we seem to have more than our share. Why is this? Why is bad teaching so persistent?

A SOURCE OF BAD MATH TEACHING

I'm talking about teachers at universities with PhD. programs in math. Liberal arts colleges seem to be different

For many of us the passage through graduate school was deeply imprinting. We were apprentices, struggling for our thesis adviser/supervisor's approval. This apprenticeship stamped many of us with our adviser's way of thinking and teaching. (Occasionally the imprint was reversed. After a "stormy" advisership, a student sometimes teaches and thinks in a style opposite to her adviser's.)

In research this tendency is well known. The experienced reader recognizes the writing, not only of Professor X, but also of X's students. It's natural that something similar happens in teaching. This is rarely mentioned, because teaching is semi-private. (Not strictly private, since students are present. But to the professor's colleagues, it's private. Mathematician A generally doesn't know much about the teaching of mathematician B.)

Graduate math teaching seeks to produce mathematicians. If some students get Ph.D.'s, publish and become recognized mathematicians, the program is a success. If others fail to follow the lectures or complete the program, that's of little consequence.

A successful graduate professor is embedded in research. In his graduate teaching he may use the language, assumptions, viewpoints he does with research colleagues. Then the graduate student must somehow leap into the gestalt of research level talk.

There's a connection between teaching and writing. A tragic policy of some math research journals is to severely limit motivation and heuristics. Authors are not encouraged to write much about why their problem is interesting. Even less may they describe the blind alleys that ultimately led to success. From a certain "rigorous" point of view, it's necessary only to state theorems accurately and prove them correctly (rigorously.) Where they come from, what they're good for, aren't part of the mathematics. Indeed, the graduate professor himself need not have a deep understanding of where his subject came from, or what it's good for, if he was educated in the abstract, dogmatic style he perpetuates.

His lectures can be as bare of heuristics and motivation as his articles. Consciously or unconsciously, his students can take him as a model. While taking his course they work as teaching assistants. Often they are given no training in teaching or lecturing. They're just handed a textbook, a classroom number and a meeting time. The graduate lectures they attend every day affect how they teach their calculus or pre-calculus students.

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Some TA's are naturally good teachers. Some others learn in time to listen to students and communicate with them. This is a personal matter. The typical university neither requires it nor rewards it.

Later, as assistant professors, they are free to continue teaching in the style they started as TA's. After all, nobody says to do different. Their first concern now, of course, is tenure, not teaching. (Their students do evaluate their teaching. But students usually can't explain very well what they don't like. Anyhow, evaluations don't matter much if they aren't catastrophic.)

This description of untenured assistant professors doesn't apply to participants in "Project NExT". This exemplary MAA activity brings them together and helps them exchange ideas and experiences about teaching and other professional concerns.

To be sure, some graduate math professors are great teachers who love to explain the heuristics behind

their discoveries. Inspiration from such a professor can persist in teaching by his student, just as the dogmatism of another professor can persist in teaching by his student.

Not all graduate math teachers are inspiring. To join a graduate faculty of math you're not necessarily required to be a great teacher. What you do in class is pretty much your own business. Take pains with your teaching or don't take pains, most of your colleagues will be neither delighted nor upset.

I conclude that major obstacles to reform of math teaching are the teaching styles we absorb in graduate school, and the policies of our institutions that under-value teaching quality in hiring, tenure and promotion.

WHAT CAN BE DONE?

Can we change this story, where bad teaching propagates from one generation to the next?

We could concentrate on the leaders of American math. Top math professors in top grad schools, research managers in top industrial labs, top math bureaucrats in the U. S. Office of Education and the 50 State Departments of Education, editors of math texts in top math text publishing companies, math ed. professors in top Colleges of Education, top officers and staff of AMS, MAA, SIAM.

These men and women might come to agree and declare "It matters *how* math is taught, not just *what* math is taught. Treat math students as human beings, to avoid math avoidance. Independent work is important, K through 20. Realistic, credible applications are important, K through 20."

Such an agreement and declaration would result in improved math teaching.

Can we bring such a solution about? It doesn't seem easy. Recall Aesop's fable. To be safe from Kitty's claws, the mice must hang a bell on her neck. But which mouse will bell the cat?

Maybe we can improve mathematics education by organized effort, by education, by long-continued lobbying and agitation. That's how change is usually achieved in the U.S. Who will do that organizing, educating, lobbying and agitating? Those who care enough. I know of one small organization* in this work today (reference below.) If enough people care, more organizations may appear. More people may join. Something may happen.

But while we try to transform math education in the large, let's change it in the small. Let's teach students (not merely "teach the material"), by knowing them and caring about them (as far as class size permits!) Let's understand where the math came from and where it's going, and share this information with our students. Let's insist on interaction in the classroom, not tolerate passive classes that just copy formulas off the blackboard.

To change an old saying, "Let's light a candle or two while we curse the dark."

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Editor's Note: HMNJ #8 includes three articles about Potsdam College.

PROPOSED RECOMMENDATIONS, FROM THE "MATHEMATICS AND THE MEDIA CONFERENCE", TO EDITORS OF MATHEMATICS JOURNALS AND MAGAZINES.

1. The *Monthly*, the *Notices*, and other publications should have a monthly column on teaching mathematics. The privilege of writing a column would be awarded to teachers in all sorts of institutions who are nominated and selected as outstanding teachers.

The purpose is not only for the value of the columns, but especially as national acknowledgment of teaching as a high-prestige activity. National recognition would foster local recognition of teaching as a high-prestige activity. Local recognition would be an incentive for people to pay attention to their teaching.

2. "Suggestions on the teaching of college mathematics," produced 25 years ago by Don Bushaw and a committee, and published by the MAA, is an outstanding guide book on college math teaching. It's out of print and almost forgotten. It should be reprinted, possibly with some additions, and marketed enthusiastically. The *Monthly*, the *Notices*, etc., should print a reminder every August that a conscientious mathematics teacher ought to reread this booklet at the beginning of the school year.