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Review: Classification of Four and Six Dimensional Drinfel'd Superdoubles

Gizem Karaali
Pomona College

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Eghbali, A. [Eghbali, Ali] (IR-AZTMS-P); **Rezaei-Aghdam, A.** (IR-AZTMS-P);
Heidarpour, F. (IR-AZTMS-P)

Classification of four and six dimensional Drinfel'd superdoubles. (English summary)

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Lie superbialgebras are the graded analogues of Lie bialgebras. The paper under review is a detailed classification of all two- and three-dimensional Lie superbialgebras obtained from decomposable Lie superalgebras (augmenting and completing the authors' work in [J. Math. Phys. **51** (2010), no. 7, 073503; MR2681095 (2011h:17029)]). The authors then use these as a way to obtain all possible four- and six-dimensional Drinfeld superdoubles. Thus they continue on their trek to exhaustively construct all low-dimensional Lie superbialgebras.

Reviewed by *Gizem Karaali*

References

1. Andruskiewitsch, N., "Lie superbialgebras and Poisson-Lie supergroups," Abh. Math. Semin. Univ. Hambg. **63**, 147 (1993). MR1227871 (94g:17004)
2. Beisert, N. and Spill, E., "The classical r-matrices of AdS/CFT and its Lie bialgebras structure," e-print arXiv:0708.1762 [hep-th].
3. Eghbali, A., and Rezaei-Aghdam, A., "Poisson-Lie T-dual sigma models on supermanifolds," J. High Energy Phys. 09 (2009) 094; e-print arXiv:0901.1592 [hep-th]. MR2580709 (2011b:81219)
4. Geer, N., "Etingof-Kazdan quantization of Lie super bialgebras," e-print arXiv:math.QA/0409563.
5. Juszczak, C. and Sobczyk, J. T, "Classification of low dimensional Lie super-bialgebras," J. Math. Phys. **39**, 4982 (1998). MR1643318 (99k:17031)
6. Juszczak, C., "Classification of $osp(2|2)$ Lie super-bialgebras," e-print arXiv:math.QA/9906101.
7. Karaali, G., "A new Lie bialgebra Structure on sl(2,l)," Contemp. Math. **413**, 101 (2006). MR2262367 (2008e:17019)
8. Eghbali, A., Rezaei-Aghdam, A., and Heidarpour, F., "Classification of two and three dimensional Lie super-bialgebras," J. Math. Phys. **51**, 073503 (2010); e-print arXiv:0901.4471 [math-ph]. MR2681095
9. Backhouse, N., "A classification of four-dimensional Lie superalgebras," J. Math. Phys. **19**, 2400 (1978). MR0506713 (80c:17004)
10. Hlavaty, L. and Vysoky, J., "Manin supertriples and Drinfel'd superdoubles in low dimensional," e-print arXiv:0908.0997 [math-ph].
11. DeWitt, B., *Supermanifolds* (Cambridge University Press, Cambridge, 1992). MR1172996 (93j:58008)

12. Rezaei-Aghdam, A., Hemmati, M., and Rastkar, A. R., ”Classification of real three-dimensional Lie bialgebras and their Poisson-Lie groups,” J. Phys. A **38**, 3981 (2005); e-print arXiv:math-ph/0412092. MR2144530 (2006c:17031)
13. Jafarizadeh, M. A. and Rezaei-Aghdam, A., ”Poisson-Lie T-duality and Bianchi type algebras,” Phys. Lett. B **458**, 77 (1999); e-print arXiv:hep-th/9903152. MR1715374 (2000g:17029)
14. Snobl, L. and Hlavaty, L., ”Classification of 6-dimensional real Drinfeld doubles,” Int. J. Mod. Phys. A **17**, 4043 (2002); e-print arXiv:math.QA/0202210. MR1935877 (2003j:17028)
15. Note that the bracket of one boson with one boson or one fermion is usual commutator but for one fermion with one fermion is anticommutator. Furthermore, we identify grading of indices by the same indices in the power of (-1), for example *grading*e; this is the notation that DeWitt applied in his book (Ref. 11).
16. Note that the appearance of $(-1)^{jk}$ in this relation is due to the definition of natural inner product between $\mathbf{g} \otimes \mathbf{g}$ and $\mathbf{g}^* \otimes \mathbf{g}^*$.
17. Note that as we use De Witt notation and standard basis here, the structure constants $(\widetilde{X}^i \otimes \widetilde{X}^j, X_k \otimes X_l) = (-1)^{jk}\delta^i_k\delta^j_l$ must be pure imaginary.
18. The Lie superalgebra C^B_{FF} is one dimensional Abelian Lie superalgebra with one fermionic generator where Lie superal-gebra A is its bosonization. Furthermore, $A_{1,1}$ is different from $C^1_{1/2}$ for C^1_p and we show the latter by $p = \frac{1}{2}$.
19. Note that 17 of them are two bosons-one fermion and 53 of them are one boson-two fermions Lie superbialgebras.
20. See Appendix A for definition of superdeterminant.
21. Here superscript $C^1_{p=1/2}$ stands for supertranspose.
22. Note that st is one of the dual Lie superalgebras $\widetilde{\mathcal{G}_{\alpha\beta\gamma}}$,$(A_{1,1} + 2A)^0_{\alpha,\beta,\gamma}$ or $(A_{1,1} + 2A)^1_{\alpha,\beta,\gamma}$

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.