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Review: Massey Products on Cycles of Projective Lines and Trigonometric Solutions of the Yang-Baxter Equations

Gizem Karaali
Pomona College

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Polishchuk, A. [Polishchuk, Alexander] (1-OR)

Massey products on cycles of projective lines and trigonometric solutions of the Yang-Baxter equations. (English summary)

Algebra, arithmetic, and geometry: in honor of Yu. I. Manin. Vol. II, 573–617, Progr. Math., 270, Birkhäuser Boston, Inc., Boston, MA, 2009.

This paper continues an investigation initiated by the author in [Adv. Math. **168** (2002), no. 1, 56–95; [MR1907318 \(2003d:16051\)](#)]. The main object of study is the associative Yang-Baxter equation (AYBE):

$$r^{12}(-u', v)r^{13}(u + u', v + v') - r^{23}(u + u', v')r^{12}(u, v) + r^{13}(u, v + v')r^{23}(u', v') = 0,$$

where $r(u, v)$ is a meromorphic function of two complex variables in a neighborhood of $(0, 0)$ taking values in $\text{Mat}(N, \mathbb{C}) \otimes \text{Mat}(N, \mathbb{C})$. The author showed earlier that solutions of this equation relate to triple Massey products for simple vector bundles on elliptic curves and their degenerations. In the paper under review he characterizes such solutions which come from triple Massey products of cycles of projective lines. He also classifies all nondegenerate unitary solutions of the AYBE satisfying a particular periodicity condition, extending simultaneously his earlier results and also those in [T. Schedler, *Math. Res. Lett.* **10** (2003), no. 2-3, 301–321; [MR1981906 \(2004e:16041\)](#)]. The paper presents some elegant mathematics that sheds light on the connection between the combinatorics of simple vector bundles on a cycle of projective lines and the Belavin-Drinfeld combinatorics that describes, among other things, the various solutions of the classical Yang-Baxter equation (CYBE) [A. A. Belavin and V. G. Drinfeld, in *Mathematical physics reviews, Vol. 4*, 93–165, Soviet Sci. Rev. Sect. C Math. Phys. Rev., 4, Harwood Academic Publ., Chur, 1984; [MR0768939 \(87h:58078\)](#)]. Connections with the quantum Yang-Baxter equation as well as the CYBE are also considered.

{For the entire collection see [MR2640497 \(2010k:00009\)](#)}

Reviewed by *Gizem Karaali*