1-1-2010

Review: Quantization of Hamiltonian-type Lie Algebras

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Recommended Citation

MR2485471 (2010D:17021) Song, Guang'ai; Su, Yucai; Xin, Bin, Quantization of Hamiltonian- type Lie algebras. Pacific J. Math. 240 (2009), no. 2, 371–381. (Reviewer: Gizem Karaali)

MR2485471 (2010d:17021) 17B37 (17B62)
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Quantization of Hamiltonian-type Lie algebras. (English summary)
Pacific J. Math. **240** (2009), *no. 2,* 371–381.

Let Γ be a nondegenerate additive subgroup of $\mathbb{F}^{2n}$ and pick an $\mathbb{F}$-basis $\varepsilon_1,\dots,\varepsilon_{2n}$ for $\mathbb{F}^{2n}$ from Γ. Set $\sigma_i=\varepsilon_i+\varepsilon_{\bar{i}}$ for $1\le i\le n$. The Hamiltonian Lie algebra $\mathcal{H}$ is defined to be the quotient $\overline{\mathcal{H}}/\mathbb{F}\cdot 1$, where $\overline{\mathcal{H}}$ is the Lie algebra spanned by the set $\{t^\alpha|\ \alpha\in\Gamma\}$ with the product

$$[t^\alpha,t^\beta]=\sum_{i=1}^n(\alpha_i\beta_{n+i}-\beta_i\alpha_{n+i})t^{\alpha+\beta-\sigma_i},$$

for $\alpha,\beta\in\Gamma$.

Hamiltonian Lie algebras were defined in [X. P. Xu, J. Algebra **224** (2000), no. 1, 23–58; MR1736692 (2001b:17021)] as generalizations of simple Lie algebras of Cartan type. The Lie bialgebra structures on such Lie algebras were classified in [B. Xin, G. A. Song and Y. C. Su, Sci. China Ser. A **50** (2007), no. 9, 1267–1279; MR2370614 (2008j:17042)]. In the paper under review the authors quantize these structures. In particular they explicitly write down the coproduct and the antipode of the quantized enveloping algebras associated with a given Drinfel'd twist element. The proof consists of six pages of involved computations organized into five lemmas.

Reviewed by *Gizem Karaali*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.