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Review: Set-Theoretic Solutions of the Yang-Baxter Equation, Graphs and Computations

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The set-theoretic solutions of the Yang-Baxter equation have been of interest ever since the works of V. G. Drinfeld [in *Quantum groups (Leningrad, 1990)*, 1–8, Lecture Notes in Math., 1510, Springer, Berlin, 1992; MR1183474 (94a:17006)] and A. D. Weinstein and P. Xu [Comm. Math. Phys. **148** (1992), no. 2, 309–343; MR1178147 (93k:58102)]. The latter related these solutions to Lagrangian submanifolds of a particular symplectic groupoid. P. I. Etingof, A. V. Solov′ev and R. M. Guralnick [J. Algebra **242** (2001), no. 2, 709–719; MR1848966 (2002e:20049)] classified all indecomposable, nondegenerate set-theoretic solutions. More recently, D. Hrencecin and L. H. Kauffman [Topology Appl. **134** (2003), no. 1, 23–52; MR2005847 (2005a:57004)] studied the connection between biquandles and set-theoretic solutions to the Yang-Baxter equation.

The paper under review focuses on combinatorial and algebraic properties of these solutions. The authors develop a graphical presentation of the solutions and use this method to study solutions of finite order and their automorphisms.

Reviewed by *Gizem Karaali*

References

1. Artin, M., Schelter, W., 1987. Graded algebras of global dimension 3. Adv. Math. 66, 171–216. MR0917738 (88k:16003)
2. Drinfeld, V., 1992. On some unsolved problems in quantum group theory. Lecture Notes in Mathematics 1510, 1–8. MR1183474 (94a:17006)
3. Etingof, P., Schedler, T., Soloviev, A., 1999. Set-theoretical solutions to the quantum Yang–Baxter equation. Duke Math. J. 100, 169–209. MR1722951 (2001c:16076)
4. Gateva-Ivanova, T., 1994. Noetherian properties of skew polynomial rings with binomial relations. Trans. Amer. Math. Soc. 343, 203–219. MR1173854 (94g:16029)
5. Gateva-Ivanova, T., 1996a. Skew polynomial rings with binomial relations. J. Algebra 185, 710–753. MR1419721 (97m:16047)
6. Gateva-Ivanova, T., 1996b. Regularity of the skew polynomial rings with binomial relations, preprint. cf. MR 97m:16047
7. Gateva-Ivanova, T., 2000. Set theoretic solutions of the Yang-Baxter equation. Mathematics and education in Mathematics. In: Proc. of the Twenty Ninth Spring Conference of the Union of Bulgarian Mathematicians, Lovetch, pp. 107–117.
8. Gateva-Ivanova, T., 2004a. Quantum binomial algebras, Artin–Schelter regular rings, and solutions of the Yang–Baxter equations. Serdica Math. J. 30, 431–470. MR2098343 (2005h:16045)
9. Gateva-Ivanova, T., 2004b. A combinatorial approach to the set-theoretic solutions of the

Yang–Baxter equation. J. Math. Phys. 45, 3828–3858. MR2095675 (2005h:16077)

10. Gateva-Ivanova, T., Majid, S., (2005) Matched pairs approach to set theoretic solutions of the Yang–Baxter equation. arXiv:math.QA/0507394, revised 12/06. MR2383056 (2009a:16079)
11. Gateva-Ivanova, T., Jespers, E., Okninski, J., 2003. Quadratic algebras of skew polynomial type and underlying semigroups. J. Algebra 270, 635–659. arXiv:math.RA/0210217v1. MR2019633 (2004m:16039)
12. Gateva-Ivanova, T., Van den Bergh, M., 1998. Semigroups of I-type. J. Algebra 206, 97–112. MR1637256 (99h:20090)
13. Jespers, E., Okninski, J., 1998. Binomial semigroups. J. Algebra 202, 250–275. MR1614214 (99k:20119)
14. Lu, J., Yan, M., Zhu, Y., 2000. On the set-theoretical Yang–Baxter equation. Duke Math. J. 104, 1–18. MR1769723 (2001f:16076)
15. Majid, S., 1990. Matched pairs of Lie groups associated to solutions of the Yang–Baxter equations. Pacific J. Math. 141, 311–332. MR1035446 (91a:17009)
16. Majid, S., 2002. A Quantum Groups Primer. In: L.M.S. Lect. Notes, vol. 292. 179 pp. MR1904789 (2003f:17014)
17. Manin, Yu., 1988. Quantum Groups and Non Commutative Geometry Montreal University Report No. CRM- 1561. MR1016381 (91e:17001)
18. Rump, W., 2005. A decomposition theorem for square-free unitary solutions of the quantum Yang–Baxter equation. Adv. Math. 193, 40–55. MR2132760 (2005k:81132)
19. Schenkman, E., 1975. Group Theory. Robert E. Krieger Publishing Company, Huntington, New York. MR0460422 (57 #416)
20. Takeuchi, M., 2003. Survey on matched pairs of groups. An elementary approach to the ESS-LYZ theory. Banach Center Publ. 61, 305–331. MR2024436 (2004k:16104)
21. Weinstein, P., Xu, P., 1992. Classical solutions of the quantum Yang–Baxter equation. Comm. Math. Phys. 148, 309–343. MR1178147 (93k:58102)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.