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ABSTRACT
Various relations between direct and metalevel studies of language and mathematics are examined from an interdisciplinary perspective, in order to sort out in what way these interactions may be seen as, or lead to form, a coherent pattern of ideas. After setting up a rather precise map of this interaction, it is argued that coherence is necessary besides being possible.

1. INTRODUCTION AND BACKGROUND

Relations between language and mathematics are, in a broad sense, increasingly in the focus of attention of recent studies in the philosophy of mathematics as well as in mathematics education.

In the philosophy of mathematics, two current tendencies may be identified. Both are rooted in the basic idea that mathematics represents a special form of language use, according to a more general theory on how language use is constitutive for structures of abstract meaning, represented by figures as different as Peirce, Wittgenstein and Vygotsky. The first class of developments of this idea is diachronic in nature, as it is based on what is claimed to be historical evidence for the "dialectical nature" of the special language use involved in the construction of mathematical meaning; furthermore, it insists that this dialectics can only be understood as socially situated. At times, this direction approaches what might rather be called a "sociology of mathematics." Considering it a philosophy of mathematics, the basic claim seems to be that the nature of mathematical knowledge can only be studied indirectly, mainly through the institutions, interpersonal relations, etc., that are related to the creation and dissemination of mathematical knowledge. Representative accounts are found in (Hersh, 1979), (Kitcher, 1984), (Tymoczko, 1986), and (Ernest, 1998), where the philosophical nature of this viewpoint is also explicitly defended. One characteristic feature of such an account is that it is deductive, in the sense that it infers from general beliefs (such as "knowledge is socially situated") and related theories (notably sociology and elements of sociolinguistics), to the special case of mathematics, accommodating the specific characteristics of this special case in the general picture to the extent such specifics are considered at all. In fact, reference to actual mathematical practice is typically scarce. Even in (Ernest, 1998), the main such reference is indirect, namely via Lakatos' classical study (1976) of the history of Euler's polyhedron formula.

The second class of developments is focused on structural relations (similarities, differences, dependencies, etc.) between linguistic and mathematical knowledge, taking as a basis a synchronic, interdisciplinary analysis of actual mathematical and linguistic structures. The objective of such studies, then, is to shed light on the nature of mathematical knowledge through its relations with the (more "commonplace," but clearly not completely understood) nature of linguistic knowledge. It seems fair to say that there are more claims of such structural relations (which may be found scattered throughout the literature) than actual, detailed studies based on linguistic methods and explicit mathematical content, but at least, we have presently several examples, and some tentative theoretical frameworks, cf. (Halliday, 1974), (Pimm, 1987), (Rotman, 1988), (Walkerdine, 1990), (Winslow, 1998). Any such theory will, in principle, be inductive in nature, proceeding from analysis (in a more or less formal sense) of actual texts (in a broad sense) to general hypothesis and claims about patterns and characteristics of mathematical language use. The perspective of the present paper falls mainly in this second class.

It seems clear, even from this quick sketch, that the two viewpoints are not theoretically opposed to each other; rather does the difference reflect incommensurable views of what constitutes (or perhaps, what is important in) a theory of knowledge.

Turning now towards mathematics education, the main theme has been the roles and functions of natural language in the learning of mathematics, especially the mechanisms and the learning potential of discursive interaction in mathematics classrooms and correlations between performance in mathematics learning and competency in the language of instruction. It seems clear that such roles, functions and correlations should be both relevant to, and enlightened by, the philosophical issues alluded to above. The fact is that such relations are not studied in depth, especially as concerns the second of the above-mentioned viewpoints on mathematical knowledge.

The aim of this note is to argue that this relation may be established in a coherent way, while taking seriously into account both mathematics (the centerpiece, after all!), linguistics (not just folklore beliefs about "the social nature" of language), the philosophy of mathematics (as a discipline with certain basic issues and a long history related to philosophy in general), and mathematics education (as a young and quite vital discipline addressing highly important problems for education in general). As intermediate steps, we need furthermore to accommodate certain areas of sociolinguistics and language pedagogy, which are associated to (and partially derivative of) linguistics as such.

The main point of our discussion is that leaving out the full perspective of one of these disciplines could (and does in some cases) mean turning from "communication oriented theory of mathematical knowledge and learning" to either a very narrow study of two-sided correlations, or (in the worst case) to incoherent, unprofessional gossip.

The reader may wish to consult, at this stage, the figure in Sec. 7, in order to see at once the coherence of the first six sections, despite their apparent disparity. Moreover, the reader only interested in metamathematics (not in learning theory) could omit Sec. 5 and 6, corresponding to the right third of the mentioned figure.

2. FROM MATHEMATICS TO LINGUISTICS

This is the first and easy part of the story, as it is essentially just to be recalled (not constructed anew); the way back, from linguistics to (meta)mathematics, is the difficult part.

The flow of ideas from mathematics to linguistics can be regarded at two levels, which, after the early days (Harris, 1970), seem to be increasingly distinct in practice. The first is similar to the use of mathematical models in natural science fields, and is the source of what is called mathematical linguistics. Here, one studies certain mathematical systems which are, at the outset, defined in order to model certain aspects of natural language (typically within syntax). The problem vis à vis linguistics seems to be that the study of such models quickly generates interesting questions and sophisticated techniques, while "applications" (insights about natural language) become somewhat secondary. This is of course also the case with several other "mathematical" disciplines, such as mathematical physics or mathematical biology. For a readable introduction to mathematical linguistics, see e.g. (Gross, 1972).

The second level of influence is less direct in nature. It can be described as a transfer of methodological approach (rather than concrete theory), in this case resulting in a shift from the traditional study of "local phenomena" (e.g. the historic development of a certain word or expression) to the study of language as structures which are amenable to "global" study, much like (but not as) a mathematical structure. Although apparently this is a weaker kind of impact, it has been much more important: first of all, it is crucial in the rise of modern, structural linguistics. It has also, at times, been significantly inspired by results and methods from mathematical linguistics, as in Chomsky's famous argument that language structure cannot be generated from finite evidence alone (Chomsky, 1957, 83).

It goes without saying that the transfer of research methods among fields which were, traditionally, as separate as mathematics and linguistics, is neither harmless nor exempt from controversy. Indeed, it is highly questionable whether the study of language can address its most interesting aspects with a rigid and formal approach inspired by logic and mathematics (cf. also Sec. 2); this is, by the way, not the claim of structural linguistics as such. On the other hand, such an approach has not only given a new and better understanding of syntax and other parts of formal language structure, it has also added a dimension to our understanding of what research in linguistics and more generally in the humanities may be.

Incidentally, this second level of influence is not restricted to the field of linguistics. In fact, structuralism, as a general trend in the human sciences, may to a large extent be regarded as the result of this transfer of systematic approach from mathematics, see e.g. (Piaget, 1968) and (Gibson, 1984, B1).

3. LINGUISTICS BROADENING: SOCIOLINGUISTICS AND LANGUAGE PEDAGOGY

Sociolinguistics may be roughly defined as certain domains of inquiry in which the study area and methods of linguistics are extended to embrace wider areas than the structure of natural language at phrase level. These extensions happen in different dimensions: to cover natural language use at text level (discourse analysis), to analyze human sign systems beyond written and spoken natural language (semiology), and to study the use of natural language within various segments of a human society ("sociolinguistics" in a narrow sense). A well-developed area, which has affinities with all these aspects of sociolinguistics as well as with education, is the field of language pedagogy, in which the subject of study is the teaching and learning of natural language (mother tongue or foreign), cf. (Stern, 1983) and (Stubbs, 1986), that contain also references to work done in sociolinguistics. A main fact to observe here is that all of these extensions have significant roots in structural linguistics and directly or through structural linguistics in the Saussurian view of language as a sign system. In particular, they all carry the imprint of structuralism, and, with it, of a more or less direct transfer of methodology from mathematics. Discourse analysis, being a straightforward extension of grammar in the traditional sense, clearly confirms this claim, although the structural description may (and does) in this case exhibit a more pragmatic character. The idea of sign systems, in which signifieds are more often than not represented by signifiers of other signs, may be regarded as a mathematical metaphor from the structuralist view of mathematics as concerned with abstract "relation patterns" (Resnik, 1997). The study of language use which is particular for certain societal groups may look, at first sight, less subject to such relations, yet its inevitable need for describing these particularities (e.g. in terms of syntax or lexicology) ultimately forces it to interact with structural categories of natural language (Hudson, 1986). Similar remarks apply to the area of language pedagogy, where the influence of cognitive psychology (e.g. in the Piagetian sense) only adds further imprint of structuralist approach (in the sense outlined in the introduction).

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4. RELATIONS BETWEEN MATHEMATICS AND KNOWLEDGE ABOUT IT

After having outlined the flow of ideas from mathematics as far as to language pedagogy, we return to mathematics and begin to look at it from outside; theories about the nature and structure of mathematics as a whole (assuming this "whole" makes sense in an appropriately delimited sense) will be said to belong to the domain of metamathematics. In particular, this includes theories which are (at least similar to) mathematical theories themselves, such as various forms of logic, or category theory. It also includes theories of philosophical nature, to the extent these are concerned with mathematical knowledge. In fact, one has yet to see an interesting theory on the foundation of mathematics which does not raise philosophical problems.

The issues considered here thus ranges from the technical foundations of the subject, over its functions in other fields, to ontic and epistemic questions. A main torment of such theories seems to be that a credible solution to one problem often seems to have unacceptable consequences for some of the others; for instance, the various brands of realism offer coherent ontologies but also harsh difficulties with epistemic questions, while the theories that regard mathematics as a social entity seem to be more coherent on epistemic issues than as regards foundations. What I would like to do here is to replace the abstract subject (mathematics) with its appearance in the world, which is plainly textual (communication about, or using, mathematics). It appears that many controversial issues, especially as regards ontology and foundations, arise from the trying to grasp "the thing behind," the silent, unarticulated Mathematics behind its manifestations. In this sense, what we are left with could be said to be of "social" or "human" nature, just as other communicated realms; but it seems to me that to accept this is an empty triviality that is controversial only to the extent one has not abandoned the idea of finding mathematics elsewhere than through mathematics as encountered in the world. Whether or not there is such an out-wor(l)dly thing, I think it is legitimate and even wise to restrict rational inquiry to its complement.

Settling with this, we have excluded from our perspective of metamathematics only certain forms of realism which claim that mathematical objects have existence beyond their presence in communication about mathematics. Notice that this does not necessarily exclude them from being permanent, and, as far as we can see, even eternal; the universal grammar of human languages is a perfectly assuring analogy in this respect. On the contrary, having to rely on invariant properties of a subject which is not explicit in its articulated forms, seems to me to be of rather little assurance as regards the stability of the subject. We are thus left with all foundationalist theories of this century, and almost all of the relevant analytic and continental philosophies. Mathematics itself has limited use in judging between them. Indeed, we know at least in a certain limited sense (from Gödel's work) that no foundational theory will be "proved" mathematically. However, mathematics is in various forms is taken into account in each of them although not in all of them in a way consistent with the picture we are about to set up. For instance, Brouwer's "first act of intuitionism" (intuitionistic mathematics is an essentially languageless activity of the mind, in Brouwer, 1981) suggests that this school may fall partially outside the perspective of this paper, but the main reason is its prescriptive nature, as discussed in the next section.

5. LINGUISTICS AND METAMATHEMATICS

Metamathematics is inspired from linguistics in narrow and in broader ways. In the most literal sense, the logico-foundational studies of Russell, Gödel, Tarski and others are certainly a kind of linguistic inquiry into formal (mathematical) language in a very special technical sense. I claim that these studies have really little to do with linguistics and mathematics in the plain sense. As concerns linguistics, only few, especially Montague (1974), have tried to connect theories of formal languages directly to linguistics in the usual sense (dealing with natural language), and it seems to have little or no direct impact on modern linguistics. Regarding mathematics, the problem is that the formal languages analyzed in this tradition are quite different from the way mathematics is communicated by mathematicians other than these logicians. Rather than being a descriptive (or naturalistic) account and analysis of mathematical language use, this field constructs, to a large extent, the language it analyzes, although with the intention of creating a more consistent form in which to formulate existing mathematics. It thus has a prescriptive element which is quite foreign to linguistic methodology; right or wrong in language use is relative (to the grammar etc. of a speech community), not a matter of legislation. No linguistic theory of natural language (not even that of Montague) would start by constructing alternative forms of the language in order to put our understanding of the less perfect original on firm grounds.

The broadened scope of linguistics (Sec. 2), especially semiology and discourse analysis, offers various means of interpreting mathematics as language use. The "semiotics of mathematics" was opened, as a new perspective, in (Rotman, 1988), while ideas from discourse analysis have been influential in recent work in mathematics education (e.g., Pimm, 1994 and Sfard, 1998). The major question is now: which theories, among the fairly large and varied assortment available, are more likely to provide us with tools to make this interpretation faithful? One popular notion in this respect has recently been that of metaphor (e.g. Sfard, 1994, Lakoff-N'Òez, 1997), a notion which otherwise is especially used in the analysis of literary texts. While this has led to insights regarding the construction of meaning in mathematical texts as well, it rests purely semantic; it establishes an analogy between the meaning construction of literary and mathematical text, but it does not in itself enable contact with the specifics (or, more plainly, the obvious differences) between these. In short, the notion of metaphor would be much more powerful if associated with a firm theory of the linguistic specifics of such texts. Furthermore, the field of structural linguistics seems a quite obvious place to start, given its resonance with mathematics. This certainly does not imply that we are to restrict our investigation of mathematical text to their form but that our main goal, to describe the nature of their meanings, is better served when enlightened by a natural theory of the nature of their form. I believe we are here at a crucial intersection of the argument;

the question is, why not just bypass all this formal business and stick with pragmatic analysis of how thoughts are exchanged, regardless of well-formedness? Because in doing so, we bypass what makes mathematical communication both special and possible. The point is made as follows, in the context of grammar in natural language, by Moravcsik (1976): There is no thinking without rules. Both logic and grammar conspire to make it possible for us to articulate thoughts. It is absurd to think of rules as restricting thinking; rules of coherence, consistency, and grammaticality are what makes thinking possible. An analogy with games like chess should make this clear...The very possibility of playing chess is given by the fact that the game is defined in terms of rules...Learning and following rules...enhance our lives and enable us to be free to participate in a large number of activities.

Indeed, it is obvious that mathematical texts conform to strict formal rules in the sense of structural grammar: apart from symbol strings, the text consists of phrases of natural language which are constructed more or less as in other natural language texts; the symbolic parts are also highly regulated, although not according to usual linguistic principles. The interplay between symbol language and natural language is complex but clearly also crucial to the meaning of the text both at sentence level and at discourse level. A systematic yet still somewhat sketchy description of this interplay, from the point of view of structural grammar and discourse analysis, is given in (Winslow, 1998, Sec. 3) and (Winslow, 1999, Sec. 6). The latter reference also discusses the relation to Chomsky's view of linguistic knowledge. We can summarize the main results as follows:

Mathematical texts contain a certain regulated mixture of natural language and symbol languages (the latter including figures of geometric nature). At the first level, the syntax of the text is that of natural language, with certain phrase elements (e.g., a noun-phrase) replaced by symbol strings (e.g., an equation). Then, at the second level, the symbol strings have their own (universal and context-bound) syntax, which interacts systematically with the natural language syntax of the first level, e.g. in regulating "replacement."

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Understanding a proof is mainly a question of realizing certain transformations as acceptable to the grammar of mathematical language use.

We may describe the communicated realm of mathematical texts by the specific language use they represent, i.e. as a family of linguistic registers. This description includes the syntactic observations mentioned above, but also certain patterns of discursive practice which are closely linked to the syntactic phenomenon of transformations. An analytic tool in the analysis of discourse in mathematical registers is the notion of ensemble, which is a dynamic structure of textual information around which the discourse is centered.

As in the case of natural language, the complexity of mathematical language use forces us to accept that human competency in this domain has a non-void (innate) "initial state."

Metamathematical questions, delimited as in this section, address the nature of mathematical knowledge as evidenced by performance, i.e. from communication among human beings; this knowledge consists roughly of communicating competency (analogous to knowledge of a natural language) and factual knowledge (a finite number of sentences believed on linguistic evidence to be true). Notice that it is the latter, finitary part of mathematical knowledge which has often been in focus of philosophical studies; for us, they are the trivial part, and may in principle be considered lexical material. None of the traditional problems are swept under the carpet this way, because the evidence underlying this material, as well as the individual's belief of it, is highly dependent on communicative competency. Yet there is a shift of emphasis, well in line with mathematical practice: understanding a proof is mainly a question of realizing certain transformations as acceptable to the grammar of mathematical language use. The context is built up using, but not itself constituting, the register. Thus, as with natural language, the (extended) lexicon is occasionally revised during discourse practise. The elementary logical basis of mathematical reasoning another traditional source of controversy is then viewed as an integrated part of human language capacity, much in the Wittgensteinian sense of rule following as a condition to engage in meaningful language acts.

6. RELATIONS BETWEEN THE KNOWING AND THE TEAMING OF MATHEMATICS

In any domain, theories of knowledge and theories of learning interact. In everyday language, learning is a process towards or between states of knowing; a (problematic) mathematical metaphor could be a function of time (representing knowledge) and its derivative (representing learning). The problem with teaming mathematics (and language, an analogy to which we return in Sec. 6) is that it is not an accumulative process which is easy to test in the function picture; we are talking of a function with no obvious range and means for evaluation (this, then, applies to the derivative as well!), and it is definitely not a process which may reach a final "perfect" state. Apart from the most trivial kinds of recitative knowledge, such as knowing the multiplication table from 2 to 10, it does not really help to restrict or specify our attention to certain domains of mathematical knowledge.

It is also clear that there is no easy relation between metamathematics and mathematics education, understood as the theory and practice of learning and teaching mathematics. As Ernest (1991) rightly points out, we are talking here of resonances rather than logical implications between theories of knowledge and learning. In particular, the function image is also false in suggesting that the latter are derivatives of the first in any sense. All too often, educators have been more or less explicitly drawing links between false dichotomies of (what they conceive of as) "good" vs. "bad" views of knowing and learning mathematics, often as a circular justification of the "good" views.

How are we to proceed, if we are nevertheless sustaining that substantial links exist? I believe a key is to realize that, in fact, knowing and learning mathematics are not separable phenomena; acceptable signs that an individual "knows" mathematics (his "performance") will in all interesting cases consist in the production rather than reproduction of mathematical text, hence involve an element of self-induced learning on the part of the knowing individual. To see how a disciplinary distinction may then be sustained, we have to further graduate the rough definition of mathematics education given above, to comprise at least three main layers, each with plenty of room between them:

a) Mathematics acquisition (theories about the universal aspects of individual mathematics learning)

b) Didactics of mathematics (inquiry into how individuals may support learning of other individuals)

c) Methodology of mathematics teaching (concrete methods of didactics implementation in a specific context of contents, age group etc.).

It is clear that these three are also closely interacting in practice, but at least the traditional tension between theory (ranging from philosophy to psychology) and implementation (such as classroom teaching) can now be located as the span from a) to c), the former being the main site of interaction with metamathematics. In (Winslow, 1999) I have given an example of how metamathematics and a) interact, in particular how Chomsky's version of Plato's problem arises and may be approached in ways similar to what is found in linguistics models (of initial states and so on).

7. SOCIOLINGUISTICS, LANGUAGE PEDAGOGY, AND MATHEMATICS EDUCATION

There are two senses in which language theory may occur in mathematics education research. The first is related to natural language use in mathematics teaching, the second to mathematical language use as a central aim, and often also difficulty, of learning. In classroom discourse, one may typically find a maze of intertwining phenomena related to these two categories, but it is seems clear to me that to study the first without the second (that is, completely neglecting the linguistic specifics of the subject matter taught) is not a task for mathematics education, but rather for general educational discourse analysis. By now, there is an abundance of studies (not to speak of collected data) regarding communication in mathematics classrooms, and I think it is fair to say that few of these are explicitly grounded in coherent theories of both mathematical and natural language discourse. Because of the complexity of mathematics classroom discourse, in particular the delicate mixture of registers, only parts of which are mathematical, this leads to a situation where the results of research become incommensurable analyses of special cases, with reproducibility in other contexts far out of sight. Notice that this is not meant as a critique or denial of the value of those works; indeed, in a pre-paradigmatic phase, such a situation is an inevitable and even necessary step towards the formation of coherent research programs.

A perspective suggested by our discussion is the potential parallels between mathematics education and language education, and furthermore the necessity of making this explicit has so far been almost completely ignored in the study of linguistic aspects of mathematics education. The following are quite obvious domains of inquiry in which this parallel can be pursued:

How does knowledge of grammar behind "known language" uses (natural language mother tongue as well as known mathematics) affect learning of the new "grammar" (of mathematical language use)? Research partially along this line may be found e.g. in (UNESCO, 1974) and (Saxe, 1988).

In what ways does mathematical discourse competency develop from other forms of discourse in mathematics learning environments? See e.g. (Pimm, 1994) and (Sfard, 1998).

What are the roles of "learner factors" (such as affective factors, maturity factors and aptitude, cf. (Stern, 1983)), as studied in language education, in the learning of mathematical language use? This parallel is discussed e.g. in (Winslow, 1998, Sec. 4.3.2) and, at a much larger scale, in (Clarkson and Ellerton, 1996).

What diagnostic teaching forms (including tests) can be used to address particular language-related troubles in the learning process? This issue also has affinities with recent neuro-psychological research on the cognitive structures behind linguistic and numerical activity (Dehaene, 1997).

Are there mathematics-specific language disabilities, and how are they related to the classical types of (natural) language disorder? Research along this line may be found e.g. in (Donlan and Hutt, 1991), cf. also (Stubbs, 1986, Chap. 10) for background on "conversation disorders."

For all of these (and similar) points, it should be acknowledged that additional complexity (and, presumably, difficulty) arises in learning which is meant to include mixed discourse abilities, that is, the understanding and production of text in which mathematical language use occurs in contexts other than "pure mathematics," that is, in applied contexts. This, on the other hand, also parallels obvious issues in foreign language teaching, which is seldomly restricted to technical acquisition of the language (but includes also e.g. cultural and literary elements).

8. THE GRAND PICTURE AND THE NEED FOR IT

Viewing the preceding sections separately, I hope to have given the reader an impression of the deep links between the study of natural language structure on the one side, and the study of mathematics as a domain of knowledge on the other side. Both come in three more or less consecutive layers: the field itself, its metaaspects and its learning. Putting them together and drawing the flows of intellectual current which were described, we arrive at the diagram of Figure 1. It is important to note that not all arrows have the same status at present; especially the downward arrows are only now appearing in tentative ways, and as mentioned in the introduction, these ways are partially incompatible if not incommensurable.

The question is now: to what extent do these flows add up to a coherent picture? And, even more importantly: to which extent could and should they? By coherence, I do not mean strict commutativity in the

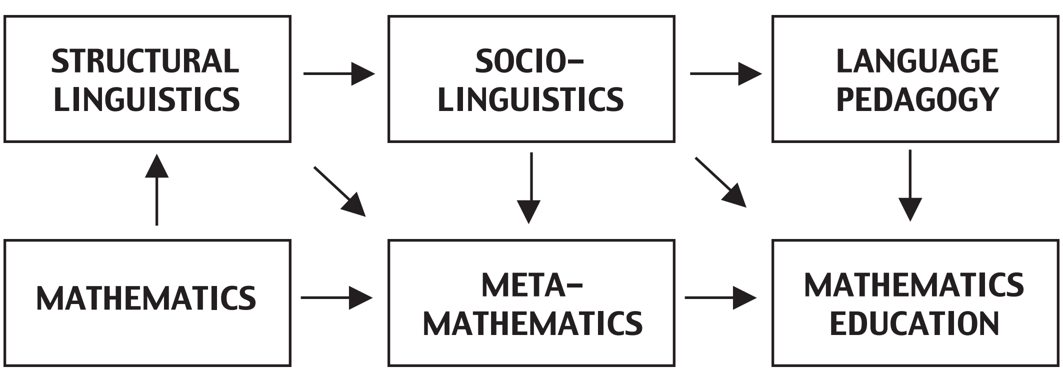

Figure 1

mathematical (mapping) sense, but rather that the flows are conceived as coordinated around each of the four triangles, representing (from the left): the influence of mathematics on metamathematics and linguistics, metatheories inspired by linguistics, the uses of sociolinguist perspectives in metamathematics and mathematics education, and the (emanating) flow of ideas from sociolinguistics through language pedagogy to the theory and practice of mathematics education.

I will close this note by sketching my main arguments that coherence is crucial for these interactions to play a significant role in metamathematics and mathematics education. First, assuming that our inquiry addresses issues (epistemology, foundations, etc.) which are specific to mathematics, it is evidently important that mathematics is in a substantial way taken into consideration. If we are to make the case that language type phenomena are, in a non-superficial way, crucial for these issues, then mathematics itself must be accommodated in our framework from the outset. This particularly concerns the left triangle of Figure 1, without which the rest has no direct link back to mathematics (this is to a large extent the case in much of the current relevant literature).

A crucial issue is the role of conversation in the learning and creation of mathematical knowledge (cf. e.g. Ernest, 1998). Here, of course, mathematics is taken into consideration, but usually only through (a few) specific historical instances. This is highly unsatisfactory if one is interested in the general case, but in the absence of a coherent and general linguistic theory of mathematical language use, there is no better option. This is why we are forced to substantially involve the apparatus of modem linguistics. Interdisciplinarity is obviously a necessity for the study of relations between as disparate domains of inquiry as dealt with here. Because of the traditions of professional training, few if any agents in the research communities will be experts in each of the six participating fields. The result of having no consensus or common overall perspective may be a wealth of bidisciplinary efforts, in which important input from other parts of the pattern is ignored.

On the other hand, a coherent understanding of interdisciplinarity in this area may lead to a new type of professionalism. This phenomenon is well known for bidisciplinary work, as the examples discussed in Sec. 1. My vision for the linguistic study of aspects of mathematics is exactly that this could be the case for the six-discipline interaction mapped out in Fig. 1, engaging mathematicians, linguists, philosophers of mathematics etc. and particularly "combinations" thereof in an explicitly articulated enterprise. If language use in mathematics is subject to defining rules of linguistic nature, then this enterprise is essential for our understanding and dissemination of mathematical knowledge.

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"As far as the laws of mathematics refer to reality, they are not certain; as far as they are certain, they do not refer to reality."

--Albert Einstein