

1-1-2008

# Review: Some Remarks on Quantized Lie Superalgebras of Classical Type

Gizem Karaali  
*Pomona College*

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## Recommended Citation

MR2344579 (2008G:17029) Geer, Nathan Some remarks on quantized Lie superalgebras of classical type. *J. Algebra* 314 (2007), no. 2, 565–580. (Reviewer: Gizem Karaali)

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**MR2344579 (2008g:17029)** 17B67 (17B37)

**Geer, Nathan (1-GAIT)**

**Some remarks on quantized Lie superalgebras of classical type. (English summary)**

*J. Algebra* **314** (2007), no. 2, 565–580.

In [Selecta Math. (N.S.) **2** (1996), no. 1, 1–41; [MR1403351 \(97f:17014\)](#)], P. I. Etingof and D. A. Kazhdan constructed a quantization for an arbitrary Lie bialgebra over a field  $k$  of characteristic zero. In his earlier work [Adv. Math. **207** (2006), no. 1, 1–38; [MR2264064 \(2007g:17019\)](#)], the author developed a similar theory for the graded case, proving in particular that the Drinfeld-Jimbo type superalgebra  $U_h^{\text{DJ}}(\mathfrak{g})$  associated to a classical Lie superalgebra of type  $A-G$ , with distinguished Cartan matrix (which was explicitly defined by H. Yamane [Publ. Res. Inst. Math. Sci. **30** (1994), no. 1, 15–87; [MR1266383 \(95d:17017\)](#)] in terms of generators and relations) is isomorphic to the Etingof-Kazhdan quantization of the Lie superalgebra. In this paper, he extends this result to arbitrary Cartan matrices  $A$  and thus to arbitrary Drinfeld-Jimbo algebras  $U_h^{\text{DJ}}(\mathfrak{g}, A, \tau)$ .

This is a very important result in its own right. However, the paper also includes some interesting consequences. The first of these describes all highest weight modules of a Lie superalgebra of type  $A-G$  as deformations of appropriate modules of the corresponding Drinfeld-Jimbo type superalgebra. The proof does not automatically follow by a mere superization of the proof of the non-graded version, because the original proof of the non-graded statement makes use of the vanishing of a particular cohomology whose graded counterpart does not vanish in general.

The second interesting corollary is the super version of the Drinfeld-Kohno theorem [T. Kohno, Ann. Inst. Fourier (Grenoble) **37** (1987), no. 4, 139–160; [MR0927394 \(89h:17030\)](#); V. G. Drinfeld, Algebra i Analiz **1** (1989), no. 6, 114–148; [MR1047964 \(91b:17016\)](#)] relating the monodromy of the Knizhnik-Zamolodchikov equations to a representation of the braid group arising from the universal  $R$ -matrix of  $U_h^{\text{DJ}}(\mathfrak{g}, A, \tau)$ .

Reviewed by *Gizem Karaali*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*