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Review: Degenerate Series Representations of the q-deformed Algebra soq'(r,s)

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Degenerate series representations of the q-deformed algebra $\mathrm{so}'_q(r,s)$. (English summary)
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This paper describes in detail certain representations of $\mathrm{so}'_q(r,s)$, which is a real form of the complex q-deformed universal enveloping algebra $U'_q(\mathrm{so}(n,\mathbb{C}))$ [A. M. Gavrilik and A. U. Klimyk, Lett. Math. Phys. **21** (1991), no. 3, 215–220; MR1102131 (92k:17021)]. The latter algebra differs substantially in structure from the standard quantum algebra $U_q(\mathrm{so}(n,\mathbb{C}))$ as defined by Drinfel′d and Jimbo; for instance it is not a Hopf algebra. However, as it is defined in terms of the generators $I_{k,k-1}=E_{k,k-1}-E_{k-1,k}$ for $\mathrm{so}(n,\mathbb{C})$, it has the nice property that

$$U'_q(\mathrm{so}(n-2,\mathbb{C}))\subset U'_q(\mathrm{so}(n-1,\mathbb{C}))\subset U'_q(\mathrm{so}(n,\mathbb{C})),$$

which in turn allows one to work with Gel′fand-Tsetlin type bases for finite-dimensional representations of $U'_q(\mathrm{so}(n,\mathbb{C}))$, which have been studied in [N. Iorgov and A. U. Klimyk, Int. J. Math. Math. Sci. **2005**, no. 2, 225–262; MR2143754 (2006c:17021)].

In the paper under review, the author focuses on certain infinite-dimensional representations of $\mathrm{so}'_q(r,s)$. These representations are described in terms of the finite-dimensional representations of $\mathrm{so}'_q(r)$ and $\mathrm{so}'_q(s)$, the compact real forms of $U'_q(\mathrm{so}(r,\mathbb{C}))$ and $U'_q(\mathrm{so}(s,\mathbb{C}))$, respectively. Explicit action formulas are given. An irreducibility criterion for the general construction is provided, along with a description of the irreducible constituents of the reducible ones coming from the same construction.

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References

1. Gavrilik A.M., Klimyk A.U., q-deformed orthogonal and pseudo-orthogonal algebras and their representations, *Lett. Math. Phys.* **21** (1991), 215–220. MR1102131 (92k:17021)
2. Drinfeld V.G., Hopf algebras and quantum Yang–Baxter equation, *Sov. Math. Dokl.* **32** (1985), 254–258. MR0802128 (87h:58080)
3. Jimbo M., A q-difference analogue of $U_q(gl(N+1))$ and the Yang–Baxter equations, *Lett. Math. Phys.* **10** (1985), 63–69. MR0797001 (86k:17008)
4. Klimyk A.U., Schmüdgen K., Quantum groups and their representations, Springer, Berlin, 1997. MR1492989 (99f:17017)
5. Klimyk A.U., Kachurik I.I., Spectra, eigenvectors and overlap functions for representation operators of q-deformed algebras, *Comm. Math. Phys.* **175** (1996), 89–111. MR1370088 (96m:33011)
6. Nelson J., Regge T., $2+1$ gravity for genus s¿1, *Comm. Math. Phys.* **141** (1991), 211–223. MR1133265 (93b:81266a)
7. Noumi M., Macdonald's symmetric polynomials as zonal spherical functions on quantum

homogeneous spaces, *Adv. Math.* **123** (1996), 16–77. MR1413836 (98a:33004)

8. Noumi M., Umeda T., Wakayama M., Dual pairs, spherical harmonics and a Capelli identity in quantum group theory, *Compos. Math.* **104** (1996), 227–277. MR1424556 (99c:17020)
9. Iorgov N.Z., Klimyk A.U., The q-Laplace operator and q-harmonic polynomials on the quantum vector space, *J. Math. Phys.* **42** (2001), 1326–1345. MR1814692 (2001m:39051)
10. Bullock D., Przytycki J.H., Multiplicative structure of Kauffman bracket skein module quantization, math.QA/9902117.
11. Twietmeyer E., Real forms of $U_q(g)$, *Lett. Math. Phys.* **49** (1992), 49–58. MR1162899 (94e:17029)
12. Dobrev V.K., Canonical q-deformation of noncompact Lie (super)algebras, *J. Phys. A: Math. Gen.* **26** (1993), 1317–1329. MR1212005 (94d:17012)
13. Celegini E., Giachetti R., Reyman A., Sorace E., Tarlini M., $SO_q(n+1, n-1)$ as a real form of $SO_q(2n, \mathbb{C})$, *Lett. Math. Phys.* **23** (1991), 45–44.
14. Raczka R., Limic N., Niederle J., Discrete degenerate representations of the noncompact rotation groups, *J. Math. Phys.* **7** (1966), 1861–1876. MR0206146 (34 #5971)
15. Molchanov V.F., Representations of pseudo-orthogonal groups associated with a cone, *Math. USSR Sbornik* **10** (1970), 353–347.
16. Klimyk A.U., Matrix elements and Clebsch–Gordan coefficients of group representations, Naukova Dumka, Kiev, 1979. MR0548464 (80j:22019)
17. Howe R.E., Tan E.C., Homogeneous functions on light cone: the infinitesimal structure of some degenerate principal series representations, *Bull. Amer. Math. Soc.* **28** (1993), 1–74. MR1172839 (93j:22027)
18. Gavrilik A.M., Klimyk A.U., Representations of q-deformed algebras $U_q(\mathrm{so}_{2,1})$ and $U_q(\mathrm{so}_{3,1})$, *J. Math. Phys.* **35** (1994), 3670–3686. MR1279333 (95k:81060)
19. Kachurik I.I., Klimyk A.U., Representations of the q-deformed algebra $U_q(\mathrm{so}_{r,2})$, *Dokl. Akad. Nauk Ukrainy, Ser. A* (1995), no. 9, 18–20. MR1374675 (97h:17018)
20. Schmüdgen K., Unbounded operator algebras and representation theory, Birkhäuser, Basel, 1990. MR1056697 (91f:47062)
21. Ostrovskyi V., Samoilenko Yu., Introduction to the theory of representations of finitely presented $*$-algebras, *Reviers in Math. and Math. Phys.* **11** (1999), 1–261. MR1997101 (2005a:46123)

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.