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The Prospects for Mathematics in a Multi-Media Civilization

Philip J. Davis
Brown University

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Philip J. Davis
Division of Applied Mathematics
Brown University
Providence, RI

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I

I am honored and delighted to be here this evening in conjunction with ICM 98 as part of this most distinguished series at the Urania Theater. I wish to thank the Invitation Committee of the ICM for proposing me as a speaker.

At the very beginning, let me explain my use of the phrase "multi-media" civilization. I mean it in two senses. In my first usage it is simply a synonym for the year 1998 and for the next decade or so. In the my second usage I refer to the widespread and increasing employment of computers, fax, e-mail, CD-roms, chips, videos in all mixtures. I mean it to designate the world that embraces such terms as interface design, cybercash, cyberlaw, virtual-reality assisted learning and V-R-A-surgery, cyberfeminism, tele-immersion, interactive literature, interactive cinema and animation, hand-held electronic books, 3D conferencing, spam.

And I might even include as media the U-Bahn (the subway system), the automobile and modern air transportation that have transmitted all our brains here to Berlin.

I personally cannot do without my word processor and my mathematical software. I find I can check conjectures quickly and find phenomena accidentally. (I find too many trivialities!) I am not yet fully into the Web.

Yes, the computer has become a universal and no longer strange attractor from which none of us are able to escape. Soon we will not be able to read anything without a mouse in our hands. We have been seduced, we have benefited, and we have become addicts.

And what aspects of mathematics shall I consider? The logical chains from abstract hypotheses to conclusions? Other means of arriving at mathematical conclusions and suggesting actions? The semiotics of mathematics? Its applications (even to multi-media itself!)? The psychology of math creation? The manner in which math is done, linked with itself and with other disciplines, published, transmitted, disseminated, taught, supported financially, applied? What will the job market be for its young practitioners? What will be the public's understanding of mathematics? Ideally, I should like to consider all of these. But, of course, every topic that I've mentioned would deserve a week or more of special conferences and would result in a large book.

Poincaré's Predictions

Our proximity to the millennium inevitably suggests that a speaker project forward in time. While such projections, made in the past, have proved notoriously inaccurate, I would be neglecting my duty if I do not make projections even though it is guaranteed that they will become the objects of future humorous remarks.

An example from the past. Ninety years ago, at the Fourth International Congress of Mathematicians held at Rome in 1908, Henri Poincaré undertook such a task. In a talk entitled "The Future of Mathematics," Poincaré mentioned ten general areas of research and some specific problems within them, which he hoped the future would resolve. What strikes me now in reading his article is not the degree to which these areas have been so developed—they have—but the necessary omission of a multiplicity of areas which we now take for granted and which were then only *in utero* or not even conceived.

Though the historian can always find in the past the

seeds of the present, particularly in the thoughts of a mathematician as great as Poincaré, I might mention as omissions from Poincaré's prescriptive vision, the intensification of the abstracting, generalizing and structural tendencies, the developments in logic and set theory, the pattern-theoretic, the emerging of new mathematics attendant upon the physics of fluids, materials, relativity, quantum theory, communication theory. And of course, the computer, in both its practical and theoretical aspects; the computer which has altered our lives almost as much as the "infernal" combustion engine and which may ultimately surpass it in influence.

Poincaré's omission of all problems relating immediately to the exterior world—with the sole exception (!) of Hill's theory of lunar motion—is also striking.

> ***The object of computation is not numbers but insight.***
>
> ***--Richard Hamming***

How then can the predictor with a clouded vision and limited experience in such matters proceed? Usually by extrapolating forward linearly current tendencies that are obvious to the most imperceptive observer.

What will pull mathematics into the future?

Mathematics grows from external pressures and from pressures internal to itself. I think the balance will definitely shift away from the internal and that there will be an increased emphasis on applications. Mathematicians require support; why should society support their activity? For the sake of pure art or knowledge? Alas, we are not classic Greeks or 18th century aristocrats, and even their material was pulled along by astronomy and astrology and geography. Society will now support mathematics generously only if it promises bottom-line benefits.

Now focus on the word "benefits." What is a benefit? Richard Hamming of the Old Bell Telephone Laboratories said in a famous epigraph to his book on scientific computation, "The object of computation is not numbers but insight."

Insight into a variety of physical and social processes, of course.

But I perceive (forty years later and with a somewhat cynical eye) that the real object of computation is neither numbers nor insight, but to make money—often via computations that are authorized by project managers who have little technical knowledge. If, by chance, humanity benefits, then so much the better, everybody is happy. And if humanity suffers, the neo-Luddites will cry out and form chat groups on the Web or their hackers will attack computer systems or humans. The techno-utopians will explain that you can't make omelets without breaking a few eggs.

And pure mathematics will follow along, moving closer to applications while justifying its purity to the administrators, politicians and the public with considerable truth that one never knows in advance what products of pure imagination can be turned to society's benefit. Employing that most weasel of rhetorical expressions: "in principle," in principle, all mathematics is potentially useful.

I could use all my time this evening in describing a few applications that seem now to be hot and are growing hotter. I will mention a few and comment very briefly on some of them.

Mathematics and the physical and engineering sciences

Classical. These have been around since Galileo, but only in the past, say, hundred years, has theoretical mathematics been of any great use to technology.

Mathematics and the life sciences

Mathematical biology and medicine are booming. There is automatic diagnosis. There are many models around; most are untested. One of my old PhD students has worked in biomolecular mathematics and designer drugs. He and numerous others are now attempting to model strokes via differential equations of fractional order. Good luck!

Mathematics and the military sciences

Then there is mathematics and the death sciences: war, both defensive and offensive. For the past sixty years this has been a tremendous engine pulling both pure and applied to new achievements. And, of course, defense will be with us as long as aggression is a staple of human behavior.

Mathematics and entertainment

There is mathematics and entertainment, through animation, simulation and computer graphics. The (ex)-executive of Silicon Graphics recently opined that the future of the United States lay not lie in manufacturing nor in the production of food, but in producing a steady flow of entertainment for the rest of the world. Imagine it, a future President of the United States may have to warn us against the media-entertainment complex as Eisenhower did with the military-industrial complex.

But wait! Through animation and simulation, the world of defense joins up with the world of entertainment and the world of medical technology. These worlds find common problems and can share computer software. (There was a recent conference on this topic.) Mickey Mouse flies the Stealth Bomber and performs virtual surgery via the same sort of software products. Young mathematicians take your results not to the ICM but to Steven Spielberg!

Mathematics and money

Marriages of business and mathematics are booming. Business and trade have always been tremendous consumers of low-level mathematics. But now it is no longer low-level. Zebra stripes. (Product identification.) Playing the market by clever statistical strategies. Portfolio management. The United States government spends billions on economic and social modeling and projections.

Mathematics and the graphic arts

Graphic art is being revolutionized along mathematical lines, a tendency—would you believe it?—that was present 3000 years ago in the art of Egypt when some of their art was pixelized.

Computer art shows are now commonplace. Is such art a kind of "soft mathematics?"

Mathematics, law, legislation and politics

Law is just beginning to feel the impact of mathematizations. Leibniz and Christian Wolff talked about this three centuries ago. Nicholas Bernoulli talked about it in 1709. Read his inaugural dissertation "On the Use of Probability (Artis conjectandi) in Law."

Statistics are more and more entering the courts as evidence. There are DNA identifications. Most recent and most notorious identification is the long conjectured Thomas Jefferson—Sally Hemmings liaison. This shows how little one can trust historians who wear blinders fashioned by their preconceptions. But can statisticians be trusted? The "experts" are often found testifying on both sides of a question.

There is epidemiology. There are class action and discrimination suits. Automated multiple regression makes it all feasible.

We spend hundreds of millions on polls of voters, consumers. The census. How to count? The simplest of mathematics operations turns out to be a practical impossibility. Sampling is recommended, but reduces variance, but increases the discrepancy.

On Jan 13, 1998, a demographer, head of the Census Bureau, resigned. She was in favor of sampling. The thought is that sampling will increase the power of the minority party, hence the majority party is against it. On Nov. 30, 1998, the case was argued before the U.S. Supreme Court.

Despite all these developments, we are as far as ever, perhaps further than ever, from Leibniz' dream of settling human disputes by computation.

All of the above major areas are intersected by

Mathematics and education

Consider education, for example. One of my colleagues writes me as follows:

> "My teaching has already changed a great deal. Assignments, etc. go on the web page. Students use e-mail to ask questions which I then bring up in class. They find information for their papers out there on the web. We spend one day a week doing pretty serious computing, producing wonderful graphics, setting up the mathematical part of it and dumping the whole mess into documents that can be placed on a web page. I am having more fun than I used to, and the students appear to be having a pretty good time while learning a lot. Can all this be bad?"

A very distinguished applied mathematician of my acquaintance is spending part of his time producing

cd-roms to publicize his theories and experiences.

The classic modes of elementary and advanced teaching have been amplified and sometimes displaced by computer products. A good computer store has more of these products for sale than there are brands of cheese in the famous Ka-Da-We department store in Berlin. Will flesh and blood teachers become obsolete?

Working habits and the working environment

Is all this perceived as good? Apparently not. Another colleague has written me:

> "On balance, I believe that science will suffer in the multi-media age. My experience is that true thinking now goes against the grain. I feel I have to be rude to arrange for a few peaceful hours a day for real work. Saying no to too many invitations, writing short answers to too many e-mail questions about research. A letter comes to me from a far corner of the world: 'please explain line six of your 1987 paper.' I stay home in the mornings hiding from my office equipment.
>
> Big science projects, interdisciplinary projects, big pushes, aided and abetted by multi-media and easy transportation have diminished my available time for real thought. I am also human and succumb to the glamour of today's technoglitz"

Is all this really for the better?

Teachers? Who needs them? The built-in HELP statements, so it is said, can impart plug-and-chug mathematics far more efficiently than any human.

Dissemination

Mathematicians used to relegate certain jobs to other disciplines or crafts or professions; we have become our own typists, typesetters, draftsmen, library scientists, book designers, publishers, jobbers, public relations agents, salesmen. For all that the computer is rapid, these activities absorb substantial blocks of time that were formerly devoted to thinking about problems.

In 1597, Tycho Brahe lugged his own heavy printing press from Copenhagen to Prague when he took a job there. We have become matheNETicians.

Teachers? Who needs them? The built-in HELP statements, so it is said, can impart plug-and-chug mathematics far more efficiently than any human.

Journals? Who needs them when you can download papers by the hundreds? The dissemination of mathematics through textbooks and learned journals is threatened in favor of on-line electronic publishing. Every man and woman his own journal. And we may be writing papers that might look like a code or a flowchart, that expose links more easily and prominently.

Whereas people such as Copernicus and Newton waited years before they published, today's scientists, under a variety of pressures, go on line with electronic publishing before their ideas are out of the oven, often unchecked, and they can get equally rapid and equally half-baked feedback.

Self-publication is rife. Refereeing has diminished. The authority that once attached to the printed page has vanished. Are books obsolete?

A collaborator working with me and much in love with mathematical databases, picked up (after clever filtering) more than 100,000 references to a key word that was relevant to our work. This produced an immediate blockage or atrophy of the spirit in him. We wondered whether we could afford the time to assess this raw, unassimilated information overload or simply plow ahead on our own as best we could.

Semioticist and novelist Umberto Eco wrote, in "How to Travel with a Salmon," "...the whole information industry runs the risk of no longer communicating anything because they tell too much."

Nonetheless, my eyebrows were raised recently when I learned that as part of a large grant application to the NSF, the applicants were advised to include a detailed plan for the dissemination of their work. In the multi-media age, mathematics is being transformed into a product to be marketed as other products.

The public understanding of mathematics

Every aspect of our lives is increasingly being mathematized. We are dominated by and are accommodating to mathematical machines and the arrangements they prescribe. Yet, paradoxically, the nature of technology makes it possible, through chipification, for the mathematics itself to disappear into the background and for the public to be totally unaware of it.

It is probably the case that despite the claims of educational administrators, the general population needs now to know less mathematics than at any time in the last several hundred years. What civilization needs is a critical education that brings an awareness and judgement of the mathematics that dominates it, and is able to react with some force to evaluate, accept, reject, slow down redirect, reformulate these abstract symbols that are affecting their concrete lives. Technology is not neutral. It fosters certain kinds of behavior. Mathematics is not neutral.

Individual mathematicians are aware of this. Groups calling themselves "technorealists" have web sites. But with some exceptions, the awareness has not yet penetrated the educational process.

Benefits

To all these developments there will surely be benefits, and the advancing waves of new technologies that are sweeping over us hardly need a public relations agent to trumpet the benefits. The size of Bill Gates' personal fortune derives from the public saying "yes" to all this. "Bring on more." The mental gridlock has only begun to appear. (Emboutaillage mentale.)

In making up the previous list, I have ignored pure fields out of personal incompetence. I simply do not have the knowledge or authority to single out from a hundred expanding subfields the ones with particularly significant potential. We have already heard about many of them in other Urania Talks and in the more than 1400 talks and poster-presentations given at ICM 98.

II

The inner texture (or soul) of mathematics

Let me now go up a metalevel and ask: how will multimedia affect the conceptualization, the imagery, the methodology of future mathematics? The metaphysics or philosophy of the subject? What is mathematics going to be doing other than presenting displays of its own narcissism? This is for me a tremendously interesting but difficult question. This is really what I want to discuss this evening. Here again, all I can do is to describe what I see and to project forward.

I shall make my attempt in terms of what I call

The tensions of texture

The discrete vs. the continuous

This is the point of William Everdell's "The First Moderns." To be up to date, apparently, is to be discrete, discontinuous. Everdell shows how this has operated in literature and art as well as in science and mathematics. Not so long ago, there was a movement afoot in the USA, asserting that continuous mathematics ought to give way in education and in philosophy to discrete mathematics. This movement seems to have quieted down a bit.

The deterministic vs. the probabaiistic

This split applies not only to the modeling of the exterior world, but resides interior to mathematics itself. It relates to such questions as: are the truths of mathematics probabalistic, or to the question of the extent to which we are able to live with such truths. Yet again, one little old lady residing in Rhode Island, winning $17,000,000 in the powerball lottery, does more to question and destroy the relevance of probability theory in the public's mind than all the philosophical skeptics such as myself.

The two dichotomies just mentioned are old, but they persist. The last goes back surely as far as the philosophic discussions of "free will."

And now for some new dichotomies.

Thinking vs. clicking

I have heard over and over again from observers that "thinking increasingly goes against the grain."

Is thinking obsolete or becoming more obsolete? To think is to click. To click is to think. Is this the equation for the future?

Did not mathematician/philosopher Alfred North Whitehead write in one of his books that it was a mis-

take to believe that one had constantly to think? Do not the rules, the paradigms, the recipes, the algorithms, the theorems and generalizations of mathematics reduce the necessity for thought? Did not Descartes write that his specific goal was to bring about this condition?

And in the historic past was not thinking confined to a special class of people? Have not thoughts over the whole of history been controlled--otherwise one might be declared a dangerous heretic or a traitor? Were not women forbidden to think and to study?

Experimental mathematics, visual theorems, are increasing in frequency. There are now two types of researchers: the first try to think before they compute; others do the reverse. I cannot set relative values on these strategies.

A researcher in AI has written me:

> "Your question 'Is Thinking Obsolete' is very much to the point. This has certainly been the trend in AI over to the past ten years (just now beginning to reverse itself)—trying to accomplish things through huge brute-force searches and statistical analyses rather than through high-level reasoning."

We can also ask in this context, is traditional advanced mathematics obsolete? For example, what portions of a theory of differential equations retain value when numerical solutions are available on demand; when, in many fields, computation is far in advance of success in explaining what is going on by analytic or theorematic mathematics?

Words or mathematical symbols vs. icons

A Semanticist, Mihai Nadin, now teaching in The University of Wupperthal, has written a large book, *The Civilization of Illiteracy*, on the contemporary decline of the printed word; how the word is being displaced by the hieroglyphic or iconic mode of communication. There is now computer induced illiteracy and innumeracy.

There is no doubt in my mind but that this displacement will have a profound affect on the inner texture of mathematics. Such a shift already happened 4000 years ago. Numbers are among the oldest achievements of civilization, predating, perhaps, writing. In his famous book, *Vorgreichischer Mathematik*, Otto Neugebauer "explains ... how hieroglyphs and cuneiform are written, and how this affects the forms of numbers and the operations with numbers."

Another such shift occurred in the late Middle Ages when algebraic symbolisms began to invade older texts.

Mathematics as objective description vs. mathematics by fiat, or the ideal vs. the constructed and the virtual

Applied mathematics deals with descriptions, predictions and prescriptions. We are now in a sellers' market for all three. Prescriptions will boom. There may indeed be limits to what can be achieved by mathematics and science (there are a number of books on this topic), but I see no limits, short of the willingness of humans to endure them, to the number of mathematizations that can be prescribed and to which humans are asked to conform.

> ***All products, all human activities are now wide open to prescriptive mathematizations.***

In the current advanced state of the mathematization of society and human affairs, we prescribe the systems we want to put in, from the supermarket to the library to the income tax to stocks and bonds to machines in the medical examination rooms. All products, all human activities are now wide open to prescriptive mathematizations. Prof. David Mumford anticipates a great increase in the invention of new mathematical structures. The potentialities and the advantages envisaged and grasped by the corporate world will lead it to pick up some of the developmental tab. And, as it does, the human foot will be asked, as with Cinderella's sisters, to fit the mathematical shoe. If the shoe does not fit: tough for the foot.

What is proved vs. what is observed

This is the philosophical argument between Descartes and Giambattista Vico. I venture that as regards the generality of users of mathematics, its proof aspect will diminish. Remember: mathematics does not and never did belong exclusively to those who happen to

call themselves mathematicians and who claim to pursue Mathematics with a capital M. I would hope that the notion of proof will be expanded so as to be acknowledged as simply one part of a larger notion of mathematical evidence.

The whole present corpus of mathematical experience and education has come under attack from at least two different social-political directions:

Euro or Western mathematics vs. other national or ethnic mathematics

We have today's ethno-mathematicians to thank for reminding us that different cultures, primitive and advanced, have had different answers as to what mathematics is and how it should be pursued and valued. (E.g., ancient oriental mathematics was carried on in a proof-free manner. Ancient Indian mathematics expressed itself in verse.) More important than drawing on ancient, "non-Western" material is the possibility that new "ethnic" splits, to be described momentarily, will emerge from within current practices. Will a civilization of computer-induced illiteracy compel major paradigm shifts in mathematics? Extrapolating from Nadin's book, one might conclude that this might arrive sooner than we think and perhaps more rapidly than is good for us.

Male vs. female mathematics

Mathematics has been perceived as an expression of male machismo. Margaret Wertheim is a TV writer as well as a former student of math and physics. Let me quote from her recent book *Pythagoras' Trousers*:

> "One of the reason more women do not go into physics is that they find the present culture of this science and its almost antihuman focus, deeply alienating. ... After six years of studying physics and math at university, I realized that much as I loved the science itself, I could not continue to operate within such an intellectual environment." (p. 15)

The bottom line of this book is that if more women were in mathematics and science (particularly in physics), then they would create

> "an environment in which one could pursue the quest for mathematical relationships in the world around us, but within a more human ethos." ... "The issue is not that physics is done by men, but rather the kind of men who have tended to dominate it." ... "Mathematical Man's problem is neither his math nor his maleness per se, but rather the pseudoreligious ideals and self-image with which he so easily becomes obsessed."

More women are entering mathematics and science, and it will take at least two generations to observe whether or not Wertheim's vision will materialize and what it implies.

The apparent vs. the occult

In a somewhat disturbing direction, we have the concern on the part of some mathematicians and physicists with hermeticisms, apocalypses of various sorts: final theories of everything, secret messages hidden in the Bible, everything under the sun implied by Goedel's Theorem.

I was shocked recently to read that one of the Mathematical Societies in the USA had published some of this kind of material—even though it was in a spirit of "fun."

The old marriage of literacy and rationality, in place since the Western Enlightenment, seems to be ending in divorce. Rationality has shacked up with fanaticisms. Are these part of the breakdown of a literate civilization or merely the age old and temporary anxiety that accompanies the arrival of a new millennium?

Soft mathematics vs. traditional mathematics

I have picked up the term "soft mathematics" from Keith Devlin's popular book *Goodbye, Descartes* which describes the difficulties of the relationship between natural language, logic, and rationality. These difficulties, Devlin asserts, cannot be overcome by traditional mathematics of the Cartesian variety, and he hopes for the development of a "soft mathematics"—not yet in existence—that

> "will involve a mixture of mathematical reasoning, and the less mathematically formal kinds of reasoning used in the social sciences." Devlin adds that, "perhaps most of today's mathematicians find it hard to accept the current work on soft mathematics as 'mathemat-

ics' at all."

Nonetheless, some see the development as inevitable, and Devlin uses as a credentialing authority the mathematician-philosopher Gian-Carlo Rota. Rota comes to a similar viewpoint through his phenomenological (Husserl, Heidegger) orientation.

After listing seven properties that phenomenologists believe are shared by mathematics (absolute truth; items, not objects; nonexistence; identity; placelessness; novelty; rigor), Rota goes on to say:

> "Is it true that mathematics is at present the only existing discipline that meets these requirements? Is is not conceivable that some day, other new, altogether different theoretical sciences might come into being that will share the same properties while being distinct from mathematics?"

Rota shares Husserl's belief that a new Galilean revolution will come about to create an alternative, soft mathematics, that will establish theoretical laws through idealizations that run counter to common sense.

And what is "common sense?" It may be closer than we think to what George Bernard Shaw wrote in "Androcles and the Lion," "People believe not necessarily because something is true but because in some mysterious way it catches their imagination."

Platonic (Deistic) philosophies of mathematics vs. humanistic philosophies

British analytic philosophy died from "dead-end-itis." Is philosophy, in general, obsolete in today's world? Perhaps, but let us have a bit more of it before we say Nunc Dimittis.

The vaunted and (I think) mythic Unity of Mathematics is further threatened by self-contained, self-publishing chat groups. It was already threatened in Poincaré's day by the sheer size of the material available. The riches of mathematics, without contemplative judgements, would, in the words of Poincaré, "soon become an encumbrance and their increase produce an accumulation as incomprehensible as all the unknown truths are to those who are ignorant."

The classic Euclidean mode of exposition and teaching: "definition, theorem, proof" has come under serious attack as not providing a realistic description of how mathematics is grasped, utilized or created.

Platonism and its various offspring, which have been the generally accepted philosophies of mathematics, have come under serious attack. Here are a few quotes that bear on this.

> "By giving mathematicians access to results they would never have achieved on their own, computers call into question the idea of a transcendental mathematical realm. They make it harder and harder to insist as the Platonists do, that the heavenly content of mathematics is somehow divorced from the earthbound methods by which mathematicians investigate it. I would argue that the earthbound realm of mathematics is the only one there is. And if that is the case, mathematicians will have to change the way they think about what they do. They will have to change the way they justify it, formulate it and do it."
> — Brian Rotman

> "I know that the great Hilbert said 'We will not be driven out of the paradise that Cantor has created for us.' And I reply: 'I see no need for walking in.'"
> — Richard Hamming

> "I think the Platonistic philosophy of mathematics that is currently claimed to justify set theory and mathematics more generally is thoroughly unsatisfactory, and that some other philosophy grounded in inter-subjective human conceptions will have to be sought to explain the apparent objectivity of mathematics."
> — Solomon Feferman

> "In the end it wasn't Goedel, it wasn't Turing and it wasn't my results that are making mathematics go in an experimental direction. The reason that mathematicians are changing their habits is the computer."
> — G. J. Chaitin

III

A Personal Illumination

Here, then, are some of the "tensions of mathematical texture" that I perceive. Today's scientist/mathematician spends his or her days in a way that is vastly different from 50 years ago, even 20 years ago. Thinking now is accomplished differently. Science is now undergoing a fundamental change; it may suffer in some respects, but it will certainly create its own brave new world which will proclaim new idealisms.

I think there will be a widening to what has been traditionally been considered to be valid mathematics. In the wake of this, the field will again be split just as it was in the late 1700's when it split into the pure and the applied. As a consequence, there will be the "true believers" pursuing the subject pretty much in the traditional manner, and the "radical wave" pursuing it in ways that will raise the eyebrows and the hackles of those who will cry "they are traitors to the great traditions."

Elias Canetti, Nobelist in Literature (1981), in his autobiography speaks of an illumination he had as a young man. Walking along the streets of Vienna, he saw in a flash that history could be explained by the tension between the individual and the masses.

Walking the streets of my home town, I got an illumination: the history of future mathematics will be seen as the increased tension and increased interfusion, sometimes productive, sometimes counterproductive, between the real and the virtual. How these elements will play out is now a most excellent subject for writers of mathematical fantasies.

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metallic numbers whose continued fraction expansion was only periodic (the copper number and the nickel number).

6. CONCLUSIONS

In analyzing, from a mathematical point of view, the similarities as well as the differences among the members of the MMF, it is obvious that these characteristics are strongly linked with the transition from periodic to quasi-periodic dynamics. But simultaneously, from the beginning of humanity, there have been philosophical, natural and aesthetic considerations that have given them primacy in the establishment of geometrical proportions based on some members of this family. Such a broad range of applications opens the road to new multi-disciplinary investigations that undoubtedly will contribute to clarifying the relations between art and technology, building a bridge that should join rational scientific thinking with aesthetical emotion. Hopefully, this new perspective could help us to confer on technology, from which we depend every day more and more for our survival, a more human character.

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