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To My Students

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One of the distinguishing characteristics of Harvey Mudd College is the requirement that the study of engineering and science should be accompanied by the study of the humanities and social sciences. This requirement is not only to help HMC graduates to be well rounded people, but also to help you achieve a better understanding of engineering, science and their applications and interactions with the rest of the world. A narrow focus on the technical content will decrease understanding of that technical content. In fact, if the focus is too narrow, it may be impossible to understand anything at all!

The humanities and a humanistic point of view are not extra additions to the technical content, but are part of the technical content. A humanistic point of view permeates the effort to understand all subjects, including mathematics.

The syllabus seems so crowded, that one might assume that there is no room for anything but a narrow focus on technical content, and that a humanistic approach will require that some of the technical content will have to be dropped. I assure you that that assumption is not correct. My experience has shown that, actually, the students learn at least as much, and usually more than other students in more narrowly focused courses. Learning is not effortless or automatic, but the students have found the course to be less stressful and more leisurely.

SCIENCE IS MORE THAN FACTS AND FORMULAS

The facts and formulas are really a small part of a subject, although in high school most class time is spent on them. College is different. Here we are dealing with general principles and how the ideas and concepts fit together. That which is beyond the facts and formulas may be difficult to describe, but it can be discussed in general terms. It seems paradoxical that concentrating on that which is beyond the facts and formulas will give us a greater mastery over those very facts and formulas that have occupied most previous exams.

THE MODERN WORLD

The invention of the Calculus is one of the most important events in the history of civilization. It separates medieval from modern times. Our whole mentality and mode of thinking are decisively influenced by modern science. Newton's ideas set the program for science and social science for 200 years. The success of the quantitative approach suggested to physiologists and psychologists that they look for explanations of their problems in mechanical terms instead of in terms of astrological portents, soul, mind, spirit, humors, and other vague notions. Appreciation of the already amazing power of mathematics and science imbued thinking men with enthusiasm for a sweeping reorganization of all knowledge. They exalted human reason as the most effective instrument for the attainment of truths. Because they regarded mathematical reasoning as the embodiment of the purest, deepest, and most efficacious form of all thought, they urged the use of mathematical methods and mathematics proper for the derivation of knowledge. The chief characteristic of this new approach to knowledge was unbounded confidence in reason and in the validity of the extension of mathematical methods throughout the physical and formal sciences and beyond them to all fields of knowledge (Kline).

MEANS AND ENDS

So learning mathematics brings one right to the heart of things. The subject is intertwined with the translation of perceptions from our senses to science and technology, and with fundamental philosophical and historical questions. We shall study not only the facts, formulas, and techniques, but also the spirit and structure of the subject as one of the humanistic disciplines. The facts, formulas, and techniques will be the means, not the ends; they will be the building blocks for the edifice of science. Our ends or goals will be to comprehend the principles of mathematics and science and to enter into the spirit of creative understanding

and discovery.

LEARNING TO QUESTION

The essay "Learning to Question" asserts, "The cutting edge of knowledge is not in the known but in the unknown, not in knowing but in questioning. Facts, concepts, generalizations, and theories are dull instruments unless they are honed to a sharp edge by persistent inquiry about the unknown." The author's advice will be a guide for us. Not only will we ask questions about how to solve a problem or how to prove a theorem, but also such questions as can the solution to the problem be used in any other way; or is this problem special or general; or why is it interesting? Questions about why is a theorem significant and what are the essential ingredients of a proof will concern us. Albert Einstein felt that his imagination and curiosity were more important than his knowledge of certain facts. One of our goals will be the strengthening of our imagination and questioning ability. Another goal, which depends on imagination and questioning, is to learn to approach problems and subjects as experts rather than as novices. We shall become aware of our own growth toward expertness.

PRODUCTIVITY AND UNDERSTANDING

The article from the National Academy of Sciences is about science education. It is also about a real measure of knowledge and understanding. The last page of the article discusses the acquisition of language which distinguishes humans from the other animals. The "...extraordinary output of new sentences by men out of their stock of words and syntax. It is this that makes human language such a remarkably revealing mirror and so potent a tool of the mind and spirit. The same test is most congenial for any real understanding of mathematics and science at every level. What can the student do with what he knows to make a new sentence?" This class will follow the recommendation of the National Academy of Sciences. We shall try to make "new sentences" out of the facts, formulas, and techniques that we learn. Everyone will have the opportunity to be creative by asking questions or finding answers, or by distorting a textbook problem until it is no longer solvable and then studying the difference between solvable and unsolvable problems. There are, of course, many ways to be creative; finding them is an act of creativity.

ROLE OF UNCONSCIOUSNESS

One of the best essays on creativity is *The Psychology of Invention in the Mathematical Field* by Jacques Hadamard. Although the title mentions mathematics, the ideas apply to all fields. There are idea gems throughout the book, and everyone will discover profitable insights by reading it. A central idea is the role of the unconscious in discovery or problem solving. Many people have had the experience of trying to solve a problem, or even to remember something. After much effort the struggle is abandoned; then the solution or answer pops into consciousness at a moment of relaxation.

This surprising situation is common and is described by Hadamard at length. A summary follows:

- First there must be *recognition* of a problem or puzzlement.
- This is followed by *preparation* or study.
- The important third stage is *incubation*. This is the stage when our unconsciousness is working efficiently. The stage of *incubation* is often neglected by students, perhaps because most previous problems were trivial.
- The moment when the solution pops into consciousness is called *illumination* or *revelation*.
- The final stage is *verification*, which is a working out of the details that the sudden insight revealed.

This five stage sequence has been described by many people who have thought about creative discovery or problem solving:

0. Recognition
1. Preparation
2. Incubation
3. Illumination
4. Verification

This scheme has been described by chemists, physicists, novelists, poets, composers, psychologists, and others who create and solve problems. An awareness of the scheme will help in all of your college subjects as well as beyond the classroom.

ROLE OF BEAUTY

G.H. Hardy writes, "The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is

no permanent place in the world for ugly mathematics."

Hadamard, discussing the general direction of research, asks how the important choice of direction is to be made. He answers emphatically, "The answer Is hardly in doubt: it is the same which Poincaré gave us concerning the means of discovery, the same for the *drive* as for the *mechanism*. The guide...is that sense of scientific beauty, that special esthetic sensibility, the importance of which he has pointed out."

Although the quality of beauty seems to be a personal, subjective concept, general agreement somehow occurs, and many people refer to it with no hesitation. Heisenberg's essay *The Meaning of Beauty in the Exact Sciences* explores the question in a broad sweep from Pythagoras to Einstein. Following the Classical Greek tradition, he defines beauty as the proper conformity of the parts to one another, and to the whole. He considers what motivated Kepler, Newton, Pauli, and Carl Jung. His writing is compelling. He asks, "But are we dealing here with knowledge merely, or also with the beautiful? And if the beautiful is involved, what role did it play in the discovery of these connections?... What is it that shines forth here? How comes it that with this shining forth of the beautiful into exact science the great connection becomes recognizable, even before it is understood in detail and before it can be rationally demonstrated? In what does the power of illumination consist, and what effect does it have on the onward progress of science?"

Freeman Dyson quotes Hermann Weyl, "My work always tried to unite the true with the beautiful; but when I had to choose one or the other, I usually chose the beautiful." [World of Mathematics, p. 1831].

This point of view which is so strongly stated by so many creative people, will be a guide for our own creative and learning efforts.

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"Cantor's Coat"

Lawrence Mark Lesser

In his coat,
Cantor kept a note
his father wrote
Urging him
that faith within
will sustain him.

He picked his path
the freedom of math
but met with wrath
For counting infinities
real as the Trinity
uncountably vast.

Cantor believed this was Divine Plan
But his mentor said, "God made whole numbers,
the rest is by man!"

So his mentor withdrew,
said "Your renegade view
corrupts the youth!"
Kept from his goal,
Cantor searched his soul
to fit part to whole...

"Father, do I hear cries of Galileo?
For this fight, was I right to give up the violin?"

Some say he went mad
from the quest he had
or was he just sad
When his heart went still--
sanitorium swill
against his will.

But in his coat,
they found a note
his father wrote.