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# Al-Khwārizmī and the Hermeneutic Circle: Reflections on a Trip to Samarkand

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## Synopsis

In this paper we discuss al-Khwārizmī's life and aspects of his work and suggest a possible hermeneutic avenue into his contributions to mathematics.

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Recently I attended the *al-Khorezmiy14* conference at Samarkand, Uzbekistan. Even though the conference was not on the history of mathematics or on al-Khwārizmī, many speakers, especially the ones from Uzbekistan, emphasized al-Khwārizmī's contributions to mathematics. Clearly and perhaps justly they are very proud of the fact that al-Khwārizmī was from the "East". Although it is clear to me that one cannot characterize mathematical development solely in terms of European developments, the mathematical renaissance in the Arab world was surely not the only one, either. One has to consider the contributions and impact of the rest of the "East"; namely, Indian and Chinese mathematics. So was there anything special about the part of the east where al-Khwārizmī comes from?

Samarkand is a special city. It is situated in the southern part of Uzbekistan, a region that produced many mathematicians and scientists after the decline of Greek civilization but before the modern period. The Mongol conqueror Tamarlane made Samarkand his capital city, and his grandson, Ulugh Beg, built an observatory in Samarkand where the renowned al-Kashi worked. In this observatory a great catalogue of stars was compiled. While Arabic star names have a complicated history and predate Ulugh Beg, one can still wonder why many of our modern star names are of Arabic origin.



Ulugh Beg working with other scientists in Samarkand

Ali Kuscü, a 15<sup>th</sup> century mathematician, was also educated in Samarkand and moved to Istanbul in 1474 [22], where he presented his mathematics book *Muhammediye* to Fatih Sultan Mehmet, the conquerer of Constantinople (Istanbul). Khwarizm was the name of the region in the southern part of the Aral sea. Two major figures come from this area: one was the great al-Biruni, who was born in 973; the other one was simply known as al-Khwārizmī, named after his place of origin. His name is metamorphosed into the word “algorithm” and the title of one of his books gave us the word “algebra”. It is well known that al-Khwārizmī’s book on “Indian Arithmetic” - translated into Latin - led to the adaptation of “Arabic numerals” in the “West”. Thus there is ample evidence that al-Khwārizmī’s impact on mathematics is huge.

However, even though the boundaries between the East and the West are always shifting, the myth that modern mathematics comes from men, in particular, from white Christian men in the Greco-European tradition still persists (see Grabiner [4], page 243). The polarization between the East and the West is not just a problem of politics or geography but a problem of knowledge, ideas, and thought. It may not be useful to overemphasize the contributions of “Islamic eastern mathematics” to mathematics, since mathematics is built on past mathematics and belongs to all of humanity.

I cannot help but wonder how the impact of ancient texts written by al-Khwārizmī, an Easterner and a Muslim scholar, should be interpreted so we come to close enough to truth, “within  $\epsilon$ ”. Can hermeneutics help? After all, it has been used to interpret texts in literature, law, philosophy, even in economics [1]. In this paper, I share some of my observations on al-Khwārizmī’s work and explore possible avenues where one can get some help from hermeneutics in interpreting ancient texts.

## 1. What is hermeneutics?

According to E. Klein, [7], the word *Hermeneutics* comes from the Greek word *ἐρμηνεύω* “hermeneia”, meaning “interpretation” or “explanation”. Hermeneutics derives its meaning from the Greek god *Hermes*, son of Zeus, the messenger of gods, god of trade, even god of thieves. Hermes was given the function of translating and transmitting the message of gods to human beings. Here “translation” does not only mean between languages but also within the same language. Hermes fulfills this function of translating by inventing language and writing. Since words have the power to conceal or twist meaning, it is fitting that he was also considered a “liar”. Hermeneutics is known as the theory of textual interpretation, especially the interpretation of biblical, literary and philosophical texts. It is interesting to note that Greeks were using techniques of interpretation to assist in understanding and criticism of Homer.

Friedrich Schleiermacher, [19], widely regarded as the “father” of hermeneutics, believed that, in order for an interpreter to understand the work of another author, they must familiarize themselves with the historical context in which the author published his/her thoughts. Schleiermacher develops several canons of interpretation. One of his canons [19, page 12] is:

Before the art of hermeneutics can be practiced, the interpreter must put himself both objectively and subjectively in the position of the author.

Here “objective” aspects refer to the author’s language and “subjective” refers to inner and outer aspects of the author’s life. Schleiermacher emphasizes the *original intent of the author*, and states:

a more precise determination of any point in a given text must be decided on the basis of the language common to the author and his original public.

Schleiermacher proposes another principle of hermeneutics which deals with the relationship between the *whole* and the *parts* of the text. This work was the inspiration for Heidegger's *hermeneutic circle* (in jurisprudence it is also called *totality of the meaning*), a frequently referenced model that claims one's understanding of individual parts of a text is based on their understanding of the whole text, while the understanding of the whole of the text is dependent on the understanding of each individual part.

In a **narrow sense**, this is a simple relationship between two sets, for example we think of a set say  $A$ , consisting of al-Khwārizmī's complete works, and if the set  $B$  denotes one of his works, say his text *Al-Kitab al-Jabr wa'l Mukquabala*, then trivially  $B$  is contained in  $A$ . However the point to recognize is that understanding *Al-Kitab al-Jabr wa'l Mukquabala* (The Compendious Book on Calculation by Completion and Balancing), is not independent of understanding *Kuyosh soati hakida Kitob* (Book on Sundials).

In a **wider sense** the hermeneutic circle also points to the relationship between an author's work as a whole and his era, including intellectual, and historical circumstances. Thus one needs to understand the works of the scholars at the *House of Wisdom* to understand al-Khwārizmī's intellectual creativity. I will say more about the House of Wisdom in Section 2, but it is possible that few of the books from it would have survived war, ransacking, and general destruction. In any case translations and copies of some of the books have survived. For example in [16], it is claimed that there exist in Hagia Sophia (Istanbul) treatises plausibly attributed to al-Khwārizmī which give solutions of various standard problems of spherical astronomy.

There are several works focused on hermeneutics and mathematics. For example, J. M. Salanskis, in [18], studied the characteristic procedure of geometric reasoning which has been called "analysis". He claimed that the procedure of analysis in the classical, technical sense of the term that it has acquired since the Greeks, is closely related to hermeneutics. Salanskis also argued that analysis is the regressive method of all transcendental inquiry and he tries to determine the essence of contemporary mathematical analysis. If one looks at C. B. Crowley [2], one will again come across the concept of "economy of thought": the methods of establishing unity among various mathematical sciences. This topic has also attracted the attention of influential thinkers including Hilbert, Tarski, Bourbaki, and Mac Lane.

## 2. Al-Khwārizmī and the House of Wisdom



Al-Khwārizmī and the House of Wisdom

Abu Jafar Muhammad ibn-Musa al-Khwārizmī was born in 783 in Khiva which was a major city in a region now called Khorezm. He received his initial education in madrasas of Khiva, Kiat, and Gurgench (Urgench), all major cultural centers of the time. He studied mathematics, astronomy, and geography. He was able to read and study books written in Arabic, Persian, Syrian, and Sanskrit. Soon he established himself as a leading scholar of Khorezm and his fame spread through the East. During the reign of the al-Ma'mun as a governor for the eastern lands of the caliphate, between the years 813 and 833, al-Khwārizmī worked as a scholar in the *Bayt al-Hikma* or what is known in the west as the *House of Wisdom* in Baghdad. He was the head of the Bayt al-Hikma starting from the year 829 until his death.

The House of Wisdom was a research library together with an observatory founded by al-Mamun, who had a dream of translating all the Greek texts that could be found. Controversy remains as to its size, (some estimate its holdings around 400,000 books) and its scope and location. For more about the House of Wisdom, readers can check out [6, 8, 9].

The Arabs did not have a good relationship with the Byzantine Empire; nonetheless they negotiated the acquisition of texts through a series of treaties, and at the House of Wisdom, complete versions of Euclid's *Elements* and Ptolemy's *Almagest* were translated. This process of copying and translating has preserved many Greek classics which otherwise would have been lost. The observatory became a meeting place of Indian, Babylonian,



Hellenistic, and probably Chinese astronomical traditions [5]. Al-Khwārizmī collaborated with many great scientists and mathematicians from the region, such as the Banū Mūsā brothers, Al-Farghani (Alfraganus), and others [17]. One of the projects al-Khwārizmī and others worked on there was an estimate of the earth's circumference. Using their estimate they concluded that the shape of the earth is spherical [11]. Thus, in the early ninth century in the world of Islamic science, the spherical shape of the earth was accepted.

The word 'algebra' is derived from the title of al-Khwārizmī's book called *Al-Kitab al-Jabr wa'l Mukqabala* (The Compendious Book on Calculation by Completion and Balancing). This was a book on the "science of equations" [21] and discusses linear and quadratic equations. What al-Khwārizmī did for algebra with this book could be compared to what Euclid did for geometry with his *Elements*. These two works remained the best elementary treatments of their respective subjects until modern times.

In the opening part of his book, al-Khwārizmī considers six types of algebraic equations. These are, in modern notation:

$$ax^2 = bx, \quad ax^2 = c, \quad bx = c, \\ ax^2 + bx = c, \quad ax^2 + c = bx, \quad \text{and} \quad bx + c = ax^2$$

Like Diophantus, al-Khwārizmī considered only whole numbers in equations, but unlike Diophantus, he insisted on positive numbers as solutions whereas Diophantus allowed negative numbers. However, al-Khwārizmī went beyond merely providing an algebraic recipe for solutions of these equations, adding the Euclidean style of geometric proofs to algebraic facts. Euclid's propositions were entirely geometric, and al-Khwārizmī was the first one to apply them to quadratic equations.

If one wants to claim that al-Khwārizmī was the "father" of algebra, then one has to acknowledge several grandfathers, such as Diophantus and Euclid. There could be others. As Parshall writes in [12]:

... because his treatment of practical geometry so closely followed that of Hebrew text, Mishnat ha Middot, which dated around 150 AD, the evidence of semitic ancestry exists.

Some historians think that al-Khwārizmī would not have been aware of Diophantus or his *Arithmetica*, for the first Arabic translation of this book

was not made until several decades after al-Khwārizmī wrote *al-Jabr*. However, an early translation of the *Elements* by al-Hajjāj ibn Yusuf was available to al-Khwārizmī. For more on the sources of al-Khwārizmī's algebra we refer the reader to [3]. Even though it is of interest to understand how much the mathematical legacy al-Khwārizmī inherited from Euclid or Diophantus, it is clear that Diophantus was more interested in the theory of numbers and Euclid's interest was geometry, whereas al-Khwārizmī concentrated on algebra for the very first time as a separate area from either geometry or arithmetic.

What sets al-Khwārizmī apart from all mathematicians before him is that instead of solving particular equations, he gives the rules for solutions step by step for a general quadratic equation, thus introducing the algorithm. This was explained nicely in [6] where following quote from Ian Stewart is cited:

It is the difference between on the one hand supplying lots of specific examples and leaving the reader to conclude that the same procedure works on similar ones and, on the other hand, explaining the procedure in general terms in its own right. So, when al-Khwārizmī says 'māl (meaning  $x^2$ ), he does not refer to the specific square like 16, say. He means the square of his unknown, his *shay*', which does not represent any number in particular at all. He may use specific numbers that illustrate the method later, but the method itself is conceived as a general procedure.

Today we think of algebra in terms of symbols. Al-Khwārizmī did not have them. In his context then, to completely classify quadratic equations and describe an algorithm for solving them is immensely harder. When one describes an equation in words, it is much harder to see how many roots it has and why the roots depend on coefficients. The book by K. Parshall and V. Katz, *Taming the Unknown* [13], has a section on al-Khwārizmī explaining how he transforms his verbal algebraic results to geometric interpretations.

al-Khwārizmī's other notable work is devoted to Arithmetic, and is called *Al-Hisop Hind* (Indian Arithmetic), which also played an important role in developing mathematical sciences both in the East and West. For a long time *The Big Siddhanta*, an Indian book on astronomy, was used as a manual in the East. Al-Ma'mun, realizing that it contained mistakes and was not easy to understand, asked al-Khwārizmī to revise it. He not only produced the famous *Sindhind*, but also another book, *Indian Arithmetics*. Some of al-Khwārizmī's work include:



- Zidj al- Khorezmi (Sinhind), (Astronomical tables of al- Khorezmi),
- Kitob Surat al-arz (The Picture of the Earth),
- Kitab at-Tarija (A History book),
- Kuyosh soati hakida Kitob (Book on Sundials),
- Kitab ar-Rhuma (A Book of Sun Watch),
- Kitab al-Amal Asturlab (A Book of Working with Astrolabe).

For the present day researcher in mathematics this list of books might seem eclectic, but this was the way scholars researched then. They studied mathematics along with astronomy even with astrology. This was a common practice of the learned man.

### 3. Al-Khwārizmī and Hermeneutics

Following Schleiermacher's canons of hermeneutics, we can ask what was the original intent of al-Khwārizmī in writing his book *al-Jabr*?



The Compendious Book on Calculation by Completion and Balancing

In the very first page of this book there is a dedication to al-Ma'mun where he states [14, 15]:

That fondness for science, by which God has distinguished the Imam al-Ma'mun, the Commander of the Faithful . . . has encouraged me to compose a short work on calculating by (the rules of) completion and reduction.

Clearly al-Khwārizmī was a religious man and his patron al-Ma'mun had some practical concerns. Figuring out complex laws of inheritance, and trade relations motivated the calculation of proportions and made algebra, like geometry, a tool worth developing. The fact that his book contains solutions to practical problems of inheritance and obligations is in line with Islamic traditions. The prophet Mohammed himself was a merchant actively involved in arbitrating conflict concerning inheritances, and migrated from Mecca to Medina as an impartial arbitrator (Al-amin:= Trustworthy, Al-sadiq:= Truthful) and helped to write the Medina constitution. This aspect of al-Khwārizmī's work is in line with the “totality of the meaning” in the wider sense. He was concerned with the intellectual influences of the time and motivated by practical problems common to the Islamic tradition of the time. Although the tension between “applied” versus “pure” did not exist then, one can call al-Khwārizmī as an applied algebraist as well.

If we ask the same question about “original intent” about his second notable book on Indian Arithmetic, al-Khwārizmī begins this book with the words [14]:

Let's praise the merciful and kind Allah with beyond praises, let's express our nobility, praise to the skies, let's beg alms him lead us to the justice and right way, when I saw the Indians to make any number by placing and composing 9 letters, if the Allah blesses, in order to make easy for the learners I decided to show what can be composed from those 9 letters. If the Indians want just this case and the meanings of the 9 letters are the same in their understanding and in my comprehension, let the Allah direct me to it. But if they make in another way, in this case they're sure to find the right way easily in my composition and it will be visible clearly for both the learner and the teacher.

Below he introduces the natural numbers with a certain depth of abstraction and understanding for the very first time. In his words [15]:

When I consider what people generally want in calculating, I found that it always is a number, I also observed that every number is composed of units, and that any number may be divided into units. Moreover, I found that every number which may be expressed from one to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled just as before the units were: thus arise twenty, thirty, etc. until a hundred: then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand;....so forth to the utmost limit of numeration.

Here we see his observation on Indian mathematics with the idea of constructing any number with the help of numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 . During the 8<sup>th</sup> and 9<sup>th</sup> centuries various systems of addition and multiplication were in use. For example Romans' 12, 20, 60 numeration system was not practical; there was a definite need for the manual *Small Siddhanta*. Al-Khwārizmī explained how to add, subtract, divide, and multiply with this system and added 0 to the decimal numeration system in order to complete it. Here we see his creative side again, even though there were practical reasons, he was the one who took the knowledge one-step further. This manuscript became the rarest manual for the scholars up to the thirteenth century. It opened new horizons in mathematics both in Europe and in the East. In the twelfth century it was translated to Latin. Another translation of this manuscript, completed in the fourteenth century, is kept in a library at Cambridge University.

According to Schleiermacher, the objective aspect of the interpretation is the *language*. The language al-Khwārizmī wrote was Arabic. However, it is not clear it is even correct to use the terms “Arabic Science” or “Arabic Scholars”. Since the science was not cultivated by the Arabs only. Most “Arabic Scholars ” were Persians, Tadjiks and other Turcic people, Egyptians, Jews and Moors. As for the the scholars at the House of Wisdom, according to Yu Ruška, [17] the majority of scientists were from Khorezm, Ferghana, Shash and Horasan. It is been claimed that al-Khwārizmī was an Iranian Zoroastrian who converted to the muslim religion. None of this matters: he wrote in Arabic and “Arabic” was the language of the science of the Islamic world as “Greek” was for the Eastern and as “Latin” was for the Western Christian world. Al-Khwarzmi also invented “terms” in mathematical language: his mathematics is done entirely in words with no symbols

being used. For example he used *unit* for a number, a *root* was  $x$  and a *square* was  $x^2$ . Two operations of *al-Jabr* and *al-muqabala* means completion and balancing respectively. For example “al-jabr” transforms  $x^2 = 10x - 3x^2$  to  $4x^2 = 10x$  and “al-muqabala” reduces  $5 + 2x + x^2 = 2 + 9x$  to  $3 + x^2 = 7x$  (in his terminology one application to deal with numbers and a second one to deal with the roots). Just like *Hermes*, al-Khwārizmī was inventing language to communicate with others. Al-Khwārizmī’s algebra and arithmetic was used in the discovery of various mathematical sciences, thus serving as a *method of establishing unity among various mathematical sciences*.

His intellectual circle was in the House of Wisdom, and his impact was mixed with the impact of the other scientists that came from the traditions he helped to develop. Even though the collection of books at the House of Wisdom was lost, at the same time, some of the great scholars of the day were not only spared, but set up in research facilities by the conquerors. This includes the astronomers who moved to what is today western Iran, where a new observatory was created for their use. In the East, Omar Khayyam following the contribution of al-Khwārizmī, and using different methods, gave the general solutions for cubic equations. It is highly possible that tens of thousands of extant scientific manuscripts in the main Islamic languages – Arabic, Persian, Urdu, and Turkish – have never been examined by modern scholars. It could be the case that we simply don’t yet know the full scope of Muslim Eastern science. A good source for the reader in English, containing three surveys on the history of Islamic mathematics is in [20].



Two Circles

One of the scholars of al-Khwārizmī, R. Rashed, claims that modern interpretations are somewhat “anachronistic” representing the material in terms of things other than its historical time [14]. This very point is criticized by J. Oaks in [10]. J. Oaks claims that, Rashed applies modern algebraic symbols to both medieval algebra and medieval arithmetic. He also claims:

... symbolic algebra serves as the foundation upon which he [Rashed] interprets indeterminate problems using terms of modern algebraic geometry.

This points to Heidegger’s understanding of the hermeneutic circle, where the interpreter encircles the material with his or her subjective understanding. The hermeneutic circle and other interpretive canons provide very valuable and useful information about the intellectual climate of the author and a consistency check between the parts or whole of his/her work. Ironically, the principle of the hermeneutic circle has become a critique to formalism that every interpretation involves understanding and understanding, as a whole is a subjective phenomenon. Every one’s understanding is shaped by his/her own historicity, prejudices, preconceptions, and any interpretation starting from this subjective foundation is doomed to be subjective in general. In this connection the “hermeneutic circle” came to be regarded as simply enclosing the interpretable object within the interpreter’s preexisting belief system. Thus, there can be no objectivity whatsoever because every beginning is already an interpretation. Perhaps this interpretation of the hermeneutic circle explains the failure of understanding al-Khwārizmī’s impact on mathematics both in the East and the West.

*Photo credits.*

Photographs of Ulugh Beg and al-Khwārizmī were taken by me when I was visiting Uzbekistan. Photographs of the book al-Jabr and the House of Wisdom are from Wikipedia. Photograph of the book depicting two circles is from the library of Süleymaniye mosque in Istanbul.

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