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Inquiry Based Learning from the Learner's Point of View: A Teacher Candidate's Success Story

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Abstract

The goal of this paper is to review current research on Inquiry Based Learning (IBL) and shed some light, from a student's perspective, on the challenges and rewards of this pedagogy. The first part of the article provides an extensive review of the literature on IBL. The second part focuses on one student's experiences in an IBL classroom.

In particular, a graduate secondary mathematics student reflects upon his experiences in a college mathematics class where the instructor implemented an Inquiry Based Learning model. His experience is validated by current research on IBL educational methodology which structures the classroom environment for student centered learning. This student reported increased critical thinking skills, higher levels of motivation and engagement, and more retention of content than in other mathematics courses taken. Furthermore, this student believes that he was more invested in his learning, experienced improved conceptual understanding of content, and had higher levels of confidence in mathematics as a result of the IBL experience. As a preservice teacher candidate, he hopes to integrate IBL practices into his future teaching career to improve student learning.

1. Introduction

Inquiry-based learning (IBL) has been employed extensively in science for many years. It is widely accepted that learning science means investigating and exploring when developing a hypothesis. However, such has not always been the case in mathematics [32]. Students have often been encouraged to get "the right answer" and practice fundamentals over creation, investigation, and discovery. Teachers of mathematics promise students they will use the skills someday [34]. Few teachers use modeling or real world situations to make mathematics relevant, even though such approaches can positively impact student motivation [25, 32, 34].

Students need to take responsibility for their learning. They should construct meaning that is "powerful, memorable and effective" [25, page 4]. When students are invested in their education, they are more engaged [25, 32]. Even group work, which should be exciting and challenging, is often structured to solve problems that many students can solve on their own, and so ends up leading to boredom. According to Blair, "Inquiry is compatible with the learning of mathematics, leads to higher levels of motivation and results in a deeper understanding of concepts and of how to learn those concepts" [3, page 32].

In this article we first review studies which explore the benefits of IBL over traditional instruction methods. Then we aim to capture IBL challenges and benefits by documenting one student's journey in an advanced, collegiate mathematics course delivered with IBL techniques. Our unique qualitative approach seeks to capture a richer insight than survey data collection methods which dominate existing research.

2. Definition of IBL

IBL is an instructional practice where students explore content by posing, investigating, and answering questions. Students are at the center of the learning experience and take ownership of their own learning [4]. They often work independently and in small collaborative groups.

As Mahavier *et al.* state, in an IBL classroom, "the instructor plays the role of coach, mentor, collaborator, guide, and occasional cheerleader" [20, page 1]. More specifically, the teacher's role in IBL is to guide students and promote thinking and curiosity [32]. This takes purposeful planning to manage multiple student investigations simultaneously. Teachers monitor the progress of each student and provide immediate feedback [12, 17]. IBL does not indicate less guidance from the teacher, but rather delivers instruction in such a way that the student constructs their own meaning [25]. The teacher serves as the facilitator who plans, instigates, and observes the student learning process.

Currently, there are many definitions of IBL and a variety of approaches. The Academy of Inquiry-Based Learning states that IBL engages students and requires them to: solve problems, conjecture, experiment, explore, create, and communicate [1].

3. Theoretical Framework

Marshall and Horton use the four E's to describe IBL: Engage, Explore, Explain, and Extend. The learner needs to be "perturbed" and challenged to think critically and analytically [21, page 92]. In a similar vein, for the Center for Inspired Teaching [4], the "Wonder Experiment Learning Cycle" represents the student learning process, see Figure 1 on the next page. In the figure, at the center of the cycle, students achieve with the 4 i's: intellect, inquiry, imagination, and integrity. The outer cycle includes five "core elements of inspired teaching": 1) mutual respect, 2) student as expert, 3) purpose, persistence, and action, 4) joy, and 5) evidence of learning.

Constructivists believe that individuals create their own meaning and knowledge of the world through experiences and reflecting on said experiences [8]. Social constructivists further this idea that learning occurs through our interaction with others. Vygotsky's [29] zone of proximal development is described as, " . . . learning awakens a variety of internal developmental processes that are able to operate only when a child is interacting with people in his environment and in cooperation with his peers" [29, page 90]. Learning is an active process, not a passive one.

IBL in mathematics is often associated with "the Moore Method," which was developed by Robert Lee Moore during the early 1940s. Moore expected his students to discover mathematics on their own presenting their work to their peers in class [20, 24]. Moore believed that "[t]he student that is taught the best is told the least" [24, page 390]. Over eighty years after Moore, his

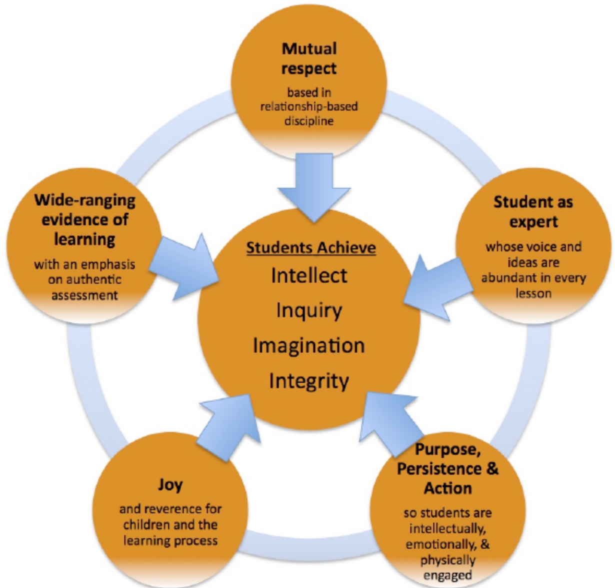

Figure 1: *5 Core Elements of Inspired Teaching*, Center for Inspired Teaching. Image available at http://inspiredteaching.org.s136195.gridserver.com/about/our-model/5-core-elements, accessed on June 9, 2017.

method is still not widely known, adapted, or adopted.[1]

The Moore Method includes several crucial elements that lead to student inquiry, such as the student struggling on their own and presenting their findings to their peers. However, the peer interaction is important in IBL and this was missing in Moore's practice. Modern IBL researchers Laursen *et al.* [17] describe the "twin pillars" of IBL to include students' deep mental

[1]Moore's association with racism and other discriminatory ideals and practices probably accounts for at least some of the reluctance from the mainstream mathematics community. Today many IBL practitioners within mathematics recognize that it is important to acknowledge this past: "It is time that we in the community of mathematics educators who teach with inquiry similarly acknowledge that the legacy of RL Moore is a symbol that includes racism and misogyny — even if every member of this community comes to it because they find value in teaching with inquiry and proactively rejects the bigoted elements of this symbol" [14]. We must therefore emphasize that even though the Moore Method was probably the first step to IBL in mathematics instruction at the college level, today the community of IBL practitioners does not see Moore as a representative of their values and ideals.

engagement with challenging mathematical tasks and peer-to-peer collaboration whether whole class or in small group. Through their research, they realized that both pillars are necessary since students saw both engagement and collaboration as integral to their learning. "When students engaged deeply with the mathematics on their own, they came to class well-equipped to contribute meaningfully to group work. In communicating their ideas and listening to others, they deepened and crystallized their own knowledge" [17, page 137].

We synthesized the above frameworks by using some aspect of each of them. Our consolidated framework defines IBL as an instructional practice that actively engages students collaboratively through challenging tasks whereby students take ownership of their learning while respecting one another in intellectual exploration. Further, students develop conceptual meaning through verbal communication, both written and oral, with their peers. As they reflect upon their learning, new deeper understanding is achieved and increased self-efficacy is possible.

4. Literature Review

Research has shown that students who participate more in classroom discussions obtain higher scores on standardized testing. Talking more means higher scores, in general, regardless of race, poverty, and/or gender differences. In fact, traditional gaps of subgroups is significantly reduced when students are required to participate in active learning exercises [5]. Cohen and Lotan [5] suggest that teachers must be explicit that everyone participates and everyone assists each other. The quality of the discussion predicts final performance on individual and group assignments [6]. When students share ideas and self-assess individually and as a group, the product is enhanced [5].

A number of studies seek to compare IBL and non-IBL (e.g., traditional lecture) classrooms [11, 16, 17, 18, 35]. Laursen *et al.* [17] prepared a report for the Educational Advancement Foundation and the IBL Mathematics Centers Assessment & Evaluation Center for Inquiry-Based Learning in Mathematics in which four university mathematics departments sponsoring IBL Mathematics Centers were studied during 2008. Classroom observations, surveys, and student interviews were conducted to gather data. Laursen *et* *al.* [17] reported that undergraduate students that take IBL-taught mathematics courses enroll in more future mathematics courses than students in traditional mathematics classes, also see [11, 16, 18]. Further the study reports that, "IBL students reported higher gains in understanding concepts, mathematical thinking, confidence in doing and communicating about mathematics, persistence, and positive attitude about mathematics learning" ([11, 3-74] and [17, page 49]). This study also revealed that women, in particular, benefited the most from IBL methods [11, 3-75], as compared to all other groups (non-IBL men, IBL men, and non-IBL women) on all cognitive and affective domains (see Figure 2).

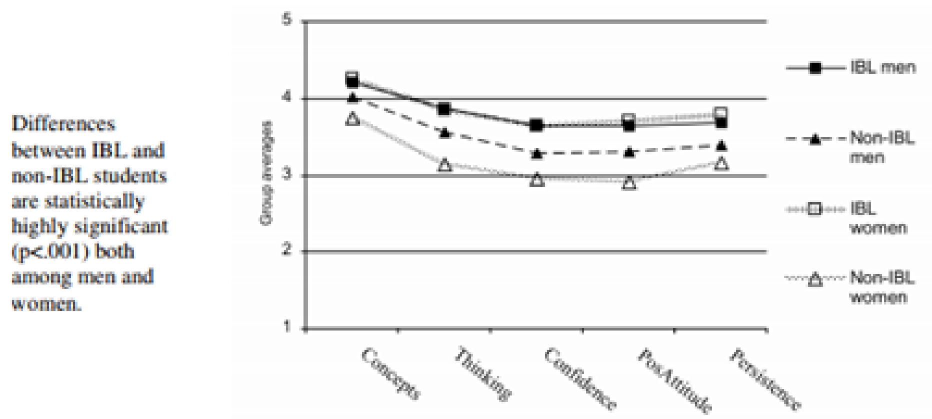

Figure 2: Differences between IBL and non-IBL men and women. Image from [11], used with authors' permission. Also see [18].

Furthermore, low achieving students earn higher average grades in future mathematics courses when compared to their non-IBL peers. This indicates that IBL instruction may have lasting effects. These findings could have important implications for the way undergraduate mathematics courses are structured and taught.

In the same study, Kogan and Laursen stated, "On average, about 60% of class time in IBL courses was spent on student-centered activities such as small group work, student presentation of problems at the board, or whole-class discussion, while in non-IBL courses over 85% of class time consisted of the instructor talking" ([16, page 185], also see [17, page 163]). Both male and female IBL students pursued further IBL courses at higher rates.

IBL women took, on average, a full elective course beyond their non-IBL female peers. Medium and high achievers earned subsequent grades similar to those of their non-IBL peers, but low achievers from IBL sections earned consistently higher grades than their low-achieving peers in non-IBL sections. Lower achieving students' grades improved in later courses.

Another study found that IBL students succeeded at least as well as their peers in later courses, challenging concerns that the slower pace of IBL, through omitted material or other structural differences may hinder student success in the future [35]. Blair [3] reported that teachers use IBL less often due to lack of time with the pressure of standardized exams. Some teachers also believe that students do not have the skills necessary to do independent work. These individuals feel that students need clear guidelines and instructions to stay on task and behave [28].

Freeman *et al.* [10] reported that active learning yielded increases in examination performance over traditional lecturing. Additionally, lecture formats yielded 55% higher failure rates. These increases in achievement held across all of the STEM disciplines and occurred in all class sizes, course types, and course levels. Active learning was found to be particularly beneficial in small classes and yielded increased performance on concept inventories.

English Language Learners (ELLs) exhibited higher proficiency on standardized tests in science, mathematics, English language, and reading the longer they were enrolled in IBL science classes [2]. One reason is that students are able to construct context with hands on activities. They also developed a positive attitude toward learning while engaging in classroom discourse. Group work provides students, particularly ELLs, the opportunity to hear the directions multiple times, ask questions, practice what to say, and communicate, at times, in their native language thus gaining access to the task and meaningful discourse [5].

According to the Mid-Atlantic Equity Center [23], minority students, for example, perform best in mathematics classes when the content is related to their previous experience. Activity-based programs have been demonstrated to significantly improve minority student performance in mathematics. In Virginia, at Hollin Meadows School, the Standards of Learning (SOL) test scores improved, the achievement gap diminished, and qualitative assessments showed that students were developing identity and confidence in their educational achievements. Teachers used an interdisciplinary approach that was inquiry-based. Students outscored division and state performance in mathematics on the 2010 Virginia SOL tests (State 87%, our division 84%, Hollin Meadows 95%). The passing rate for all students in mathematics increased from 81% in 2007 to 95% in 2010. Of particular note was that black students' scores increased from 76% to 88%; Hispanic students' scores increased from 72% to 96%; white students' scores increased from 93% to 100%). In 2010, there was no gap between low-income students' scores and scores of other students [25].

Kahle *et al.* [13] offer further support. Their study showed improvement of test scores among black middle school students in classes where teachers received inquiry teacher training versus students in traditional settings. They further determined that the achievement gap narrowed between male and female students participating in IBL. Teachers also reported that students were more interested in the content.

Li *et al.* found higher scores and understanding of material in IBL groups [19, page 11]. Qualitative evidence in responses of teachers and students supported students overall higher degree of understanding and favorable attitudes for IBL [19, pages 12-15].

Based on prior research of science courses using concept inventories, Epstein [9] conducted research in college calculus classes that implemented traditional instruction versus Interactive Engagement (IE) using a concept inventory. The lowest gain score of any group of students at the University of Michigan was 0.21 in IE classes compared to traditional instructional methods.[2] Most strikingly, the highest gain score in the traditional sections was equal to the lowest gain score in the IE sections.

Despite such studies that promote IBL over traditional instructional approaches, there are also critics of IBL. They claim that IBL has minimal

[2] "The class performance is measured by the *normalized gain*, defined to be

$$\langle g \rangle = \frac{\mu_f - \mu_0}{100 - \mu_0},$$

where μ_0 is the mean score of the class (not of individuals, hence the angle brackets) at the start and μ_f is the mean score at the end (in percent correct). This measures the gain in the class's performance on the [concept inventory] as a fraction of the maximum possible gain." (p.1019-1020)" [9, pages 1019–1020].

teacher guidance during instruction. "The past half-century of empirical research on this issue has provided overwhelming and unambiguous evidence that minimal guidance during instruction is significantly less effective and efficient than guidance specifically designed to support the cognitive processing necessary for learning" [15, page 76]. Prior knowledge is applied to new problems. The more experience and expertise teachers have, the more the students will be able to solve new problems. Such critics write:

> The goal of instruction is rarely simply to search for or discover information. The goal is to give learners specific guidance about how to cognitively manipulate information in ways that are consistent with a learning goal and to store the result in long-term memory. [15, page 77]

Their research shows that students learn better when guided [15]. Kirschner *et al.* [15], clearly not proponents of IBL, state that tests performed over the decades seeking data on IBL students show that overall effectiveness is less than students who were taught in traditional ways. Post-testing scores are lower for IBL.They further purport that with more than fifty years of investigation, there is no body of research supporting the minimal instruction technique. They argue that research almost uniformly supports direct, strong instructional guidance rather than constructivist based minimal guidance during the instruction of novice to intermediate learners [15].

More recent research seems to imply consistently that student performance benefits from the IBL approach. However given contradicting work such as [15], one might consider other tangible outcomes besides performance. Within the past ten years, a growing body of research has suggested that IBL has numerous benefits to students in addition to improved performance. In particular, recent research has emphasized attitudinal or behavioral changes in the students as a result of taking an IBL course. Davidson [7] found that students reported greater confidence in their mathematical abilities. They also reported a preference for in-class activity as opposed to lecture. In Davidson's study, the professor noticed that the students were more engaged and energized. Matthews and Hodge [22] also studied IBL implementation in a history of mathematics college course for pre-service teachers and found that students reported higher levels of confidence. Additionally, students were inspired by the experience.

In a deviation from much of the existing research, these studies address qualitative student affects stemming from IBL techniques. In an attempt to augment research on student benefits, Caroline, a mathematics education researcher, approached Derek, a graduate pre-service educator, about his first experience with IBL in a collegiate, linear algebra course. The rest of the article reports on the collaboration that resulted.

In what follows, we specifically emphasize the student experience. Students have strong feelings about their instructional experiences and we believe their voices need to be a part of our investigations. There have been a number of studies on the efficacy of teachers, but few on the efficacy of students. In the following we work with the principle that if we ask them, they will tell us what they learn and how they learn best. It is our conviction that asking students to tell their stories has power and meaning, that test scores and assessments do not tell the whole story, and that a students journey is of value to the learning process.

5. The Study

The co-authors in this study are advisee (Derek) and advisor (Caroline) in the Mathematics and Computer Science Department. Derek is a graduate student earning an MAT in Mathematics while obtaining his secondary teaching licensure. Caroline is a faculty member who teaches practicum and student teaching courses for secondary mathematics teacher candidates. During an advising session, the co-authors decided to examine the implementation of inquiry-based instruction in an upper level mathematics course, linear algebra, from his perspective.

In what follows, Derek writes about his experiences throughout the semester (Fall 2015) and reflects a year later (July–August 2016) on his overall impressions. To supplement the student perceptions, the Linear Algebra course instructor was interviewed and student feedback forms were used to validate the student's findings.

The class met four (4) hours per week for fourteen weeks. There were two (2) exams, five (5) problem sets, weekly in class problems and homework culminating to a final project with presentation.[3] The problem sets were graded, required proof writing, and a deep understanding of the material that was being covered in class. The course was held at Rhode Island College in Providence, RI.

The student, Derek, was enrolled in the course with six other matriculating students. The majority of the students were Mathematics majors. Two were Secondary Mathematics majors; of which both were second bachelor's degree candidates, which means they had both earned a prior undergraduate degree, not in mathematics.

6. A Student's Reflection of IBL

Throughout his studies, Derek had noticed a dichotomy between the education courses and the mathematics courses he was taking. In general, his mathematics classes were all predictable in their pedagogical processes. Each class was teacher centered and students were expected to write notes, by copying what the teacher dictated and recorded on the board. Education courses meanwhile employed ideas about techniques such as "Understanding by Design" [33] and IBL.

Education instructors expected students to write lesson plans that encouraged their pupils to think critically and work cooperatively to develop solutions that are meaningful. He liked the academic and professional support from his instructors. From his educational experience, now pursuing a master's in teaching mathematics, the education department seemed to be on the right track. Yet he observed that the pedagogy he was learning via the education department was not realized in his mathematics courses. He did not see these best practices being implemented to increase students' learning. He was constantly wondering why he was not seeing these more innovative IBL lessons in his mathematics courses. Why this dichotomy?

It was during his fifth semester that he was scheduled to take linear algebra. On the first day, the professor announced to the class that they would be learning the course content through IBL methods. The instructor explained that everyone would be learning the material for the most part on their

[3] L. Pinheiro, course syllabus, 2015.

own and would be responsible for preparing material for class discussions on a given topic each day. At first, Derek was really apprehensive because he knew that a higher level mathematics course would be rigorous. Even though he had always been a top performing student in the past, he did not feel confident that he could excel in an IBL classroom in this particular course. Derek felt like he was going to fail without the professor's direct guidance and support. In other words, even though he knew of the value of the particular pedagogy from prior experience, Derek himself noted that he was hesitant and had qualms about its practice in a mathematics class.

Higher level mathematics courses provide a challenge for any student. As mentioned earlier, the class was small, less than 10 students. It met twice a week for two hours. From the very beginning, the students were expected to read the book and do the homework on their own without being shown any previous examples. As many would agree, reading a mathematics text book, especially one on linear algebra, is not an easy task.

Nonetheless Derek took on the challenge. He spent multiple hours preparing for each class. Most of the time he felt like he had no idea what he was doing. Unfortunately, he did not understand how to use the textbook to help him. Thankfully the expectations for accuracy on proof and calculations that were done for homework were extremely relaxed. Students were allowed to leave their thoughts and comments blank when the answers eluded them. At the beginning, he consistently left questions unanswered. Deciphering mathematical text and coming up with proofs for concepts he had never seen was quite challenging and often seemed impossible.

The cohort reviewed homework together and all the problems were discussed as a group. Encouraging students to work together not only deepens the "teaching" student's comprehension of the content, but affords the peer with an alternate explanation [7].

The professor discussed his approach as one that did not give the students solutions to problems. He would steer the students in the right direction if something was missing or if their logic was unsound. From the professor's perspective, "I would fill in the gaps."[4]

[4]L. Pinheiro, personal communication, 2016. See Appendix B.

Eventually the students began to ask each other for assistance instead of the professor. It was reassuring for Derek to know that other students were also experiencing difficulties. As the group went through the assigned problems, they were asked to mark any comments or corrections in colored pen so the professor could see the progression of thinking. The assignment was passed in at the end of class. Derek and other students (as could be seen from the student feedback forms from that semester) expressed value in this group approach to learning.

In one class, for example, the assignment required students to show that the zero vector, **0** of a vector space V is unique. Derek attempted to solve the question based on his prior knowledge of algebra, see Figure 3 below.

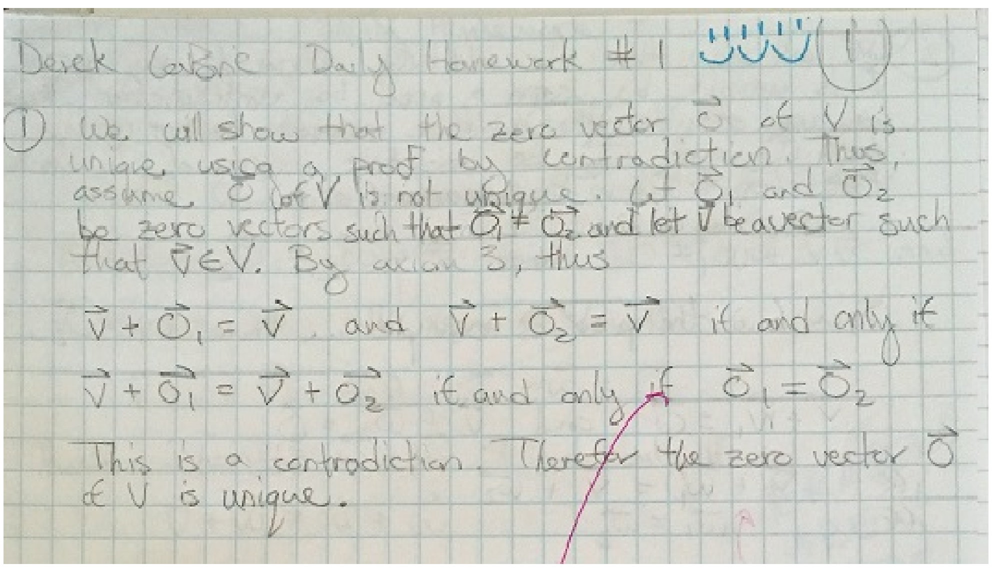

Figure 3: Derek's original attempt to prove the zero vector of a vector space V is unique.

When he presented his proof to the class, the professor said that more justification was necessary (shown by the arrow in the figure). A classmate then presented a solution which justified the proof. As required by the instructor, Derek wrote this down in pen, see Figure 4 on the next page.

Over the first couple of months, Derek went to the professor during office hours and met with classmates for whatever insights he could muster. He constantly felt defeated by the subject matter. But as time went on, he

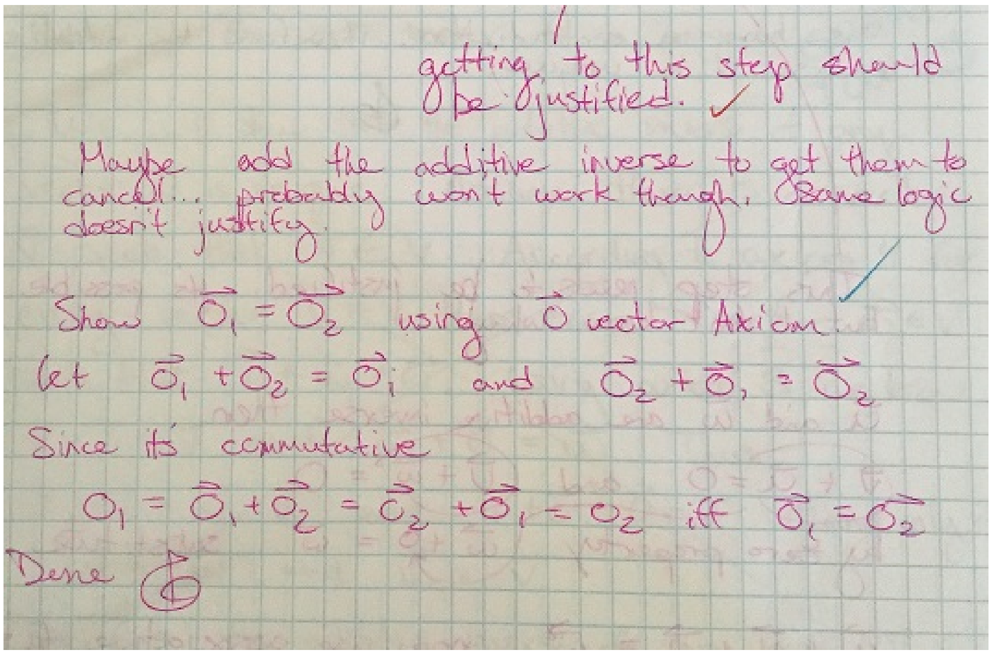

Figure 4: Derek's revision of the zero vector of V is unique proof.

noticed that his knowledge of the subject slowly improved. He recognized that he was building strategies for proof writing and problem solving and actually starting to understand the material.

It should be noted that Derek's extraverted nature facilitated the IBL process. He was not afraid to go to the board to present a proof that was unfinished. He also felt very comfortable engaging with other students even though he often had very little knowledge of the subject to offer when learning something new.

This content in this class was extremely challenging. Derek and the other students in the class (as seen in the student feedback forms from that semester) spent countless hours solving for every problem they were given. It was through this determination to find the answer that he noticed a positive change in his ability to think critically about new concepts. As he became better versed in the language of linear algebra, he found himself able to look at solutions from a variety of standpoints. Derek became a much more flexible thinker in mathematics.

Looking back a year later, Derek still remembers and understands the content. According to the instructor, the students may not have covered as much as a typical linear algebra class, but the students understanding went much deeper than a traditional mathematics course. This can be attributed to a student centered learning environment in which participants dictate the pace. Smaller content scope combined with a student managed pace enhanced retention of the material. In fact, Derek reported recall of content more than one year later for the majority of topics that were covered in class (i.e. vector spaces, linear transformations, determinants, linear independence and dependence, bases, span, eigenvectors, and kernel).

In contrast to a traditional lecture approach, the students in Derek's class spent considerably less time taking notes, memorizing theorems and algorithms, and taking exams. Derek's overall perception was that this experience was a strong impetus for him to implement IBL in his future classroom. Finally, experiencing it personally and participating in its structure within the mathematics classroom was an extremely beneficial experience.

7. Discussion

When students learn conceptually, a more significant emotional and intellectual engagement occurs. According to Epstein [9] most students leave introductory courses without a solid grasp of the basic [mathematical] concepts. Wathall [30] explains that conceptual understanding is more important than the memorizing facts and formulas. One way to approach conceptual learning is through IBL. Derek's experience with IBL was both emotionally and intellectually challenging. He felt that he learned the mathematics more deeply by first struggling with the content on his own and then with his classmates.

When developing inquiry tasks, the stimulus should be just above the current knowledge of the student, according to Vygotsky's Zone of Proximal Development construct [3, 26, 28]. The task is meant to stimulate interest and spark curiosity. It should evoke some level of frustration, but not to the point where the student gives up. In the context of an IBL classroom, it is common for students to be frustrated in the beginning.[5] Matthews and Hodge [22] reported students were challenged and felt out of their "comfort

[5] L. Pinheiro, personal communication, June 2, 2016. See Appendix B.

zone" [22, page 35] at first. Derek also reported feeling lost at first and extremely frustrated. Over time, his confidence increased so he could achieve success beyond what he believed he was capable of.

According to Blair, "[i]nquiry is not about independent discovery, but about a collaborative process in which the teacher introduces mathematically valid forms of reasoning at a point when they are necessary and relevant" [3, page 35]. Inquiry leads to higher levels of motivation and deeper levels of conceptual understanding [3, 30, 32]. Derek's own experience in working with his peers solidified his own conceptual understanding and strengthened his ability to write proofs. His professor infused key concepts as necessary to propel the group forward.

Some teachers believe that there is not enough class time to implement IBL practices; we do know, however, that research such as [3] disagrees. Other teachers assert that some students are not able to handle this type of rigor and will not be successful. The studies we referenced above refute that claim. Furthermore in our study, the professor found that the students were successful in spite of his reservations. As educators, we need to expect more of our students by providing them with challenging inquiry tasks, encouraging them to collaborate with classmates, and supporting them to achieve amazing results. Marshall and Horton [21] report that students who sit and listen to lectures and participate in rote learning are less likely to be involved in critical thinking activities. Their study showed higher-level thinking and learning takes place when traditional class time is substituted with student exploration activities in mathematics or science classes. Upon observing over a hundred middle-level classrooms, the researchers found students that were involved in IBL activities were required to "verify, justify, develop, and formulate" [21, page 99] during exploration activities deepening their understanding of conceptual mathematics involving higher levels of critical thinking. Furthermore, students typically enjoy the challenge. Derek reported enjoying the inquiry tasks while deepening his conceptual learning.

According to Scruggs and Mastropieri [27], when using IBL in place of a textbook approach in two science units, high school special needs students yielded significantly higher achievement than their peers who used a textbook. Ninety-six percent enjoyed using the IBL approach, while 80% of the students found IBL activities to be more helpful in the learning process and were more motivated. Pre- and post-tests revealed that when students were taught with IBL, they learned more and remembered more than their peers taught through traditional instruction [27]. This finding is consistent with Derek's own experience with IBL. Derek believes that the IBL instructional method is the reason he has retained the mathematical content for more than one year. As a future teacher, he wants to employ instructional strategies that yield long term benefits for students.

Derek learned many lessons while enrolled in an IBL course. Some of the lessons were about mathematics and some were about him. He learned to be persistent in working through challenging content. Another lesson learned was that he does not have to do it all alone. Solving challenging tasks may require support from peers. There are benefits to thinking and talking through problems together. In the end, his acknowledgement of owning his learning with long-term recall and increased self-efficacy was life changing.

Instructors should be aware that students need to be prepared and instructed to learn in new ways.[6] This takes time and patience on the part of the teacher. The instructor should realize that, at times, direct instruction may be useful, particularly when a group of students needs support on a particular concept; this was noted in the student feedback forms from the semester. A common behavior, however, is to relapse into old comfortable habits of direct instruction. Students may themselves welcome the traditional approach as it is familiar and predictable, but instructors should persevere.

Content is often "learned" for the test and then forgotten. If we want our students to become lifelong learners, then we need to give them challenging content where they learn to think for themselves. Students will then develop the ability to learn content with a greater sense of responsibility, autonomy, and confidence.

Having an IBL textbook is optimal for ensuring that the tasks assigned to students are of IBL scope.[7] Today the IBL community in mathematics, organized around the Special Interest Group of the Mathematical Association of America on IBL (IBL SIGMAA) and the *Journal of Inquiry-Based Learning in Mathematics*, is thriving. As a result, there are resources available to instructors at any level, and it is possible to use IBL for any mathematics course; see Appendix A, for instance, for a list of IBL Calculus texts.

[6] L. Pinheiro, personal communication, June 2, 2016. See Appendix B.
[7] L. Pinheiro, personal communication, June 2, 2016. See Appendix B.

8. Limitations

This paper incorporates one student's perspective of one specific college class with a small group of students. Because all of the students were mathematics or mathematics education majors, this is hardly a random sample. Additionally, at a class size of n=7, these observations are not valid for statistical inference. The class was comprised of a homogeneous group of students. Nor it is particularly generalizable for all students. However, to date, little work has been done on a student's reflection on his/her learning with the IBL instructional methodology. We hope that our study will encourage others to explore student experiences in similar ways in the future.

9. Conclusion

Current research documents the benefits of active, group learning over traditional teaching methods across multiple disciplines [32]. IBL is one method that employs active group learning [21]. IBL reduces the gap between subgroups of students (gender, race, socioeconomic status, learning needs, and language learners). IBL also improves student engagement, motivation, and self-confidence [17].

However, existing research seldom explores qualitative data on IBL techniques, ignoring a dimension of measuring its efficacy. This study examined key impacts of IBL through documenting a student's personal experience with a graduate course in linear algebra.

Derek's personal experience, which supports and augments existing research, indicated that benefits of IBL include greater content retention, confidence in mathematical reasoning, and increased group and personal responsibility for material. Additionally, Derek observed that his desire to engage in challenging tasks independently and with peers was increased. It is particularly interesting to note that Derek is a mathematics educator. The findings of the Laursen *et al.* study [17] did not support mathematics educators to have significant affective changes. Yet, Derek and the students in his class experienced greater feelings of efficacy. Perhaps this is due to the fact that mathematics educators take similar courses to Bachelor of Arts mathematics majors so their mathematics content exposure is comparable.

The significance of this study's findings is two-fold: 1) in the IBL context, the students took responsibility of their learning, which yielded long-term benefits in content retention, and 2) this teacher candidate is determined to integrate inquiry-based learning in his future classroom instruction.

Unfortunately, IBL is not yet widely applied in mathematics instruction. The existing literature in the field of IBL, coupled with the unique approach of our study to measuring the improved learning outcomes in the IBL setting, should encourage its broader application. Our results will hopefully be deemed worthy of consideration for mathematics faculty and others interested in the types of pedagogy where students take ownership of their own learning.

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A. Open Source Calculus Textbooks for IBL

Below is a list of some Calculus I open source textbooks available. All URLs made available were last accessed on June 9, 2017.

Boelkins, M. (2015) *Active Calculus: single variable*. American Institute of Mathematics. Available at http://scholarworks.gvsu.edu/books/10/ as well as at http://faculty.gvsu.edu/boelkinm/Home/Active_Calculus.html.

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B. Professor Interview - June 2, 2016

To supplement Derek's reflections that this study revolves around, Caroline (CJC) also decided to interview Leo Pinheiro (LP), the assistant professor of Mathematics who was teaching Derek's linear algebra course, about his first experience teaching using inquiry-based learning (IBL).

CJC: What made you decide to teach using IBL?

LP: I was selected as a fellow in the Project NEXT program [http://www.maa.org/programs/faculty-and-departments/project-next] through the Mathematics Association of America (MAA). The professional development at the conferences during that year focused on IBL. So I thought I would give it a try.

CJC: What is Project NEXT?

LP: It is a year long fellowship. There are three conferences during the year: summer, winter, and summer.

CJC: What was your initial reaction to implementing IBL?

LP: This is not going to work. I was teaching a linear algebra course in the fall of 2015 and did completely understand how the students would be able to write proofs. I did not teach them how to [write proofs].

CJC: How did you introduce it to the students?

LP: I asked them questions like:

- How does a person learn something new?
- What do you reasonably expect to remember from your courses in 20 years?
- What is the value of making mistakes in the learning process?
- How do we create a safe environment where risk taking is encouraged and productive failure is valued?

CJC: Were there any benefits to using IBL?

LP: It was an incentive for students to come to class. The teacher is no longer the focus. The students actually learn by doing.

CJC: What were you most valuable resources?

LP: The Academy of Inquiry Based Learning was very helpful. [Available at http://www.inquirybasedlearning.org/, last accessed on June 9, 2017.] I recommend Dana Ernst's book. [Available at https://zenodo.org/record/29875#.WTs5G5Lyvcc, also check out Ernst's blog at http://maamathedmatters.blogspot.com/2013/05/what-heck-is-ibl.html, both last accessed on June 9, 2017.]

CJC: What is different using IBL than the traditional direct instruction or lecture?

LP: The communication is two way now and my role is different. I fill in the [learning] gaps, you know, the fundamental things they would not have learned.

CJC: What surprised you most by using IBL?

LP: I had a family emergency one morning so I cancelled class. The next class, I found out that the class had met in my absence. They wanted to continue the work. At that moment, I realized that it is less about the professor teaching what they want. We are so used to hearing ourselves talk. Now, I find myself mediating class discussions, but I don't give them answers. The students are actively learning every day.

CJC: What concerned you the most when the course was completed?

LP: I wondered how they would fare in Abstract Algebra, the next required mathematics course.

CJC: Now that you have taught IBL for two semesters, what do you feel strongly about?

LP: It was extremely helpful having an IBL textbook in the second semester. It made such a difference.

CJC: Is there any course that you would not consider using IBL?

LP: I cannot imagine how it would work in Pre-calculus or Calculus I. [See Appendix A for open source IBL Calculus textbooks.]