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A Tour of the Calculus David Berlinski. Vintage Books, 1997.

At first reading this book comes across as a continuing commercial for the marvels of mathematics. The author is so eager to dazzle you with spectacular displays of wit and imagination that the stated goal—to develop mathematical ideas from the real numbers up through the fundamental theorem of the calculus—is often obscured rather than illuminated by the fireworks. Referring to the mathematician's inclination toward abstraction, he notes that, "This turning away from various particulars requires great discipline. Revisiting the facts, the mathematician must *resist* the tug of those rich, very voluptuous descriptions of reality that the novelist or the physician might favor..." (p. 63). Unfortunately the author didn't resist very hard, and he is aware of it; a few pages later he writes, "So much for purple prose, the stuff beginning to wear, even on me..." (p. 69).

On going through the book a second time, however, I could see that it may well be useful to the teacher of mathematics, supplying a flavor and richly colorful human dimension to mathematical developments. The author's deft use of language and frequent flights of metaphor give added insight into mathematical ideas, and extended digressions often contain biographies and rich, imaginative reconstruction of the lives, personalities and thinking of major mathematicians.

For example, leading up to the definition of the slope of the line tangent to a curve at a point, he remarks that, "The mathematician intent on donating the line's slope to a curve, and the skeptic bemused by the fact that lacking a slope the tangent line is mathematically undetermined, may be satisfied alike by a procedure that assigns a slope to the tangent line, the neutral idea of an assignment conveying, I think, the odd commingling of discovery and definition that is involved in any mathematical advance" (p. 181). Again, "In nature, some things are close (the lion and the tiger, cats both) and some things far apart (the tiger and the flatworm, different animals, different phyla even), the concept of *distance* one of the crucial, if generally hidden and obscure, instruments by which we assess the world and find our way within it" (p.18).

His soaring rhetoric and flair for the dramatic are illustrated in the following comment on Newton's towering work. "The *Principia* is the supreme expression in human thought of the mind's ability to hold the universe fixed as an object of contemplation: it is difficult to reconcile its monumental power with a number of humanly engaging but anecdotal accounts of its composition: the disheveled and halfdressed Newton, so the stories run, his crumb-filled wig askew, shambling about the evil-smelling room in which he lived and worked, muttering to himself, his thin lips half forming words, stiff with attention or slack and slumped indifferently on his unmade bed, entirely absorbed, forgetting to eat and sleeping in weak, disorganized fits, an apple rotting on the desk, the *Principia* taking shape in stages, vellum sheets piling up on the wooden desk" (p. 5).

Berlinski says that, "I have written this book for men and women who wish to understand the calculus as an achievement in human thought." I do not recommend it for someone who is not already well acquainted with the calculus. Such a person will not grasp its core ideas here. There are too few examples, the lengthy digressions mentioned above may be offputting for the novice, and some of the mathematical discussions are confusing, even misleading, The author seems at times to sacrifice accuracy for imagery. I doubt that the manuscript was reviewed prior to publication by another mathematician.

An example of notational confusion is offered by a proof of the Mean Value Theorem (p. 212). A function $y = f(t)$, $a \leq t \leq b$, is under consideration. An accompanying figure indicates clearly that b denotes the right-hand endpoint of a t-interval. It is then recalled (floridly) that, "straight lines submit to the discipline of an equation expressing their innermost nature: $y = mt$

+ b, where m is the line's slope and b its y-intercept..." Suffice it to say that the ensuing discussion employs b in both its meanings.

I was even more disturbed by an extended discussion (pp. 179-183) in which "slope" and "curvature" appeared to be used interchangeably. Was I being overly critical? Then comes this sentence: "Curvature is assessed at a point by reference to the slope of a line tangent to the curve at that point, the curve acquiring its slope at second hand, it is true, but acquiring nonetheless a slope and so a number embodying and then expressing its curvature."

The mathematician will find here amusing anecdotes, charmingly expressed, and rich metaphors, to enliven mathematical ideas. But if he or she wishes to assign readings from this book to students of calculus, my advice is to select such readings carefully.

Book Review: *Mathematical Reflections* by Peter Hilton, Derek Holton and Jean Pedersen

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Mathematical Reflections. Hilton, Peter; Holton, Derek; and Jean Pedersen. Springer, 1997.

I believe the following book is particularly suited to study by those who are or will be teachers of math and science: *Mathematical Reflections* by Peter Hilton, Derek Holton, and Jean Pedersen. After 8 chapters of delightful math, the short 9th chapter contains some very useful and thought provoking reflections on "how math should be done" and "principles of mathematical pedagogy." The following comes from a section on the special role of geometry in secondary and undergrad math.

"In fact, geometry and algebra, the two most important aspects of math at these levels, play essentially complementary roles. Geometry is a source of questions, algebra is a source of answers. Geometry provides ideas, inspiration, insight; algebra provides clarification and systematic solution."

"Thus it is particularly absurd to teach geometry and algebra in separate watertight compartments... Geometry without algebra leaves the student with questions without answers, and hence creates frustration; algebra without geometry provides the student with answers to questions nobody would ask, and hence creates boredom and disillusion. Together, however, they form the basis for a very rich curriculum, involving discrete and continuous math."

In arguing against the separation and restriction to synthetic geometry methods, they point out that completing geometric problems by Euclidean means almost always exploits some clever trick, an ingenious construction—so, roughly speaking, each problem "requires its own special idea. And none of us is bright enough to function, in any aspect of our lives, with such an enormous idea-to-problem ratio; we have to make a good idea go a long way. ... So remember: All Good Ideas in Maths Show Up in a Variety of Mathematical and Real-World Contexts."

Finally, I note that Peter Hilton (mathematician, SUNY Binghampton) has collaborated with Stephen Willoughby (former NCTM Pres., Prof. Univ. AZ), Carl Bereiter (cognitive psychologist), and Joseph Rubinstein (biologist) over a period of many years to develop the series of elementary math texts called Real Maths for K-8 originally published by Open Court, and its latest version now called Math Explorations and Applications published by SRA/McGraw-Hill currently available for K-6. I'd like to hear commentary on this series from those who know it well. Also, can someone point me to reviews of this series, with comparison to others?