Mathematicians Versus Philosophers in Recent Work on Mathematical Beauty

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Synopsis

Recent attempts at defining mathematical beauty fall roughly into two schools of thought. One takes its starting point in the subjective experience of the mathematician and characterises mathematical beauty in cognitive terms. The other seeks to reduce beauty to objective notions such as truth, symmetry, or simplicity. This second approach is popular among analytic philosophers, who are committed to seeing mathematics and science as prototypically rational enterprises. I criticise this stance on the grounds that this commitment makes its supporters approach beauty in mathematics not with a genuine desire to sympathetically understand it, but with the preconceived goal of explaining it away and playing it down.

Hermann Weyl once said:

> My work always tried to unite the true with the beautiful; but
> when I had to choose one or the other, I usually chose the beau-
> tiful. [5]

Say this in the company of mathematicians and chances are that you will be met with knowing nods and warm smiles. But say it in the company of philosophers, and chances are that they will spit out their coffee and stare at you as if you had just sworn in church.

It is wise to keep this disciplinary divide in mind when studying scholarly attempts at clarifying the elusive concept of mathematical beauty. In recent years a number of studies have been devoted to this question.

Some studies have been by mathematicians, some by philosophers. The latter are certainly comfortably in the lead in terms of word count (and according to themselves in other respects as well, as we shall see). In this essay I shall offer a critical overview of some of the philosophers' attempts and come down on the side of the mathematicians.

When reading modern analytic philosophy, it is important to understand the nature of the beast. The analytic philosophy movement, which has dominated anglophone philosophy for the past century, is deliberately modelled on mathematics and science. It aims to be a logical, rational, precise pursuit of truth. This is why Weyl's statement is blasphemy to analytic philosophers. They need Weyl to be wrong about this, not only to protect their view of mathematics but in fact, by extension, to preserve their very *raison d'être* and self-image of themselves as doing the equivalent of logical, rational mathematics in the domain of general thought.

In their view, "aesthetic judgements and the evaluation of scientific theories are odd bedfellows, and their conjunction a just object of suspicion ... [since] it looks like appeals to aesthetic criteria are simply irrational" [15, page 62]. Analytic philosophers approach the subject of mathematical beauty exclusively from this adversarial standpoint, with the purging of any semblance of irrationality as their sole goal. In short, their attempts to define mathematical beauty aim not at explaining this notion but at explaining it away.

Mathematicians, meanwhile, have no horse in the race, at least not in the same way. They have nothing to fear from beauty one way or the other, so they are free to simply tell it like it is. Mathematicians approach the question of beauty with an open mind and an insider's appreciation, whereas philosophers approach it with an ideological aversion to everything it stands for and a predetermined agenda to sweep it under the rug.

My purpose in this paper is to survey recent work on mathematical beauty in light of the above contrast, and to argue that it gives us reason to trust the mathematicians rather than the philosophers, at least in this context.[1]

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[1] The literature on mathematical beauty includes much that falls outside the scope of my analysis. I am looking only at works that attempt to define what mathematical beauty is. For other dimensions of mathematical aesthetics, including the perspectives of other interest groups such as mathematics educators, see for instance [14] and the special issue of *Journal of Humanistic Mathematics* on The Nature and Experience of Mathematical Beauty published in January 2016.

To be sure, philosophers and mathematicians are not reducible to two homogenous antipodes. But my caricature has enough basis in reality. While the philosophers I shall discuss disagree on various points, as philosophers are wont to do, they nevertheless share a common core commitment as card-carrying supporters of the philosophers' view as outlined above. In particular, the only two recent monographs from academic presses devoted to trying to capture the meaning of beauty in science and mathematics, [8] and [11], fall squarely within this philosophical camp. Few or no challenges have been forthcoming to their outlook, which is starting to acquire an aura of established consensus. Meanwhile, modern mathematicians are perhaps all too happy to say, with Poincaré, that "mathematical beauty ... is a real aesthetic feeling that all true mathematicians recognize" [12, page 59], and leave it at that. But we should be wary of outsourcing any further explication to the philosophers. This plays into their hands and enables them to craft a narrative in which they are the only ones with sophisticated, systematic understanding of the matter while mathematicians' views can be written off as mere naive intuitions. It is my goal to counter this narrative. I wish to restore due regard for the mathematicians' intuitive sense of what beauty means, and question the philosophers' emerging consensus by shining a light on their ideological preconceptions and ulterior motives.

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McAllister's work on mathematical beauty [8, 9] captures in a nutshell the analytic philosophers' approach. He declares right upfront that he "aims to defuse the threat posed to the rationalist image of science" by scientists' talk of beauty [8, page 9]. In other words, he sees beauty as the enemy and has already decided in advance what his conclusion is going to be. Indeed, reaching this predetermined conclusion—that beauty is no real "threat" to rationality—is the only reason he took up the subject in the first place.

McAllister's solution to the problem of beauty is the following *thesis of aesthetic induction*:

> Scientists attach to each aesthetic property a weighting roughly proportional to the degree of empirical adequacy that they attribute to theories exhibiting that property. The table of weightings constructed in this way constitutes the scientists' aesthetic canon, used thereafter to evaluate theories. [8, page 203]

In other words, beauty is simply empirical success with a time delay. A philosopher whose professed goal it is to defend science as solely devoted to empirical success could hardly have wished for a more ideologically agreeable answer in his wildest dreams. Which suggests that a good dose of wishful thinking may have gone into devising it.

To analyse the matter further, let us separate McAllister's thesis into two sub-claims:

(1) Whenever a scientist expresses *positive* (i.e., appreciative) aesthetic judgement of a theory, this is due to his identifying properties of the theory that have been correlated with empirical success in past theories.

(2) Whenever a scientist expresses *negative* aesthetic judgement of a theory, his evaluation will be reversed if the theory proves empirically successful.

In other words, these theses show, respectively, why neither *pursuing* nor *rejecting* theories on aesthetic grounds is a serious violation of rationality. McAllister's evidence for his thesis consists in two pairs of case studies, corresponding to these two claims; namely, Copernicus' astronomical theory and Einstein's relativity theory for the first; and Kepler's law of ellipses and quantum mechanics for the second.

Let us consider McAllister's evidence for (1). McAllister notes that Copernicus adhered to the Platonic emphasis on uniform circular motion and criticised Ptolemy for violating this principle in his use of the equant (i.e., in allowing circular motion to be uniform only with respect to a point other than its centre). Let us allow for the sake of argument that these were aesthetic judgements (though I do not think that McAllister has any good evidence for this). Then, sure enough, these aesthetic judgements were a form of conservatism. But (1) is not supported but rather flatly contradicted. According to McAllister's theory, the empirical success of Ptolemy's theory should have led scientists to induce aesthetic *appreciation* for the equant, which is precisely the opposite of what happened. McAllister skirts this point and instead offers the following explanation:

> The sixteenth century's strong preference for theories that attribute uniform circular motion to celestial bodies is easily explained by the aesthetic induction. To Copernicus and his contemporaries it would have seemed that Aristotelian natural philosophy had built up an impressive empirical track record.

> This appraisal was largely justified: ... everyday experience accords well with Aristotelian theories in mechanics and biology. Through the operation of the aesthetic induction, the community came to attach great weight to the requirement that theories in the various sciences should show allegiance to the metaphysical claims of Aristotelianism. (page 174)

Copernicus theory had many virtues, but "allegiance to the metaphysical claims of Aristotelianism" was most emphatically not among them. In Aristotle's cosmology, there is a hard and fast distinction between the sublunar and heavenly realms. Circular motion belongs to the ethereal element of the heavens only. The terrestrial realm is governed by the rectilinear motions of the four elements (earth, water, air, fire), whose natural motion is either toward or away from the center of the universe (which is of course the center of the earth). Why on earth then, if Copernicus was so impressed by Aristotelianism, did he not induce aesthetic appreciation for all these things, rather than merely the principle of circularity? In fact, the former would make much more sense since McAllister brings up "everyday mechanics" as the empirical support for Aristotle's theory. But of course this would have precluded heliocentrism, so McAllister must opportunistically cherry-pick the principle of circularity only to make his account work.

The second case of positive aesthetic judgement discussed by McAllister is Einstein's relativity theory. Einstein was not motivated by empirical concerns but by aesthetic ones, as McAllister notes. In particular, "Einstein held strongly to a relationist view of space and motion that had originated in a criticism made of Newton by Leibniz", which "motivated [Einstein] in his work on both the special and the general theory of relativity" (p. 184). McAllister notes with approval that this "reveals what Abraham Pais sees as 'relativity's aesthetic origins'" (p. 186). Again this is not a confirming instance of (1) but rather a clear counterexample to it. Like Copernicus, Einstein held a view precisely opposite to that which should have been favoured by McAllister's aesthetic induction. Why had he not induced aesthetic appreciation for the Newtonian notion of absolute space, in light of the overwhelming empirical success of Newtonian physics? McAllister has no answer. He simply presses the point about conservatism and avoids mentioning the aesthetic induction altogether.

These two cases, Copernicus and Einstein, are McAllister's examples of scientists motivated by aesthetics rather than empiricism, in the manner of the Weyl quotation. McAllister's thesis is that the rationality of such cases is secured by (1). But, as we have seen, in these case studies McAllister downplays this crucial part of his thesis, focusing instead on his more general claim that aesthetics is a form of conservatism. He argues explicitly for (1) only in the former case, and then very unconvincingly; in fact, both cases seem to directly contradict (1).

Now let us consider McAllister's evidence for (2). As examples of innovations initially criticised as aesthetically displeasing McAllister points to Kepler's law of ellipses and indeterminism in quantum mechanics. These innovations were resisted, to be sure, but was this for aesthetic reasons? McAllister is able to say yes only by construing aesthetics very broadly. In the case of Kepler, McAllister is unable to quote anyone calling ellipses aesthetically displeasing or ugly. Instead, the criticism of Kepler was formulated in terms of circles being more befitting or apt to describe celestial motions. But, says McAllister, judgement of aptness is a form of aesthetic judgement: "a property is aesthetic if, in virtue of possessing that property, a scientific theory is liable to strike beholders as having a high degree of aptness" (page 37). It is quite the same with quantum mechanics. Einstein and others disliked its indeterministic element, which McAllister turns into an aesthetic judgement by the same trick as before, namely by extending the concept of aesthetics: "I propose to regard the allegiances that scientific theories have to metaphysical world views as aesthetic properties of them" (page 55).

It seems to me that McAllister's broadening of the concept of aesthetics makes his claim about aesthetic conservatism lose virtually all its counterfactual force. Could there be *any* conservatism that is not aesthetic by McAllister's definition? Non-aesthetic conservatism would have to be such that it could not be explicated neither in traditional aesthetic terms, nor in terms of "aptness", nor in terms of "metaphysics" (of which McAllister gives no definition). And the fact that the two last criteria are so conveniently fitted to McAllister's two case studies makes one wonder whether the list is complete even at this.

This is a sly trick indeed: since any case of conservatism is now aesthetic, any case of conservatism being overcome by empirical success becomes a

confirmation of McAllister's theory. Of course no one would deny that there are plenty of such cases in the history of science. So Q.E.D., McAllister's theory fits historical data.

McAllister's two case studies are indeed of this form: first establish that the conservatism was aesthetic, then that it was overcome by empirical data. In the case of the law of ellipses, for example, McAllister quotes Crüger writing in 1629 that "I am no longer repelled by the elliptical form of the planetary orbits" (page 179). This after he had become convinced of the value of the Rudolphine Tables based on Kepler's theory. No further evidence is provided that the repulsion in question was aesthetic in nature, even though one can surely be "repelled" by many other things besides, such as, for example, mathematical intractability.

But even if we set aside the problem of the delineation of aesthetics, another problem remains. Let us say that a particular astronomer once found circles particularly aesthetically pleasing, and placed great emphasis on this fact. Suppose that he then realised that circles are empirically inadequate and that it is necessary to use ellipses instead; and that he subsequently stopped talking about the beauty of circles. I say: nothing in this indicates that he has changed his aesthetic preferences. It may be that he still holds the exact same aesthetic views, but that he chooses not to keep expressing them since there would be no point in doing so.

To state this last point in more general terms: no one doubts that conservatism, including aesthetic conservatism, has often been overcome by empirical data; the novel part of (2) is that the aesthetic standards themselves change in the process. McAllister presents no evidence for this in the case of the law of ellipses. In the quantum mechanics case, his supposed historical evidence has the following form. Initially, some people found the theory aesthetically displeasing. Later, *other* people found the theory aesthetically pleasing on *different* grounds. Such evidence quite clearly says nothing about aesthetic induction.

Let us consider in more detail McAllister's evidence for induced aesthetic appreciation in the case of quantum theory. The first piece of evidence McAllister offers for induced aesthetic appreciation is a quotation by Gribbin, written in 1984:

> [W]ave mechanics gives an *illusion* of something familiar and comfortable. . . . Several generations of students, who have now grown up to become professors themselves, might have achieved a much deeper understanding of quantum theory if they had been forced to come to grips with the abstract nature of Dirac's approach, rather than being able to imagine that what they knew about the behavior of waves in the everyday world gave picture of the way atoms behave. (page 195)

From this McAllister concludes:

> The change in attitude illustrates how, in the wake of a revolution, scientists reassess the merits of their aesthetic commitments. As the empirical track record of quantum theory improved, its aesthetic properties reshaped the aesthetic canon of physicists. . . . Gribbin's denunciation of the requirement of visualization, which would have been unthinkable in the 1920s, shows how far this amendment of aesthetic canons has proceeded. (page 195)

The conclusion is unwarranted for the simple reason that the quotation in question has nothing to do with aesthetics. Rather, it merely makes a purely *factual* claim (i.e., that wave mechanics is misleading). McAllister's conclusion follows only if one *assumes* that factual claims go hand in hand with aesthetic commitments. But of course this assumption is precisely the thesis that McAllister is trying to justify.

McAllister's second piece of evidence for his thesis consists in two casual and inconclusive quotations from Heisenberg and Jammer (from 1970 and 1966 respectively) expressing aesthetic appreciation of quantum theory (page 201). Actually, Heisenberg does not even go that far. He merely notes that "the beauty and completeness of old physics seemed destroyed" in the early 20th century, but that later research "has again restored exact science . . . to the state of harmonious completeness." As for Jammer, McAllister's allusion to him reads in full: "Max Jammer felt able to appraise quantum theory as 'an imposing intellectual structure of great beauty.'" McAllister then asserts, without any further argument or discussion, that "The discrepancy between these statements and Planck's and Einstein's declarations of revulsion at quantum theory is a demonstration of the power of the aesthetic induction to foster aesthetic appreciation for empirically successful theories" (page 201).

The discrepancy shows nothing of the sort, for, as I argued above, the fact that a theory is considered ugly in some respects does not preclude it from being (especially at a later stage) beautiful in other respects. Therefore the quotations in question fail to establish any discrepancy at all in aesthetic standards.

Even if we put this objection aside, these statements would still have nothing to do with aesthetic induction. For suppose that there was a discrepancy in aesthetic standards between Planck and Einstein on the one hand and Heisenberg and Jammer on the other. Then this proves merely that some people like quantum theory and some people do not, which does nothing to support McAllister's thesis. What McAllister needs is for Heisenberg's and Jammer's aesthetic standards to have changed into that form by induction from empirical data. But for this there is no evidence. Only the Heisenberg quotation hints at a change in aesthetic evaluation, but this change could just as well (not to say more naturally) be interpreted as being due to improvements in the theory rather than a change in aesthetic standards.

In sum, I find that McAllister's arguments are very unconvincing, while his conclusion fits like a glove the ideology he is trying to defend (namely that of the rationality of science). This is exactly what one would expect from a "flatterer of the court", i.e., someone with no interest in approaching the matter with an open mind, and every interest in concocting arguments for a predetermined, ideologically desirable conclusion at all costs. Such, I say, is the modus operandi of the philosophers, and it is why we must treat their work on this subject with the utmost suspicion.

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Mathematicians have approached the subject of beauty differently. Their style is, as a rule, much more cheerfully candid and confessional. Emblematic of this is Gian-Carlo Rota's authoritative article titled "The Phenomenology of Mathematical Beauty" [13]. Rota's goal is simply "to try to uncover the sense of the term 'beauty' as it is currently used by mathematicians" [13, page 171]—a very innocent and open-minded goal compared to McAllister's openly professed goal of dealing with beauty only because it is a "threat" to his favoured ideology.

With their unguarded openness, mathematicians have been much more open to characterising beauty in human-centered rather than objective terms.

Rota in particular connected beauty with "enlightenment" as opposed to "mere logical truth" (page 181), and argued that it goes hand in hand with a sense of "light-bulb" insights as opposed to "an intricate sequence of logical inferences" (page 179).

Other mathematically inclined authors have tended to agree with Rota. I myself suggested in this journal that:

> A beautiful proof is one which the mind can play its way through with a natural grace, as if it were created for this very purpose. ... [Whereas by contrast:] An ugly proof resorts to computations, algorithms, symbolic manipulation, ad hoc steps, trial-and-error, enumeration of cases, and various other forms of technicalities. The mind can neither predict the course nor grasp the whole; it is forced to cope with extra-cognitive contingencies. The mind's task is menial: it can only grasp one step at a time, checking it for logical adequacy. It can become convinced of the results but it is not happy since all the work was being done outside of it. Our memory is strained, our mind distorted to accommodate some artificial logic, like a student struggling with a foreign grammar. [1, page 93]

This is very similar to Cain's recent characterisation of ugliness if proofs, framed in terms of the literary notion of *deus ex machina*:

> *Deus ex machina* (literally, "god from the machine", henceforth abbreviated to *deus*) refers to a type of plot device used to resolve a seemingly intractable situation. The term is derived from ancient Greek drama, where such a resolution might be effected by a god intervening, with the actor playing the god being lowered onto the stage by a crane (the "machine"). It has come to mean any event in a story that resolves a situation but which does not fit with the internal framework of the plot. ...
>
> [In a proof involving *deus*,] Readers cannot see why this construction or this calculation or that definition is being carried out; they cannot perceive a reason for it that is internal to the proof. In short, they have more difficulty in modifying their intuitive conceptions to include the *deus*. They can follow the proof to its conclusion, checking each step against their formal knowledge

of the objects concerned, but the *deus* is a cataract that their
intuition cannot easily navigate. [4, pages 7–8]

Both Cain and I elaborate on the clear parallels of our view with that of Rota.
I also discuss a number of notable precursors of this view in the history of
mathematics.

Altogether a clear tendency emerges from these works, namely that of linking
mathematical beauty to human cognition and intuition. I shall refer to this
as *the mathematicians' view*. It seems to square well with mathematicians'
subjective experiences and descriptions thereof. The philosophers do not
like it one bit though of course, since it does not immediately turn beauty
into something rational, which is the only kind of theory of beauty they are
interested in.

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Suppose for the sake of argument that the mathematicians are right about
the nature of beauty. How would we expect the philosophers to react? Pre-
sumably they would offer revised accounts of beauty that move closer to
the mathematicians' view, while, however, saving as much as possible of the
"rationality" of McAllister's proposal. They would also be keen to portray
McAllister's proposal as basically on the right track, and their own propos-
als as natural, incremental improvements upon it. As a consequence of this,
they would need to find an excuse to dismiss mathematicians' accounts, since
conceding that the mathematicians were right would severely undermine the
philosophers' ideological goals.

This is in fact precisely what has happened. The second major monograph
on the subject after McAllister is that of Montano [11]. In keeping with the
above, Montano praises McAllister's approach as "pragmatically appealing"
in that it "achieve[s] ambitious goals like defending the rationality of science"
(page 31). He also frequently refers to his own work as "rigorous" (pages *ix*, *x*,
73, 211), as aiming "to provide the first proper theory of aesthetic phenom-
ena in mathematics" (page 59), and so on in similarly self-congratulating
terms. The only other author who earns the epithet "rigorous" is McAl-
lister (page *ix*). In the same breath Montano expressly distances himself
from "light-hearted" (page *x*) and "anecdotal" (page 9) accounts of beauty
in mathematics.

No doubt Montano is right that much useless anecdotal fluff has been written on mathematical beauty, but it seems to me that all this eager posturing also serves a second purpose, namely as a justification for ignoring the literature of the mathematicians. It does not take much perception to see that his distinction between "rigorous" and "light-hearted" happens to coincide with the division between philosophers and mathematicians.

This rhetorical device enables Montano to claim as his own certain basic elements of the mathematicians' view. Thus he argues:

> Our active engagement in following a proof plays the central role [for aesthetic purposes]. ... I have tried to highlight the role of the inner experience of mathematicians in their evaluations of mathematical entities. ... The introduction of our subjective life into our discussion is illuminating. ... This factor ... is largely neglected in the literature. (pages 42–43)

We must presumably understand that "the literature" here means the "rigorous" literature, which in turn means the literature written by philosophers. At least that is the only way I can see to interpret Montano's claims without making them obviously false, since mathematicians stressed precisely this perspective long before.

Montano's key example and application of this perspective is his account of why the computer-assisted proof of the four-colour theorem is aesthetically unpleasing:

> The experience [of studying a computer-assisted proof] has deforming narrative gaps. A computer-assisted proof shall always give us an incomplete experience, something we cannot fully appreciate, despite the fact that the proof is a perfectly acceptable and widespread method. Thus, it is not very plausible that we shall come to regard computer-assisted proofs as beautiful. (page 203)

But this is just the currently dominant view among mathematicians in so many words. Indeed, the parallel between aesthetically unpleasing proofs and unsatisfying narrative gaps was elaborated explicitly and at length by Cain [4], whom Montano does not cite. Thus Montano has, in effect, conceded that the mathematicians' view was correct all along, though he does so under a rhetorical smokescreen that enables him to claim these insights as his own.

Of course, being an analytic philosopher, Montano does not want to place too much emphasis on subjective experience since "the presence of subjective judgements certainly constitutes a problem" (page 16) in that it "can ... be perceived as a threat to rationality" (page 43). Perhaps this is what holds him back from drawing the natural completion of his picture of his account of the aesthetically unpleasing to the positively beautiful. For when it comes to the latter he stops short of making the human experience an essential ingredient, and no longer follows the mathematicians' view as he did in his account of the ugliness of the four-colour proof. (Perhaps this explains why his account of ugliness only appeared separately in a mathematical journal, see [10].)

When accounting for positive aesthetic experiences, Montano instead insists on decidedly more objective factors. His two main examples are:

- The result that the function $y = e^x$ is its own derivative is beautiful because invariance under differentiation is a form of "symmetry", which "is an aesthetic property by itself" (page 172).
- Cantor's diagonal proof for the uncountability of the reals is elegant because of its "step-parsimony = the feature of consisting of a small number of steps" (page 183).

By explicating beauty in such objective terms Montano avoids the problem that "subjective judgements are incompatible with the norms of rationality" (page 15). But he also ends up with an account of beauty that is not in any natural way the inverse of his account of ugliness.

Lange [7] offers another recent account of beauty which mirrors these features of Montano's view. He proposes to explain beautiful proofs in terms of their explanatory power. Non-explanatory proofs are typically brute-force computations that do not illuminate why the theorem holds, but rather "makes it seem like an accident of algebra, as it were" (page 16), "supplies 'little understanding' and fails to show 'what's going on'" (page 20).

So far this is very much in line with the mathematicians' view. But when it comes to positively defining the explanatory, Lange has a different proposal. He argues that if a result has a notable symmetry to it then an explanatory proof must itself involve this symmetry in an essential way: "A proof that exploits the symmetry of the setup is privileged as explanatory" (page 18) and "only a proof exploiting such a symmetry in the problem is recognized

as explaining why the solution holds" (page 19). Lange later generalises from this to allow any "salient feature" of the theorem to take the role of symmetry in this argument (page 28). He then ties this to the notion of beauty as follows:

> I suggest that all explanatory proofs are beautiful (or, at least, not ugly). They derive their beauty from exactly what gives them their explanatory power, namely, from their exploiting precisely the feature in the setup that is salient in the theorem (such as its symmetry, unity, or simplicity). (page 45)

For Lange and Montano alike, then, beauty is tied to objective properties such as symmetry.

It seems to me that the mathematicians' view accounts more naturally for what both Lange and Montano are saying. For the mathematicians' view gives a unified explanation for ugliness and beauty as a natural inverse pair, unlike the philosophers' accounts, which have one story for the one and completely different one for the other. On the philosophers' accounts it also remains something of a mystery *why* those particular objective properties they invoke to explain beauty should be the right ones, whereas the mathematicians' view brings all aesthetic experiences back to a single, unified cause, namely the cognitive endowment of the human mind. Furthermore it is easy to understand why the philosophers would be predisposed to focus on objective properties as opposed to human-centered cognitive factors, since this goes better with their oft-professed allegiance to objective rationality.

A recent empirical study by Ingli & Aberdein [6] is relevant in this context. These authors asked 255 mathematicians to "think of a particular proof in a paper or book which you have recently refereed or read" and then rate how accurately each in a list of eighty adjectives described this proof (pages 95–96). One of the adjectives was "beautiful". The authors then calculated which other adjectives were most closely correlated with "beautiful" (page 101). The list begins: elegant, pleasing, appealing, ingenuous, striking, inspired, enlightening. Though some of these terms may be regarded as more or less trivially synonymous with "beautiful", the close correlation between the beautiful and the ingenuous, inspired, and enlightening is far from a trivial correlation that one can find in a thesaurus. But the mathematicians' view of beauty makes good sense of it. In particular, arguably all of these

concepts are closely tied up with peculiarities of the human mind and human experience. Only much further down the list does one find the kinds of objective properties that philosophers are always trying to tack beauty onto, such as efficient (#23), informative (#36), minimal (#49), general (#51), simple (#53).

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Philosophers were not always so obsessed with the disembodied rationality that analytic philosophers of today are so eager to espouse and defend. Kant's *Critique of Pure Reason* was a perceptive attack on precisely such a simple-minded ideal. Kant argued persuasively that such "pure reason" is an illusion and that in reality reason is inextricably intertwined with the particulars of our cognitive and perceptive endowments. Clearly this goes well with the mathematicians' view of beauty. Indeed, Kant made some remarks to this effect, as noted in [1, page 94]. Philosophers have subsequently recognised the same point. Thus Breitenbach observes accurately:

> For Kant, beauty . . . is experienced . . . in the process of demonstrating mathematical theorems through a creative act of the imagination. Insofar as this creative activity involves the free and spontaneous use of imagination which . . . leads to conceptual insight, it points to the fit between our intellectual capacities. And it is the awareness of this fit that, on Kant's account, is experienced with a feeling of pleasure. [2, page 969]

This is a spot-on description of what I have called the mathematicians' view. But, being a modern analytic philosopher, Breitenbach is unwilling to accept the full ramifications of her own analysis. Instead she is keen to immediately bring the issue back to a familiar refrain:

> While . . . focus on aesthetic considerations may be widespread among scientists, it raises difficult questions. Why should considerations as possibly subjective and most probably contestable as aesthetic judgements play a role in science with its prima facie aim of securing objective knowledge? [3, page 84]

This is the same conundrum we have seen other analytic philosophers tackle. Breitenbach wishes to argue that the Kantian view allows for a resolution that satisfies the analytic philosopher's demand for objective truth:

> Aesthetic considerations may legitimately appeal to scientists ...
> because, in searching for beauty, scientists aim for theories that
> provide understanding. Although aiming for understanding of the
> phenomena does not by itself guarantee a path to truth, providing
> understanding is a necessary requirement for any successful the-
> ory. Scientists may therefore have a reason to take considerations
> of beauty into account. [3, page 85]

In my view we have here, much like in the case of Lange and Montano,
another instance of a philosopher not following their own arguments to their
logical conclusion, but rather stopping short of it for the sake of preserving
rationality. For if the Kantian view is right, why should the scientist employ
it only in this subservient role?

Suppose scientists can generate beauty in the Kantian sense in a way that
cannot be justified as "legitimate" in the manner suggested by Breitenbach.
Does it then make sense that they would abandon that line of thought as "ille-
gitimate"? I think not. A large part of human life consists in enjoying things
for no rational reason. Do scientists not, for example, watch movies and read
novels that are nothing but fictions concocted for pure entertainment? Then
why would they not pursue fictions concocted for pure entertainment in the
domain of speculation about nature as well? As Poincaré put it:

> The scientist does not study nature because it is useful to do so.
> He studies it because he takes pleasure in it, and he takes pleasure
> in it because it is beautiful. If nature were not beautiful, it would
> not be worth knowing and life would not be worth living. [12,
> page 22]

As we have seen in case after case, modern analytic philosophers refuse to
admit this possibility even when their own arguments—such as Lange's and
Montano's accounts of ugliness, and Breitenbach's analysis of Kant—bring
them right to the threshold of it. It seems to me that their sole reason for
refusing to take this step is that it would clash with their zeal to defend
the rationality of science and mathematics, which is a commitment they are
not willing to abandon. If my diagnosis is right, and analytic philosophers
approach the question of mathematical beauty with doctrinal preconcep-
tions that preclude them from considering viable and plausible alternative
hypotheses, then it follows that we must view their work on this subject with
considerable suspicion.

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To philosophers, mathematical beauty is like a daughter they never wanted. Their only interest in her is to marry her away to a high-status spouse who would reflect well on them. The more objective and rational a husband they can secure for her, the happier they will be and the better they will look in front of their friends. McAllister did his best to orchestrate such a marriage by promising that beauty would always go hand in hand with empirical success and copy whatever this patriarch did, albeit with a slight delay. Others agreed that this would be the best possible outcome, but feared that beauty was rather too capricious to snare such a coveted groom. So they have made it their business to line up other highly respectable matches and try to pitch beauty as a particularly suitable partner for their proposed candidate.

Mathematicians, on the other hand, are happy to accept beauty as she is and embrace her in her own terms. To us she is the apple of our eye and the one thing we did right in life. I would urge anyone delving into the literature on mathematical beauty to keep this contrast in mind, and to ask themselves: Who, then, should sooner be trusted to give a faithful account of mathematical beauty—the philosopher or the mathematician?

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