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Cover Page Footnote
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Shakespeare, a Supernova, and a Little Green Man
Walk into a Mathematics Classroom

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Synopsis

Creativity amidst constraints is a hallmark of the STEM researcher. It is precisely what is required to see what has never been seen. It is also at the core of creative mathematics, more commonly called “research”. We in the 21st century tell ourselves that science and story are separate enterprises. One goal of this article is to tell parts of the human story—featuring Shakespeare, Tycho Brahe, visiting stars, Little Green Men, and modern astrophysics—that might erode belief in that duality and illustrate why dissolving the artificial barriers between talents within individuals is to the benefit of ourselves, our students, and society.

Curious about how one of the greatest plays in English literature may be related to the man whose work made possible Kepler’s discovery of the laws of motion of the planets in our solar system? Ever wondered who was the world’s first known science fiction writer? Read on.

Let us pretend that one night on the way home from work you see a bright light in the sky. After dismissing it as a satellite or a plane or some such thing, it is a surprise to see it again in the same place the next night. And the next. It is not your imagination; some friends comment on it as you leave a pleasant and otherwise unremarkable dinner. It begins to grow brighter,

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1I am indebted to a National Public Radio story on Shakespeare and Tycho’s Star for the inspiration for this project and to the work of Peter Usher of Pennsylvania State University for the astronomical interpretation of Hamlet.

2Sheila Miller earned her Ph.D. in mathematics in Boulder in her home state of Colorado. Today she is a professor of mathematics at City University of New York and teaches yoga and meditation at Ishta Yoga in New York City.
so bright that it is visible in the daytime sky at noon. Over the course of many months it changes color from white to green to blue to red, eventually growing dimmer until it fades from view.

This has actually happened. Perhaps not to you, unless you are very old indeed, but it has happened. In fact, there is reason to believe something like it happened to Shakespeare.

Angry ghosts of murdered heroes make good material for writers, as do potentially apocalyptic, unidentified, astral phenomena. The first act of William Shakespeare’s *Hamlet* has both. It shows the changing of the guard at Elsinore, the castle of Hamlet, prince of Denmark, during which for two consecutive nights the guards of Elsinore spot a ghost that resembles Hamlet’s father. We soon learn that the King was murdered by his brother, who subsequently took possession of both the throne and the dead King’s wife.

As one might expect, it is an angry ghost they see at Elsinore.

The first scene of Act 1 is an ominous one in which the soldiers make casual reference to “Yond same star that is westward of the pole” as if the referent is common knowledge. Ten minutes outside with a star chart and a flashlight did not offer me any strong candidates for said star, but forensic astronomers, who also sought the star, thought to wonder about the nighttime sky of Shakespeare’s time rather than ours. They found a light so conspicuous that everyone would have known of it. Indeed the very soldiers at Elsinore\(^3\) would certainly have seen it and remarked on it to one another. It was a light that appeared, soon changing color and increasing in intensity until it was visible in the daytime sky. Then it faded away. A visiting star.

This is how it appears in *Hamlet*:

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BARNARDO: Last night of all,
When yond same star that’s westward from the pole
Had made his course to illume that part of heaven
Where now it burns, Marcellus and myself,
The bell then beating one-

ENTER: Ghost

MARCELLUS: Peace, break thee off. Look where it comes again!
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—Shakespeare’s *Hamlet*, Act 1 Scene 1

\(^3\)Had Elsinore been in England, where Shakespeare was, that is.
Many modern readers of Shakespeare assume that the soldier in Act 1 Scene 1 of *Hamlet* is referring to the North Star — some modernizations of the language even declare it to be so. Forensic astronomers, however, recently concluded that it is more likely a “visiting star,” a bright light that appeared in the heavens, so bright as to be brighter than Jupiter, even brighter than Venus (each of which are brighter than stars), and finally to be so bright that it was visible in the daytime sky at noon. The visiting star to which Shakespeare almost certainly refers became visible in 1572 and remained visible for sixteen months (until 1574). Over those months it grew steadily brighter until it became visible in the daytime sky, even changing color several times, and then it faded from view to the naked eye.

The observation of the 1572 visiting star marked the beginning of the career of astronomer Tycho Brahe. His nightly naked-eye observations of the sky over decades made possible Johann Kepler’s work on the laws of motion of astronomical bodies. (More on this shortly.)

Shakespeare is believed to have been born in 1564, and newspapers from his hometown of Stratford-upon-Avon make reference to the visiting star, also called “Tycho’s Star” (for Tycho Brahe), demonstrating that it was possible, even likely, that Shakespeare saw the visiting star in his childhood. It is furthermore known that he was familiar with the work of Tycho Brahe, and certainly also with the life of the man, for it would have been difficult to know of the one without the other as Tycho Brahe had a dramatic personality.

Brahe was raised by a tremendously wealthy nobleman (an uncle) who abducted him from his (also wealthy) parents when he was two, and this background of abundance allowed him to build himself a castle that housed an observatory. He wore a prosthetic nose after getting his real one cut off by the sword of a family member with whom he had a disagreement, reportedly about a mathematical formula. At his castle he kept a tame pet elk (which some sources say was a moose), and it is widely believed that he had an affair with a powerful lady, perhaps none other than the Queen of Denmark.\(^4\)

\(^4\)There is considerable rumor and speculation about this, especially in combination with the (possibly debunked) claim that Brahe was poisoned with mercury. Some even speculate that the rumors of this alleged murder were part of Shakespeare’s inspiration for *Hamlet*. 
After years of spending every night in his observatory, squinting through his telescope, Brahe’s right eye began to protrude, appearing larger than his left. A close inspection of the names of his ancestors that surround Tycho Brahe in a portrait he commissioned of himself (Figure 1) reveals the names Rosenkrans and Gyldenstern, which have Anglicised versions Rosenkrantz and Gyldensterne. These are the very names chosen by Shakespeare for the two young men assigned by Claudius, murderer of the King and uncle of Hamlet, to ensure that Hamlet is delivered safely to England where there are orders for his execution.

There is no doubt that there was overlap in the lives of Shakespeare and Brahe, and it seems that the Bard and the Astronomer saw the same mysterious light in the sky—and that each explored this fascinating and unexplained occurrence in his own work. Furthermore it seems clear that Brahe’s life and work influenced Shakespeare’s in nontrivial ways. It is one world; one world we investigate in many ways.

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Figure 1: Portrait of Tycho Brahe, astronomer (without a hat), engraving by Jacques de Gheyn II (1586). The image is a photographic reproduction in the public domain (https://commons.wikimedia.org/wiki/File:Jacques_de_Gheyn_Ii_-_Portrait_of_Tycho_Brahe,_astronomer_(without_a_hat)_-_Google_Art_Project.jpg).
The scientific ways of thinking that we take for granted today are the product of a revolution that was taking place during Shakespeare’s lifetime, the dawn of the Scientific Revolution in Europe, a time of monsters and royal intrigue, with all its associated instability. Soon Brahe’s meticulous data were to be used in a radical new way by his student Kepler.

Kepler’s Laws describe the movement of large bodies in space (such as planets and stars), explaining that the orbits of planets are ellipses, not circles, which in turn explains why some faraway planets sometimes appear to us to move backward and other planets, those close to the sun, are never visible in the night sky at midnight. Kepler identified these laws using his late teacher Brahe’s nightly observations, and also, crucially, his own imagination. He was part of the birth of the Scientific Revolution during which the rebel behavior of testing beliefs and hypotheses against observations became accepted, unlike in the time of his immediate predecessors, Copernicus and Galileo. His scientific behavior thus differed from that of Brahe; Kepler conjectured explanations (formulas) for the behavior of stars and planets and then tested those projections against Brahe’s data, reformulating, refining, and retesting until he got it right.

For a person not persuaded that Kepler was deliberately engaging his creative imagination in his scientific work — indeed in a crucial way new to Western Science — we note that Kepler, in 1608, wrote what is believed to be the first work of science fiction, a novel titled Somnium (The Dream). In addition to magic and demons and space travel, there is Kepler’s imagined view of the earth from the moon, precisely the variety of imagination that made possible his discovery of the elliptical orbits that explain the previously mysterious behavior of planets that appear from earth to sometimes move backward.

Of course it would almost certainly have been a struggle for even Kepler to imagine the truth of what caused those lights in the night sky of 1572. It was neither demons, nor space travel, but something perhaps more shocking. Stardust.

We are, quite literally, made of stardust. The Big Bang — the sudden emergence of the universe from singularity — created only Hydrogen and Helium; all other elements are created inside of stars by nuclear fusion. The elements that are the building blocks of life as we know it, the Carbon of which our bodies are composed, the Oxygen required by every one of our cells,
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the Calcium without which our brains could not think and our muscles could not contract — every element aside from hydrogen and helium — was formed inside a star and released in the universe when the star died.

Stars die in their own ways, each according to their nature. Our sun is a yellow dwarf, and when it dies it will swell in size and (relatively) gently fade away, keeping all the elements it produces nearby. Some stars, a very special few called supergiants, die in a spectacular explosion, propelling throughout their own galaxy and beyond the elements created within them during their lives as stars and even heavier elements created during the explosion itself. It is these stars that make life possible. It is these explosions, called supernovae, that can sometimes be seen from Earth and appear as “visiting stars”.6

Tycho’s Star was not the first visiting star observed by humans on Earth. Several had been observed in Asia, the most famous of which is located in the Crab Nebula. In the location of such an event observed in the sky in 1054 CE, modern day physicists found in 1967 traces of unexplained astral phenomena. Specifically, in a survey of the sky, a University of Cambridge team found perfectly regular electromagnetic pulses coming from apparently nowhere. Generally the electromagnetic signals from the sky are what is called ‘noise’: we can’t distinguish any particular patterned signal (although a lot of people look!7). It is a fun and suggestive exercise to spend three to five minutes thinking of everything possible that has or creates a pattern. What percentage of them are caused by a life-form?

For over a month the Cambridge team monitored the patterned, regular pulses. They were not just noise. Here is what Nobel Prize winner Antony Hewish8 said in his 1974 Nobel Prize acceptance speech about the period of watching and wondering:

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6When such stars explode, they shine up to one hundred billion times brighter than normal, often brighter than all the other stars in their galaxy combined.

7SETI, the Search for Extra-Terrestrial Intelligence, analyzes as many of the signals coming to Earth from space as possible, many of them using the home computers of volunteers. If you want to be part of the search for extraterrestrial life, you can download a program that analyzes signals on your computer when you are not using it, and then sends the results to SETI. See: http://www.seti.org for information about SETI and http://setiathome.ssl.berkeley.edu/ to be part of their effort to find life elsewhere in the universe.

8It was actually his graduate student, Jocelyn Bell, who made the initial discovery. There are some who feel she should have been a co-recipient of the prize.
Having found no satisfactory explanation for the pulses we now began to believe that they could only be generated by some source far beyond the solar system, and the short duration of each pulse suggested that the radiator could not be larger than a small planet. We had to face the possibility that the signals were, indeed, generated on a planet circling some distant star, and that they were artificial. [2]

By “artificial” he means intentionally created, not the product of an inevitable process. He goes on to say “Without doubt, those weeks in December 1967 were the most exciting in my life.” For a while the team called the phenomena LGM — you guessed it: Little Green Men. The ties between Life and regularity are sufficiently strong that nothing else in the domain of physics in 1974 could explain the emission of the patterned pulses. What an exciting moment in science!

Hewish and his team had indeed discovered something previously unknown and profoundly important, not Little Green Men or another form of intelligence, but the hallmark of the cosmic mechanism responsible for life.

The cause of the pulses was the remnant of a supernova — the explosion of a supergiant that threw away all the material in the outer layers of the star and left only the core, a mass so dense that the protons and electrons were crushed together to become neutrons. This rapidly spinning mass is a neutron star, or pulsar, and the gravitational field of a neutron star is about $2 \times 10^{11}$ times that on Earth. (This means you would weigh two hundred billion times as much as you do on Earth.) Before the discovery of the first pulsar by Hewish and his team, neutron stars were hypothetical stellar entities. Thanks to the work of the Cambridge team, there is now sufficient evidence to establish the existence of pulsars as scientific fact, even though we cannot see them with our eyes. Pulsars are quite small, smaller than Brooklyn, and are observed through the radio waves they emit, not through light in the visual field. But the associated explosions, supernovae, themselves are so bright that we can sometimes see from Earth, even with the naked eye.

Because pulsars spin so rapidly and because they have such strong gravitational fields, they emit electromagnetic radiation at their poles in the form of radio waves. Visible light is also a form of electromagnetic radiation, but with a much shorter wavelength. The wavelengths of radio waves vary between a decimeter and more than a kilometer, so they can have length as
small as that of a kitten or be much longer than the height of the world’s
tallest building. Visual light, on the other hand, has a wavelength similar to
the size of the average (microscopic) bacteria. The spinning of the pulsar is
like the revolving light in a lighthouse: each time a pulsar with a pole that
points toward Earth rotates, we see a pulse of electromagnetic radiation, a
radio wave. Pulsars are very consistent in the intervals of their rotation,
which makes them excellent clocks.

It might be useful to take five undistracted minutes and reflect on this cosmic
wonder. It is precisely explosions like the extraordinary death of the star
witnessed by Tycho Brahe and William Shakespeare that are responsible for
the formation and dissemination of the elements required for the sustenance
of life in the universe.

The universe and our home here on Earth are awe-inspiring, especially when
we pause and allow ourselves to experience and be amazed by our surround-
ings. Our scientific and artistic expressions of that awe are among our great-
est and most creative contributions to one another and to our own experience.
To cultivate imagination is one of the most joyful and important activities
of this life; it enables scientific discovery, artistic expression, and, ultimately
is the heart of the solutions to all problems.

Discussion—Making Mathematics Our Own: Narrative

The narrative above is consolidated from a project I gave in four parts over
one semester in a college algebra course at City University of New York’s
City Tech (New York City College of Technology), a public college of ap-
proximately 16,000 students. It is one of the most racially and ethnically
diverse colleges in the country, and the level of preparation of students is
highly variable. The project was developed in response to a student who
asked one day during class if I knew anything about pulsars. At the time
that he asked the question, the answer was ‘no.’ I later gave a variation
of the assignment to a differential calculus course, substituting related rates
questions for some of those mentioned below.

The mathematical skills required for these projects are elementary by de-
sign, a choice to devote time to supporting students in following Polya’s ad-
monition to Understand Simple Things Deeply. My hope was to potentiate
an impression of the astounding connectedness and beauty of the universe,
mathematical and physical, and our place in it. I furthermore wished for
students to have the opportunity to display and relate multiple skills and to be rewarded for creativity and imagination.

How do we foster the impulse to imagine? When teaching mathematics it is tempting to focus on content, competencies, and preparations. Despite the widely acknowledged role of creativity in research mathematics, we rarely attempt to teach it. As an example of one way to encourage imagination in the mathematics classroom, I offer the above tale of interrelation and the related student project outlined below that utilizes, among other tools, creative writing and conscious contemplation.

It is a human tendency to pursue those things to which we feel connected. We read the sections of the newspaper relevant to the places we are from, we love and re-read the books that speak most closely to our experiences, and we listen to music that evokes in us a feeling of recognition. We construct narratives to make sense of our lives and to connect our sense of self to our circumstances. Sadly, for many students, mathematics appears lifeless, alien, or malicious; they are unable to experience it as something connected to themselves.

How does this happen? Experts see their fields of mathematics and science as cohesive stories with interacting parts rather than disjointed masses of facts and rules. As educators we focus perhaps too little on helping our students achieve this perspective. It is easy to fall under the spell of a course catalogue. If we view subjects and topics as nicely divided and categorized with pre-requisites clearly identified, the enterprise of education is manageable and the path to the future neatly outlined. Indeed many people come away from their schooling believing that we can only understand the world in one way at a time, that logic and mathematics are the domains of the left brain and literature and images the realm of the right, and that our destinies are governed by the hemisphere that is dominant in our individual brains.\footnote{The idea that there are “right brained” and “left brained” people is not one endorsed by neuroscience. For an easy read summarizing how this duality is a pop-culture myth, see [4]; for a subtle and comprehensive analysis, see [6]; for one example of the extent of lateralization in fMRI imaging, see [9].} The brain-divided position, a false dichotomy, begs to be asked: how does the moment of inspiration differ between “left-brained” activities and “right-brained” activities? In my experience, it does not.
The best and most innovative ideas strike us when we are not consciously paying attention, such as when we’re doing the dishes, taking a shower, or going for a walk. One of the goals of this project is to provide an opportunity for students to think narratively about scientific and mathematical questions in order to increase their access to the pattern recognition and construction activities of the not-quite-conscious where so many novel thoughts rise.

1. Outline of the Project

1.1. Yond Same Star That Is Westward of the Pole: Paving the way to caring about pulsars

To introduce students to the project, I asked them to watch the first five minutes of *Hamlet* and read the script of the first scene, identifying the tone and how it made them feel (not think). Given the description of the visiting star in Shakespeare’s time, they were asked to imagine such a light appearing in the present day sky, and to explain what it might be, subject to the conditions that NASA, the governments of all countries, and all religious leaders do not claim to know the origin of the light. Their explanation of the visiting star could take the form of a short story or a newspaper article.

1.2. Over the River—Understanding Cosmic Speeds and Distances: Deliberate Use of Foundational Concepts.

Consider: how many of us have a feeling for how far away the sun and stars really are? For how big Jupiter is? For how long it takes light to travel to Earth from distant stars?

Recognizing when and how to apply mathematical knowledge is in itself non-trivial, and it is a skill that is notoriously difficult to teach. It is somehow easier to urge students forward, to assign them projects that extend topics or applications of a given course. This part of the project returns to concepts students have seen before, recognizing that connecting to concepts in progressively deeper ways requires repeated exposure to those ideas.

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10 Readers can find the text of *Hamlet* at [http://shakespeare.mit.edu/hamlet/hamlet.1.1.html](http://shakespeare.mit.edu/hamlet/hamlet.1.1.html), last accessed on July 5, 2017. This website is part of *The Complete Works of William Shakespeare*, as uploaded and maintained by Jeremy Hylton and The Tech newspaper of MIT, Cambridge, MA. As for a dramatic version, here is a youtube link which I gave in class [https://www.youtube.com/watch?v=aHtacpVY8DY](https://www.youtube.com/watch?v=aHtacpVY8DY), last accessed on July 5, 2017 (Wyver, J., Grant, S., Doran, G., Tennant, D., Stewart, P., Downie, P., Davies, O. F., and others, Warner Home Video, 2010).
Many students struggle with units and unit conversions long after they have learned the technical skill. The goal of this section of the project was to use the familiar mathematical concept, unit conversions, to both reinforce the importance of this basic manipulation and to illustrate that a more-than-superficial understanding of units can reveal a genuinely astonishing reality. It is one thing to be able to recite from memory the speed of light (approximately 299,792 kilometers/second or 186,282 miles/second); it is quite another to relate cosmic speeds and distances to one’s own terrestrial life.

The universe is truly vast. The distance between us and the stars is almost beyond comprehension. It takes considerable time for starlight to reach Earth (a full eight minutes from the star closest to us, the Sun). What if light traveled at the speed of a commercial airliner, or if air travel could be completed at the speed of light?

There are many forms of connection, not only between students and the material, but also between cultures, places, and times. To encourage conscious connections in their minds, students were asked to choose a person in their family tree who was not born in the United States (which could of course be the students themselves),\(^{11}\) to find the city and country where that person was born, and to describe a culturally important object unique to that country or region.\(^{12}\)

Next they researched the total time and distance from their present day front door to their ancestor’s birthplace, allowing for approximations where necessary and reasonable. They reported the total distance of each segment (in both miles and kilometers), the time required to travel it (giving the total time in days, in hours, in minutes, and in seconds), and gave the average travel time for the whole trip (including layovers, etc) in miles per hour, miles per second, and kilometers per second, all using scientific notation.

To relate these earthly times and distances to cosmic ones, students computed how long their trip would have taken (in days, hours, minutes, and seconds) if they were able to travel at the speed of light, and found the time that would be required for light to reach the earth from the sun if it traveled at

\(^{11}\)While it is possible for no such person to be known, this did not come up with this particular group of students.

\(^{12}\)The results of this portion of the assignment might have been better if this segment of the assignment was given separately. Many students simply skipped it.
the average velocity of their trip rather than at the speed of light (again in
days, hours, minutes, and seconds). Finally they were asked to determine
how many times they, moving at the speed of light, could have completed a
round trip journey to their ancestor’s home in the time they found it would
take by human transport.

Scientific notation, estimation, and subjecting their results to scrutiny are all
skills that challenge students, particularly those with weak basic numeracy.
Each of these were called upon, providing an opportunity to revisit these
crucial forms of mathematical reasoning.

2. Challenging the Illusion of Separation: Wonders Interwoven

The goals of this part of the project were for students to use familiar mathe-
matical concepts and their visual imagination to learn about different types
of stars and their properties, and to provide students an opportunity to as-
similate information sufficiently deeply to explain a simpler version of what
they understood using analogy with physical objects. The second goal was a
response to watching students consistently struggle to relate abstract math-
ematical concepts to physical equivalents.

Stars change significantly throughout their lifetime. Stars like our sun (a
yellow dwarf) are called main sequence stars and burn by converting hydrogen
into helium via nuclear fusion. Main sequence stars (of which red dwarfs are
the most common) burn for around five billion years before they begin to die.
Our sun is estimated to be 4.5 billion years old. As stars begin to run out of
hydrogen, they swell, forming giants or supergiants, ultimately either fading
away or exploding to leave behind a very small remnant of themselves: a
white, brown, or black dwarf; a neutron star; or a pulsar. For a given set of
star types, students were asked to describe what the different types of stars
might have looked like through Tycho’s telescope and to the naked eye (most
likely using the internet to help), to perform a series of calculations about the
relative sizes of stars, and to design a lesson for fifth graders to teach them
about the relative sizes of stars using physical objects. A comparison of the
type I was hoping for from students: the radius of the sun is 100 times that of
the earth, and thus 1.12–1.3 million Earths could fit into the sun. That’s too
many for physical objects, so if I wanted to make an illustration of relative
size in a classroom, it might be better to compare the sun to Jupiter (almost
930 Jupiter-sized planets would fit inside the sun) and then Earth to Jupiter
(over a thousand, approximately 1,321 Earth-sized planets would fit inside Jupiter). To find physical, everyday objects to help explain this difference in size to fifth graders, for example, requires further computations and perhaps even some measurement. (What is the volume of a basketball? A pea?)

3. You are made of stardust: A Tale of Two Supernovae

Pulsars are still being discovered and actively studied, by both professional and amateur astronomers. The final portion of the project encouraged students to reason with unfamiliar units, distances, and time scales. Supernovae are not immediately visible from Earth; the light must travel from the location of the star in the (expanding) universe to Earth. Thus, how far a star is from Earth is a significant variable in determining which of two supernovae likely occurred first. In this part of the project students were asked to determine which explosion occurred first: the Crab Supernova or Tycho’s supernova, keeping in mind that we have only interval estimates for the likely distances of these stars from Earth.

The technical skills required for this task are trivial, yet many students were unable to do it without substantial guidance. Such struggles suggest that consistent use of assignments that present previously learned technical skills in novel situations could be of significant assistance in establishing a STEM literate society.

To conclude the project, students were asked to spend five undistracted minutes contemplating the fact that we are made of stars, and then to write a paragraph summarizing their response. The students’ reflections on being composed of stardust were engaging and sincere.

References


