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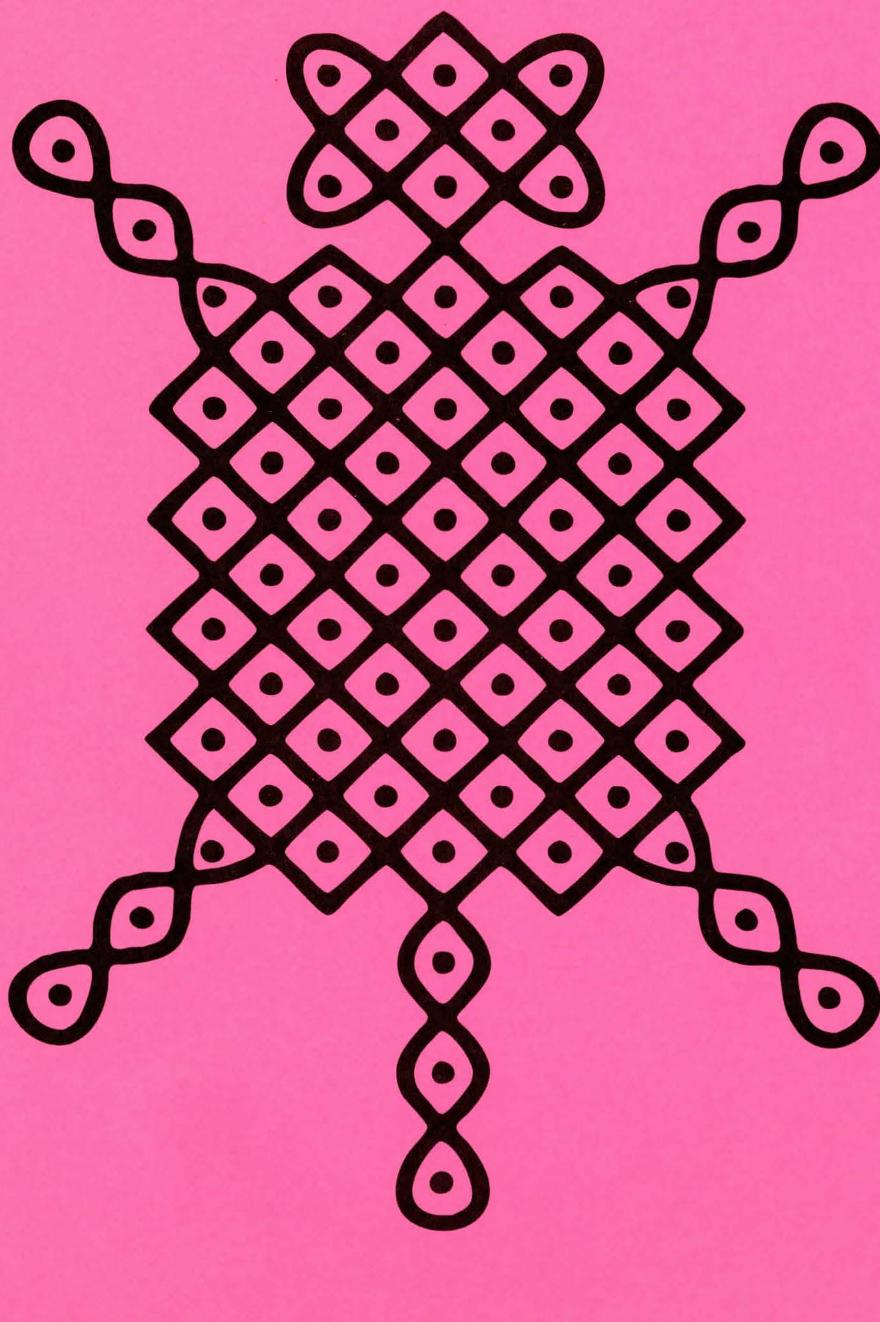

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Essays, book reviews, syllabi, poetry, and letters are welcomed. Your essay should have a title, your name and address, e-mail address, and a brief summary of content. In addition, your telephone number (not for publication) would be helpful.

If possible, avoid footnotes; put references and bibliography at the end of the text, using a consistent style. Please put all figures on separate sheets of paper at the end of the text, with annotations as to where you would like them to fit within the text; these should be original photographs, high quality printouts, or drawn in dark ink. These figures can later be returned to you if you so desire.

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SUBSCRIPTIONS

The Humanistic Mathematics Network Journal is distributed free to all libraries and individuals who request copies and subscriptions. Please address subscription requests to Prof. Alvin White.

COVER

Art or Math?: On the cover is "skin of a leopard," a *sona* sand drawing by the Chokwe people of Africa. Paulus Gerdes explores the mathematics behind *sona* in his book *Geometry from Africa: Mathematical and Educational Explorations*, reviewed by Claudia Zaslavsky on page 65.

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Humanistic Mathematics Network Journal #23
September 2000

From Newsletter #1

Dear Colleague,

This newsletter follows a three-day **Conference to Examine Mathematics as a Humanistic Discipline** in Claremont 1986 supported by the Exxon Education Foundation, and a special session at the AMS-MAA meeting in San Antonio January 1987. A common response of the thirty-six mathematicians at the conference was, "I was startled to see so many who shared my feelings."

Two related themes that emerged from the conference were 1) teaching mathematics humanistically, and 2) teaching humanistic mathematics. The first theme sought to place the student more centrally in the position of inquirer than is generally the case, while at the same time acknowledging the emotional climate of the activity of learning mathematics. What students could learn from each other and how they might come to better understand mathematics as a meaningful rather than arbitrary discipline were among the ideas of the first theme.

The second theme focused less upon the nature of the teaching and learning environment and more upon the need to reconstruct the curriculum and the discipline of mathematics itself. The reconstruction would relate mathematical discoveries to personal courage, discovery to verification, mathematics to science, truth to utility, and in general, mathematics to the culture within which it is embedded.

Humanistic dimensions of mathematics discussed at the conference included:

a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be "merely technical."
b) An appreciation for the human dimensions that motivate discovery: competition, cooperation, the urge for holistic pictures.
c) An understanding of the value judgments implied in the growth of any discipline. Logic alone never completely accounts for what is investigated, how it is investigated, and why it is investigated.
d) A need for new teaching/learning formats that will help discourage our students from a view of knowledge as certain or to-be-received.
e) The opportunity for students to think like mathematicians, including chances to work on tasks of low definition, generating new problems and participating in controversy over mathematical issues.
f) Opportunities for faculty to do research on issues relating to teaching and be respected for that area of research.

This newsletter, also supported by Exxon, is part of an effort to fulfill the hopes of the participants. Others who have heard about the conferences have enthusiastically joined the effort. The newsletter will help create a network of mathematicians and others who are interested in sharing their ideas and experiences related to the conference themes. The network will be a community of support extending over many campuses that will end the isolation that individuals may feel. There are lots of good ideas, lots of experimentation, and lots of frustration because of isolation and lack of support. In addition to informally sharing bibliographic references, syllabi, accounts of successes and failures. . . the network might formally support writing, team-teaching, exchanges, conferences. . . .

Alvin White
August 3, 1987

From the Editor

A report by the Rand Corp. (also reported by the *New York Times* on the web, Wednesday 7/26/2000) concludes that smaller classes in early grades and access to preschool appear to increase student achievement, particularly in impoverished communities, more than salaries, education or experience of teachers.

During World War II women were recruited to work in war production. Most factories had a nursery school and a preschool for the children of the working mothers on the premises. After the war there was a vigorous campaign to force women to leave the factories. Preschool for children was forgotten and discouraged. Nursery schools and preschools, if they existed, were for the wealthy and near-wealthy.

Today in Denmark nursery school and preschool teachers are professionals and well-paid. In the U.S. those are usually entry level jobs.

The Rand Corp. reports seem to be rediscovering what was well-known fifty-five years ago.

Today a prominent subject is under what circumstances children should be "retained a grade" (not promoted). The cause of the children's deficiencies or the efficacy of that idea is, at best, secondary. The fact that most children subjected to that possibility are poor is an embarrassment that politicians prefer not to discuss.

At Franklin Roosevelt's inauguration in the 1930's, he stated that, "One-third of the nation is ill-fed, ill-clothed and ill-housed." In the sixty-seven years that have passed, that fraction has not diminished.

A related story in the July 14 *Los Angeles Times* concerns the high stakes testing called The Stanford 9. This phenomenon seems to be centered in California but published and copyrighted by Harcourt Educational Measurement of San Antonio. The children who are subjected to this test are threatened with retention in grade if their scores are too low. Teachers of poor scoring children are threatened with reduction in salary.

In order to maximize profits by avoiding having to revise or correct the test, the publisher had kept the test secret and sworn to secrecy all who handle it. The *Los Angeles Times* secretly received a copy and gave it to two experts to review, W. James Popham, a professor of education emeritus at UCLA, and Robert Schaeffer, public education director for the National Center for Fair and Open testing. They concluded that the test is fundamentally flawed. "Students' scores are almost certain to be meaningfully contaminated by factors that have little to do with the effectiveness of a teaching staff's instructional efforts,"Popham said.

Popham also said that "California should not reward or punish students, teachers and schools strictly on the basis of test scores as the state plans to do." Some test questions are culturally biased. Some favor children and schools that own computers. At least one of the multiple choice questions has no correct responses!

The fate of children, teachers and schools is hanging on a threadbare string. It seems to me that profound decisions about education and educational policy are being made on the basis of seriously flawed data, secrecy, a refusal to deal with the real, substantial issues, and by an arrogant corps of politicians and bureaucrats.

Will You Still Be Teaching in the Twenty-First Century?

Theodore Panitz
Professor of Mathematics and Engineering
Cape Cod Community College

INTRODUCTION

"Computers are coming! Computers are coming!" is the cry heard around the world as the technology revolution slowly and insidiously works its way into the classroom from kindergarten through higher education. Administrators dream about the economies of inexpensive computer systems handling hundreds of students relatively independently of faculty, with the additional benefit that computers do not debate issues at staff meetings. Lost in the current rush to extol the value of having computers in every classroom, internet courses available to any student anywhere in the world, and complete degrees offered in cyberspace, is a discussion about the real nature of education, the human side. Teaching/learning paradigms at all institutions of higher education must change from lectures to interactive, student-centered inquiry approaches, in order to focus on the human aspects of learning, or else computers will do the teaching for us.

This article is not intended to criticize all uses of technology but instead to promote learning through human interactions. The author believes that appropriate uses of technology lie in supplemental instruction intended to complement classroom activities, not replace them. Computers are useful for building skills, repetition exercises, the search for information via the world wide web, and some communications via e-mail or chat rooms. But technology can never replace the affective nature of education created by face to face interaction between students, and between students and teachers. Unfortunately, the real value of computers as teaching tools has been lost on administrators who only see the apparent economics of scale when they consider the internet as a mechanism to recruit additional students beyond their immediate geographical region.

REDIRECTING THE DISCUSSION AND TRANSFORMING HIGHER EDUCATION

College professors are at a crossroads. We are under increasing pressure to incorporate technology into our courses and to offer extraterrestrial learning environments commonly referred to as cyberspace or internet courses. Simultaneously, we are expected to teach students how to think critically, solve problems and interact socially in preparation for the workplace. Something is missing from the discussion on how higher education should accomplish these goals. The question which should be driving this debate is not how much technology can we include in our teaching, but instead, "What is the underlying philosophy of education and the learning experience?"

Several questions spring to mind and should form the basis for discussions about the future of higher education. They are:

1. Should we facilitate learning through interactive, student-centered courses or focus on information transfer to students? Choosing the latter would enable computer companies to take over higher education teaching responsibilities through information delivery devices such as CD-Roms, the internet, and video courses. Professors would be required to spend all their time on research, thus removing them from contact with undergraduates.
2. Is education a matter of convenience of time and place delivered through the use of the internet where courses can be brought right into students' homes, or should we encourage students to deal with the hard work and personal responsibilities associated with student-centered, interactive learning with their peers?
3. Should we provide our students with as much information as possible, through a professor-centered expert lecture or computer program with the student as a receptor, or should we use inquiry-based, collaborative approaches to learning which provide students with the capability and desire to understand what information they need to make a decision and how to get and use it?
4. Do we wish to create learning environments where the students never see each other or "talk" to the professor except in electronic chat rooms, or

should we focus on harnessing the power of learning though social interactions within the classroom and outside of the class?

5) Can computers present lectures more effectively than professors by using self-paced programs, interactive computer activities, and interesting multimedia elements?

TECHNOLOGY: BOON OR THREAT TO TEACHING IN HIGHER EDUCATION?

The writer believes that the rush, throughout the world, to infuse technology in every course and provide asynchronous internet courses to all students seriously threatens the social aspects of learning and the need for human interaction in the learning process. Collaboration in learning assists students in becoming productive members of the various academic, social and workplace societies they wish to join. For example, people become mathematicians, historians, writers, etc., by learning the vocabulary and culture of their chosen field(s) of study. They must learn how to communicate their ideas to their peers and people outside of their field through writing and oral persuasion. The most effective learning paradigm developed to date involves argumentation, discussion, and consensus building through human interaction. Student-centered classes accomplish this in every class.

Communicating over the internet is only one small tool available to us, and, because it does not come close to providing the human interactions that classrooms do, it should not become the primary delivery system in higher education. I want to be in the classroom with my students, to observe their reactions to learning experiences. I want to observe their body language when they interact with their peers and myself. I want to have individual discussions with students in real time in order to share our experiences regarding learning and life in general. I am not impressed with internet discussions where a smiley face on a computer screen replaces a real smile or capital letters are used to emphasize shouting, etc. Textural communication is unable to convey more subtle aspects of communication, such as specific student questions. One problem arises when people "talk" across a long distance networks. The computer systems occasionally and randomly slow down, so between the time a person types something in and the next person sees it several minutes may elapse, thus delaying the response. Also, the time it takes to formulate a response and type it into the computer may take several minutes versus giving a verbal response in a face to face interaction.

Cyberspace and asynchronous distance learning are being presented as the savior for all of higher education and the future delivery system for colleges and universities. What is the driving force behind the effort to infuse technology into college courses? Initially distance learning was promoted as a way of reaching a few students in remote or inaccessible locations. This is no longer the case. Economics now drives the rush to cyberspace. College administrators envision the internet as a mechanism to reach a vast pool of applicants throughout the world. As they consider the potential market available to them, the dollar signs in their eyes grow exponentially, blinding them to the real basis for learning, human interactions. The fallacy in their reasoning is that it only takes one organization or company to develop and deliver internet courses. Once that organization is established, colleges will no longer be needed to teach students. Computers and education technicians will provide the delivery of course content, exams, paper grading, chat rooms, etc. Technicians will be hired as tutors instead of faculty. The real drive into cyberspace then is the privatization of higher education through corporate America.

In Digital Diploma Mills Parts 1-3 (1993), David Noble, a professor at York University, Toronto, Canada, documents the attempt by large technology corporations to take over undergraduate teaching at colleges and universities. He states: "Recent events at two large North American universities signal dramatically that we have entered a new era in higher education, one which is rapidly drawing the halls of academe into the age of automation." Professors at York University, Toronto, went on strike for two months to secure contractual protections regarding distance learning and technology. Also, the administration at UCLA unilaterally instituted a policy whereby all professors were required to incorporate web sites into their arts and sciences courses. This was done over the summer when most professors were not on campus. A virtual degree is available through Arizona University, and consortiums have been developed through the Western Governors Association and the California Board of Higher education to broker Internet courses. Noble has identified the key issue:

"Thus, at the very outset of this new age of higher education, the lines have already been drawn in the struggle which will ultimately determine its shape. On the one side university administrators and the myriad commercial partners, on the other those who constitute the core relation of education: students and teachers. (The chief slogan of the York faculty during the strike was "the classroom vs the boardroom"). It is no accident, then, that the high-tech transformation of higher education is being initiated and implemented from the top down, either without any student and faculty involvement in the decision-making or despite it. At UCLA the administration launched their Initiative during the summer when many faculty are away and there was little possibility of faculty oversight or governance; faculty were thus left out of the loop and kept in the dark about the new web requirement until the last moment. And UCLA administrators also went ahead with its Initiative, which is funded by a new compulsory student fee, despite the formal student recommendation against it. Similarly, the initiatives of the York administration in the deployment of computer technology in education were taken without faculty oversight and deliberation, much less student involvement." (p. 1)

It is clear that there is an effort being undertaken to commercialize higher education, not just transform it by the infusion of technology. Noble hypothesizes that technology represents the second phase of this commercialization. The first phase took place starting in the mid-1970's and involved commercialization of course content through research, patents, textbooks and degree requirements. The second phase was initiated when industry realized that information creation and use would be the next major commodity and that knowledge-based industries, such as colleges and universities, would be the next major economic area for development. Noble states:

"Within a decade was a proliferation of industrial partnerships and new proprietary arrangements, as industrialists and their campus counterparts invented ways to socialize the risks and costs of creating this knowledge while privatizing the benefits." (p. 2)

"Class sizes swelled, teaching staffs and instructional resources were reduced, salaries were frozen, and curricular offerings were cut to the bone. At the same time, tuition soared to subsidize the creation and maintenance of the commercial infrastructure (and correspondingly bloated administration) that has never really paid off. In the end students were paying more for their education and getting less, and the campuses were in crisis." (p. 2)

"The second phase of the commercialization of academia, the commoditization of instruction, is touted as the solution to the crisis engendered by the first. Ignoring the true sources of the financial debacle—an expensive and low-yielding commercial infrastructure and greatly expanded administrative costs—the champions of computer-based instruction focus their attention rather upon increasing the efficiencies of already overextended teachers. And they ignore as well the fact that their high-tech remedies are bound only to compound the problem, increasing further, rather then reducing, the costs of higher education." (p. 2)

Who is behind the effort to commercialize college instruction and materials? Noble identifies four special interest groups behind this effort. The first are the vendors of the computer software and hardware. The second are corporate training advocates who view training from an economical high speed, highly specialized perspective. Third are the university administrators who wish to be considered up to date with the most modern educational systems. Fourth are the technological "zealots" who view computers as the solution to every problem and simply enjoy working with them.

What are the implications of this attempt to shift learning onto computers and the Internet? There will be a shift away from the classroom and contact with other students and the professor toward anytime, anywhere learning. Technology will mean the extension of working time since the professor can be reached by e-mail twenty four hours a day, and students will expect quick responses. Classes may be administratively monitored more closely through record and data keeping by the computers. Once courses are on the com-

puter, the originating professor will no longer be needed. These negative consequences in part explain the strong reaction by the professors at York University. The other problem in the rush to implement technology is that students and faculty are being left out of the discussions. Perhaps administrators understand the potential negative consequences of the misuse of technology and therefore make every effort to utilize the power of their positions to implement technological strategies without input from the very constituencies who will be most effected.

In a follow-up article to Noble's article, Michael Margolis (1997) clearly identifies the driving force behind the commoditization of university instruction and the consequences this will have on the future of teaching in the university. Margolis states the " Market capitalism, not the Internet per se, is the force behind developing the wired university." He believes that students will embrace distance learning because of the financial benefits they will receive, partially through reduced tuition and elimination of other expenses associated with taking courses on college campuses. He states, "A college degree from an accredited program will suffice—the cheaper the better—as long as it increases a student's chance of securing a decent first job to help pay back his or her loans. The "hightech" universities of the next century will be hailed as yet another triumph of the free market." (p. 1)

"

The other problem in the rush to implement technology is that students and faculty are being left out of the discussions.

In order to achieve economic nirvana universities will need to implement actions to save money. Margolis states, "With proper planning, the savings generated from eliminating lecture halls, classrooms, and most undergraduate laboratories should be second only to those realized from downsizing faculty and outsourcing courses." (p. 2) In addition, costly libraries and computer centers can be eliminated by using online, digitized libraries accessible through the internet. The true intent of the technology companies is captured by Margolis in the following quotes.

> "The beauty of this power emerges not merely from customer convenience, however. It offers better quality instruction as well. As the Internet reaches a global market, local universities no longer need to limit their course instruction to their own—and let's face it—sometimes mediocre faculty, instead, they can offer choice among the world's greatest instructors online.
>
> "Once arrangements for outsourcing the desired courses have been made with the managers and instructors at the appropriate institutions, local universities can effectively become franchises of greater institutions. They can offer their customers the finest courses of instruction from Harvard, Oxford or Heidelberg, or if their customers so desire, from Hillside, Liberty Baptist or Motorola. And, because they won't need to maintain many faculty to teach on their own campuses, they can offer these courses at a fraction of their present cost. The market will determine the best courses to offer, and the economics of scale will afford even greater savings." (p. 3)
>
> "To sum up, then, the commodification of higher educational training provides the impetus for reform of costly practices of American universities. To survive in the global market universities need to implement four types of reform:
>
> 1. downsizing faculty by replacing classroom lectures with both asynchronous and simultaneous sessions on the Internet;
> 2. minimizing the need for instructional laboratories, lecture halls, and other physical spaces for teaching on campus;
> 3. cutting research costs through the use of digital libraries and networked computers, eliminating valueless scholarship, and charging a fair price for support services that universities formerly gave for free;
> 4. ending tenure as we know it and using appropriate economic criteria to evaluate each professor's teaching, research and community service.

Finally, universities can supplement these reforms with expanded investment in recreational facilities and in varsity athletic enterprises." (p. 6)

"In order to succeed with implementing all of these reforms, university managers will have to overcome the troglodytes who resist marketing higher education as a commodity. These reactionaries argue that education in the arts and sciences is also an experience that provides worthwhile non-material benefits that enrich a person's time, and they often cite philosophies of education that run back at least to Thomas Jefferson. In the global economy, however, customers see higher education as training and credentialing to secure jobs that provide better remuneration. The American public understands that every major endeavor—with the possible exception of religion—needs to be evaluated on a commercial basis." (p. 6)

ALTERNATIVES TO LECTURING: INTERACTIVE, STUDENT-CENTERED, INQUIRY LEARNING

Lecturing is used by most professors in higher education as their principal teaching strategy. This has created the rationale for replacing lectures with information delivered by computers. If we can replace professors with teaching assistants in recitation sections, then the next step is easy, replace professors with videos of the one best lecturer and use computers and technology assistants as backups. Aside from threats of obsolescence, pedagogically lecturing is a flawed approach to teaching and must be replaced by more effective teaching paradigms. David Johnson et al (1998) have identified six specific pedagogical problems associated with lecturing. They are:

1. Students' attention to what the lecturer is saying decreases as the lecture proceeds. Students concentrate and assimilate material for 10 minutes, whereupon their attention falls off rapidly.
2. For a lecture to be effective, it takes an educated, intelligent person oriented toward an auditory learning style.
3. Lecturing tends to promote only lower-level learning of factual information.
4. Lecturing is limited by the assumption that all students need the same information presented orally at the same time and at the same pace, without dialogue with the presenter, and in an impersonal way.
5. Students tend not to like lecturing.
6. Lecturing is based upon a series of assumptions about the cognitive capabilities and strategies of students. It assumes that all students learn auditorially, have high working memory capacity, have all required prior knowledge, have good note-taking strategies and skills, and are not susceptible to information processing overload.

It is clear that the simple transmission of information through a lecture is not an effective approach to meeting the goals of helping students become independent, critical problem solvers, able to interact with their peers in social and employment situations.

The Boyer Commission (1998), sponsored by the Carnegie Foundation for the Advancement of Teaching, spent several years analyzing research universities. Their efforts resulted in a report titled Reinventing Undergraduate Education. The report is highly critical of the current undergraduate teaching approaches at universities. It identified changes that have taken place in research universities which will require changes in how those institutions view education and teaching. The Boyer Commission does not address content issues but instead draws a general conclusion about the need for research universities to re-evaluate their teaching paradigms. In order to accomplish this, the report recommends ten ways undergraduate education must change to meet the needs of our students, society and the work place. The report calls for inquiry-based collaborative learning to replace lecturing as the principal educational paradigm. The report makes the following observations.

"Dr. Boyer set the tone for the deliberations by reminding the Commission that conditions in higher education have changed significantly in recent years: the American system of higher education has become less elite; students (and parents) have developed their own, often vigorously asserted, ideas about education and credentialing rather than accepting traditional modes without question; a much greater range of undergraduate professional degrees has become available; the freshman year has too often been reduced to remediation or repetition of high school curriculum, rather than an

introduction to a new and broader arena for learning." (p. 2)

"But, research universities share a special set of characteristics and experience a range of common challenges in relation to their undergraduate students. If those challenges are not met, undergraduates can be denied the kind of education they have a right to expect at a research university, an education that, while providing the essential features of general education, also introduces them to inquiry-based learning." (p. 3)

The Boyer Commission points out that for economic reasons universities and colleges have focused on research as their primary function and thus have failed their undergraduate populations. Tuition is a major source of support for research programs which support graduate students. Teaching is not the primary interest of university administrators, leading to the conclusion that:

"Some of their instructors are likely to be badly trained or even untrained teaching assistants who are groping their way toward a teaching technique; some others may be tenured drones who deliver set lectures from yellowed notes, making no effort to engage the bored minds of the students in front of them." (p. 5)

"Many students graduate having accumulated whatever number of courses is required, but still lacking a coherent body of knowledge or any inkling as to how one sort of information might relate to others. And, all too often they graduate without knowing how to think logically, write clearly, or speak coherently. The university has given them too little that will be of real value beyond a credential that will help them get their first jobs. And with larger and larger numbers of their peers holding the same paper in their hands, even that credential has lost much of its potency." (p. 5)

"The primacy of research within the espoused missions of American universities is attested over and over within the academic world. The standing of a university is measured by the research productivity of its faculty; the place of a department within the university is determined by whether its members garner more or fewer research dollars and publish more or less noteworthy research than other departments; the stature of the individual within the department is judged by the quantity and quality of the scholarship produced. Every research university can point with pride to the able teachers within its ranks, but it is in research grants, books, articles, papers, and citations that every university defines its true worth. When students are considered, it is the graduate students that really matter; they are essential as research assistants on faculty projects, and their placement as post-doctoral fellows and new faculty reinforces the standing of the faculty that trained them. Universities take great pleasure in proclaiming how many of their undergraduates win Rhodes or other prestigious scholarships and how many are accepted at the most selective graduate schools, but while those achievements are lauded, too many students are left alone to pursue them. And the baccalaureate students who are not in the running for any kind of distinction may get little or no attention." (p. 6)

What then is the answer to changing the environment of the university? The Commission suggests that,

"The ecology of the university depends on a deep and abiding understanding that inquiry, investigation, and discovery are the heart of the enterprise, whether in funded research projects or in undergraduate classrooms or graduate apprenticeships. Everyone at a university should be a discoverer, a learner. That shared mission binds together all that happens on a campus. The teaching responsibility of the university is to make all its students participants in the mission. Those students must undergird their engagement in research with the strong "general" education that creates a unity with their peers, their professors, and the rest of society." (p. 7)

In addition,

"Undergraduates who enter research universities should understand the unique quality

> of the institutions and the concomitant opportunities to enter a world of discovery in which they are active participants, not passive receivers. Although shared knowledge is an important component of a university education, no simple formula of courses can serve all students in our time. Collaborative learning experiences provide alternative means to share in the learning experiences, as do the multitudinous resources available through the computer. The skills of analysis, evaluation, and synthesis will become the hallmarks of a good education, just as absorption of a body of knowledge once was. (p. 8)

The commission states emphatically that undergraduate education will need to change to inquiry-based paradigms and move away from the lecture format of classes.

> "The inquiry-based learning urged in this report requires a profound change in the way undergraduate teaching is structured. The traditional lecturing and note-taking, certified by periodic examinations, was created for a time when books were scarce and costly; lecturing to large audiences of students was an efficient means of creating several compendia of learning where only one existed before. The delivery system persisted into the present largely because it was familiar, easy, and required no imagination. But education by inquiry demands collaborative effort; traditional lecturing should not be the dominant mode of instruction in a research university.
>
> The experience of most undergraduates at most research universities is that of receiving what is served out to them. In one course after another they listen, transcribe, absorb, and repeat, essentially as undergraduates have done for centuries. The ideal embodied in this report would turn the prevailing undergraduate culture of receivers into a culture of inquirers, a culture in which faculty, graduate students, and undergraduates share an adventure of discovery." (p. 11)

"*As a pedagogy, CL involves the entire spectrum of learning activities in which groups of students work together in or out of class.*

IDENTIFYING THE IMPORTANCE AND VALUE OF STUDENT-CENTERED LEARNING BY ANALYZING COOPERATIVE LEARNING PARADIGMS

We in higher education cannot compete with the big computer software companies in the production of technology oriented bells and whistles meant to enliven the transfer of information to students. We can compete by changing our pedagogy by moving away from the lecture format and making the students the center of the learning experience.

There are many interactive learning paradigms which could be used to create student-centered courses, giving professors a choice in their approaches to teaching. Cooperative learning, collaborative learning, problem or project base learning and inquiry-based learning are just a few of the categories of interactive, student-centered learning paradigms. Within each of these are a variety of structures available to professors. Student-centered learning is not merely a new fad or single approach to teaching that must be adopted by all professors but a philosophy which would allow professors to experiment with a variety of approaches.

As an example, cooperative learning (CL), as personal philosophy, not just a classroom technique, assumes that in all situations where people come together in groups, there are ways of dealing with each other which respects and highlights individual group members' abilities and contributions. The underlying premise of CL is based upon consensus building through cooperation by group members, in contrast to competition in which individuals best other group members. CL practitioners apply this philosophy in the classroom, at committee meetings, with community groups and generally as a way of living with and dealing with other people (Panitz & Panitz 1998).

As a pedagogy, CL involves the entire spectrum of learning activities in which groups of students work together in or out of class. It can be as simple and in-

formal as pairs working together in a Think-Pair-Share procedure, where students consider a question individually, discuss their ideas with another student to form a consensus answer, and then share their results with the entire class, to the more formally structured process known as cooperative learning which has been defined by Johnson and Johnson (Johnson, Johnson & Holubec 1990).

Nelson-LeGall (1992) captures the nature of cooperative learning when she states, "Learning and understanding are not merely individual processes supported by the social context; rather they are the result of continuous, dynamic negotiation between the individual and the social setting in which the individual's activity takes place. Both the individual and the social context are active and constructive in producing learning and understanding." (p. 52) Fogarty and Bellanca (1992) highlight the reaction that teachers have after they implement cooperative learning paradigms when they state, "Surprisingly and almost unfailingly, once the philosophical shift begins, once teachers begin implementing cooperative interactions, the evidence of student motivation becomes so overwhelmingly visible that teachers are encouraged to try more. The momentum builds for both teachers and students, and before long the "new school lecture" becomes the norm in the classroom. By then, the novelty of the models is no longer the challenge. The challenge becomes choosing the most appropriate interactive designs for the target lesson; it is choosing a design in which the final focus rests on the learner, not on the lecturer." (p. 84)

Cooperative learning is perhaps the most thoroughly studied teaching and learning paradigm (Johnson & Johnson 1989) with over 600 studies reported at all levels of education. The benefits which accrue from student-centered cooperative learning (CL) paradigms are many (Panitz & Panitz 1998, Panitz 1999). Several key benefits will be highlighted in this paper to emphasize the importance of student collaboration in the learning process.

CL DEVELOPS HIGHER LEVEL THINKING SKILLS (WEBB 1982)

Students working together are engaged in the learning process instead of passively listening to the teacher present information or reading information off a computer screen. Pairs of students working together represent the most effective form of interaction, followed by threesomes and larger groups. When students work in pairs, one person is listening while the other partner is discussing the question under investigation. Both are developing valuable problem solving skills by formulating their ideas, discussing them, receiving immediate feedback and responding to questions and comments by their partner (Johnson, D.W. 1971). The interaction is continuous, and both students are engaged during the session. Compare this situation to the lecture class where students may or may not be involved by listening to the teacher or by taking notes (Cooper, et al 1984). O'Donnell et al (1988) found that the initial benefits that accrued from a brief cooperative training experience persisted over relatively long intervals and that students trained in the dyadic cooperative approach successfully transferred their skills to individually performed tasks (McDonald et al 1985).

CL STIMULATES CRITICAL THINKING AND HELPS STUDENTS CLARIFY IDEAS THROUGH DISCUSSION AND DEBATE (JOHNSON 1973, 1974)

The level of discussion and debate within groups of three or more and between pairs is substantially greater than when an entire class participates in a teacher-led discussion. Students receive immediate feedback or questions about their ideas and formulate responses without having to wait for long intervals to participate in the discussion (Peterson & Swing 1985). This aspect of collaborative learning does not preclude whole class discussion. In fact, whole class discussion is enhanced by having students think out and discuss ideas thoroughly before the entire class discusses an idea or concept. The level of discussion becomes much more sophisticated. In addition, the teacher may temporarily join a group's discussion to question ideas or statements made by group members or to clarify concepts or questions raised by students. Nelson-LeGall (1992) comments on the value of debate in enhancing critical thinking skills in students. She states, "An awareness of conflicting viewpoints appears to be necessary in collaborative groups to engender the type of peer transactions (e.g., arguments, justifications, explanations, counter arguments) that foster cognitive growth (Brown & Palinscar, 1989)." (p. 55)

Another benefit of cooperative discussion is the effect it has on students who peer edit written work. According to McCarthey and McMahon (1992) "Research focusing specifically on revision when peers

respond to and edit writing has revealed that students can help one another improve their writing through response. Nystand (1986) found that students who responded to each other's writing tended to reconceptualize revision, not as editing, but as a more substantive rethinking of text, whereas students who did not work in groups viewed the task as editing only." (p. 19) Combining discussion with peer editing results in an important aspect of developing critical thinking skills in students.

SKILL BUILDING AND PRACTICE CAN BE ENHANCED AND MADE LESS TEDIOUS THROUGH CL ACTIVITIES IN AND OUT OF CLASS (TANNENBERG 1995)

Foundational aspects of education, the acquiring of information and operational skills, can be facilitated through the use of collaborative activities (Brufee 1993). In order to develop critical thinking skills, students need a base of information to work from. Acquiring this skills base often requires some degree of repetition and memory work. When this is accomplished individually the process can be tedious, boring or overwhelming. When students work together the learning process becomes interesting and fun despite the repetitive nature of the learning process.

Tannenberg (1995) states:

> "The most significant benefit that I have observed using CL has been for students to engage in the skills and practices of the computing discipline within the classroom. These practices include reading and understanding programs, designing and writing programs, complexity analysis, problem solving, writing proofs, scholarly debate, teaching one another, negotiating meaning, using alternate forms of representation (e.g., drawings of trees, graphs, and other data structures), and building collegial relationships. In a lecture-based setting, we are limited to the extent to which we can convey skills and practices—many of these do not lend themselves well to verbal description. And even for those that do, students appropriate such skills through active engagement, not by watching and listening. By working students can be encouraged and helped by their peers and the instructor within a small group setting, and they learn from one another by watching and imitating."

CL DEVELOPS ORAL COMMUNICATION SKILLS (YAGER 1985)

When students are working in pairs, one partner verbalizes his/her answer while the other listens, asks questions or comments upon what he/she has heard. Clarification and explanation of one's answer is a very important part of the collaborative process and represents a higher order thinking skill (Johnson, Johnson, Roy, Zaidman 1985). Students who tutor each other must develop a clear idea of the concept they are presenting and orally communicate it to their partner.

Tannenberg (1995) describes the benefit of developing oral skills which are discipline specific.

> "As in other disciplines, computer scientists use specialized language to economically and precisely communicate with one another. This involves not only mathematical symbols and programming languages, but additional terms and special uses of natural language. A consequence of having students work together in small groups is that they speak with one another and directly engage in discipline-specific language use. In trying to explain their ideas relating to the problems that they are solving, whether it be about a graph, program, algorithm, or proof, they will of necessity acquire the terms that describe these objects."

The additional benefit in having our students be fluent language users is that they can then enter into the culture of our disciplines. They will be able to understand specialized publications and talk with more knowledgeable practitioners. That is, acquiring the language of the discipline opens the portal to the vast store of knowledge within the discipline. We should therefore not minimize the value of having our students be able to talk with one another about their work in the disciplines that we teach. The social setting of CL provides this opportunity. And this is where it may be better that the students are interacting with one another rather than with experts, because they are less concerned about looking foolish, about being novices, about not being fluent in the new language and discipline, about being tourists in this foreign land—how easy it is to chat with other tourists!

CL FOSTERS METACOGNITION IN STUDENTS

Metacognition involves student recognition and

analysis of how they learn (0'Donnell & Dansereau 1992). Metacognition activities enable students to monitor their performance in a course and their comprehension of the content material. This includes detecting errors and learning how to make corrections while monitoring one's performance. Cooperative learning approaches create learning strategies which are independent of content and thus are transferable to different content areas. Cooperative learning structures encourage the development of metacognitive learning because they focus on the process of learning, which includes the evaluation of the group's work by individual group members, assessment and improvement of the social interactions which take place during cooperative activities, and efforts to make corrections in each individual's performance. The content matter is almost secondary to the learning process.

> "
> ***Students who participate in structuring the class assume ownership of the process because they are treated like adults, and their opinions and observations are respected by the authority figure in the class.***

For example, Scripted Cooperation, a cooperative structure developed by O'Donnell and Dansereau (1992), includes five generic components which are helpful in the metacognition process: 1. dividing the text into discrete and meaningful sections, 2. having both members of a dyad read the text a section at a time, 3. requiring one partner to recall the pertinent details and information, 4. requiring the other partner to monitor this oral recall to detect errors and omissions (these two roles are evenly interchanged throughout the text), and 5. having both members of the dyad elaborate on this information with methods that may include developing analogies and generating images (Hertz-Lazarowitz, Kirkus and Miller (1992) (p. 7).

COOPERATIVE DISCUSSIONS IMPROVE STUDENTS' RECALL OF TEXT CONTENT DANSEREAU (1985); SLAVIN & TANNER (1979)

When students read a text together, explain the concepts to each other and evaluate each other's explanations, they engage in a high level of critical thinking. They frame the new concepts by using their own vocabulary and by basing their comments upon their previous knowledge. Thus, they construct a new knowledge base on top of their existing base. This process leads to a deeper understanding and greater likelihood they will retain the material longer than if they worked alone and simply read and reread the text. Johnson & Johnson (1979) found that engaging in discussion over controversial issues improves recall of important concepts. Ames and Murray (1982) found that discussion of controversial ideas among pairs of nonconservers on Piagetian conservation tasks improves their recall of content material.

CL INVOLVES STUDENTS IN DEVELOPING CURRICULUM AND CLASS PROCEDURES (KORT 1992)

During the collaborative process students are asked to assess themselves and their groups as well as class procedures. Teachers who are confident in themselves can take advantage of this student input to modify the makeup of groups or class assignments and alter the mix of lecture and group work according to immediate student feedback. The teacher does not have to wait until the results of the section exam are returned to make alterations which will help the students understand the material. Students who participate in structuring the class assume ownership of the process because they are treated like adults, and their opinions and observations are respected by the authority figure in the class (Meier, M. & Panitz, T., 1996).

Marzano (1992) identifies four specific ways in which students become involved in developing class procedures when cooperative learning is the basis for class processes. The class can identify desired features of the physical environment, such as the arrangement of desks, number and type of breaks that will be taken, the display of classroom accessories—to name a few. Students can analyze the affective tone of their groups and suggest activities which will promote positive interactions or deal with conflicts or personality problems within each group. The class may be given responsibility for developing and implementing classroom rules and procedures. Students can help establish and implement rules for physical and psychological safety of their peers, such as a code of conduct which encourages students to respect each other, listen and respond attentively and generally care for their fellow students.

CL PROVIDES TRAINING IN EFFECTIVE TEACHING STRATEGIES TO THE NEXT GENERATION OF TEACHERS (FELDER 1997)

As discussed earlier, new teachers are likely to teach using the teaching style they have been exposed to during their education. The primary paradigm at universities is the lecture method combined with a competitive assessment process involving individual exams graded on a curve. If teachers had more exposure and practice using CL methods and were able to observe the significant benefits and student reactions, they would be more inclined obtain additional training and to try these techniques in their classes.

CL HELPS STUDENTS WEAN THEMSELVES AWAY FROM CONSIDERING TEACHERS THE SOLE SOURCES OF KNOWLEDGE AND UNDERSTANDING (FELDER 1997)

One reason for teacher reticence in adopting CL methods is the fact that professors have spent a lifetime developing their expertise in a subject, leading them to feel that their primary function is to impart that knowledge to their students. This, after all, is how they perceive they learned the subject material when doing their undergraduate studies. In reality, teachers become experts in their field when they teach the concepts to others and undertake research activities where they attempt to communicate their findings with their peers. Informal discussion and debate often yields more productive research breakthroughs than attending lectures.

CL approaches learning from a student-centered philosophy by encouraging students to take responsibility for their learning by involving students throughout the class and encouraging their collaboration in group efforts outside of class. The teacher serves as a resource and facilitator rather than as an expert. It is not a passive role for the teacher. CL requires a great deal of planning and preparation on the part of the teacher to develop activities which will help guide students through the curriculum. The effect is to begin to elevate students to the teacher's level and create a high expectation that they have the ability to obtain understand knowledge themselves.

CL ALLOWS STUDENTS TO EXERCISE A SENSE OF CONTROL ON TASK (SHARAN AND SHARAN)

The interactive, hands-on nature of CL exercises places the students in a position of control over the process and encourages them to take full responsibility for the outcome of particular assignments. Students receive training in social skill building, conflict resolution and team management. The locus of control is with the student because the teacher serves as facilitator, not director. Students are given a great deal of leeway to decide how they will function and what their group's product will be. CL empowers students to take control over their education.

CL PROMOTES INNOVATION IN TEACHING AND CLASSROOM TECHNIQUES (SLAVIN 1980, 1990)

Collaborative learning processes include class warm-up activities, name recognition games and group building activities, and group processing. Students work in pairs or larger groups depending upon the task at hand. Group work on content takes many forms, including pairs or groups working on individual questions, problem assignments, projects, study activities, group tests, etc. (Panitz 1996). Classes are interesting and enjoyable because of the variety of activities available for use by the teacher. In fact, collaborative learning effectively addresses the "Sesame Street" syndrome in which modern students are used to being exposed to information in short, entertaining sessions. These same students are also used to high-tech computer systems which deliver material in a variety of ways including video, text, graphical illustrations, and interactive systems. Collaborative learning effectively matches or exceeds the above approaches to learning by actively involving every student. Bean (1996) points out that CL techniques can be easily integrated with other teaching strategies.

CL ADDRESSES LEARNING STYLE DIFFERENCES AMONG STUDENTS (MIDKIFF & THOMASSON 1993)

Students working in collaborative classes utilize each of the three main learning styles: kinesthetic, auditory and visual. For example, material presented by the teacher is both auditory and visual. Students working together use their kinesthetic abilities when working with hands on activities. Verbal and auditory skills are enhanced as students discuss their answers together. Visual and auditory modalities are employed when students present their results to the whole class. Each of these learning styles are addressed many times throughout a class in contrast to the lecture format which is mainly auditory and occasionally visual.

CL ENCOURAGES DIVERSITY UNDERSTANDING (BURNSTEIN & MCRAE 1962)

Understanding the diversity that exists among students of different learning styles and abilities is a major

benefit of collaborative learning. Lower level students benefit by modeling higher level students, and they benefit by forming explanations and tutoring other students (Swing, Peterson 1982; Hooper & Hannafin 1988). Higher level students benefit by explaining their approaches. Students observe their peers in a learning environment, discuss problem solving strategies and evaluate the learning approaches of other students. Often behaviors which might appear odd when taken out of context become understandable when the opportunity is presented to students to explain and defend their reasoning. For example, Americans signal agreement by nodding vertically while students from India nod horizontally. Very little opportunity exists for students to explain their behavior in a lecture class, whereas in a CL environment discussions of this nature occur continuously. Warm-up and group building activities play an important role in helping students understand their differences and learn how to capitalize on them rather than use them as a basis for creating antagonism.

CL HELPS MAJORITY AND MINORITY POPULATIONS IN A CLASS LEARN TO WORK WITH EACH OTHER (DIFFERENT ETHNIC GROUPS, MEN AND WOMEN, TRADITIONAL AND NON-TRADITIONAL STUDENTS (FELDER 1997, JOHNSON & JOHNSON 1972)

Research into the effect of using cooperative learning with students of varied racial or ethnic backgrounds has shown that many benefits accrue from this method (Slavin 1980). Because students are actively involved in exploring issues and interacting with each other on a regular basis in a guided fashion, they are able to understand their differences and learn how to resolve social problems which may arise. Training students in conflict resolution is a major component of cooperative learning training (Aronson 1978; Slavin 1991).

CL BUILDS SELF ESTEEM IN STUDENTS (JOHNSON & JOHNSON 1989)

Collaborative efforts among students result in a higher degree of accomplishment by all participants as opposed to individual, competitive systems in which many students are left behind (Slavin 1987). Competition fosters a win-lose situation where superior students reap all rewards and recognition and mediocre or low-achieving students reap none. In contrast, everyone benefits from a CL environment. Students help each other and in doing so build a supportive community which raises the performance level of each member (Kagan 1986). This in turn leads to higher self esteem in all students (Webb 1982).

CL PROVIDES A BASIS FOR ALTERNATE FORMS OF ASSESSMENT (ROSENSHINE & STEVENS 1986) SUCH AS OBSERVATION OF GROUPS (PANITZ AND PANITZ (1997), GROUP SELF ASSESSMENT, AND SHORT INDIVIDUAL WRITING ASSESSMENTS (ANGELO AND CROSS 1993)

Collaborative learning provides the teacher with many opportunities to observe students interacting, explaining their reasoning, asking questions and discussing their ideas and concepts. These are far more inclusive assessment methods than relying on written exams only. In addition, group projects provide an alternative for those students who are not as proficient in taking written tests based upon content reproduction. Also, group tests give students an alternate way of expressing their knowledge, by first verbalizing their solution to their partner or group prior to formalizing a written response.

POLICY RECOMMENDATIONS

Administrators from the president of each college to department chairs must set a new tone in the discussion of what learning means by encouraging faculty to learn about student-centered learning paradigms and by providing the resources to make the transition from lecturing a reality. Faculty need to be encouraged to involve students in every aspect of the teaching/learning process and move away from the sage on the stage role they now play. If administrators spent half the time and energy the now use to promote technology instead to encourage faculty to use student-centered learning paradigms, we could transform our colleges and universities into true institutions of learning.

The following policies are needed for full implementation of student centered, interactive learning paradigms in our colleges and universities.

Policy 1: Support and encouragement must come from the highest policy making and financial boards and from the chief executive at the institution. Boards of trustees and presidents must embrace CL as a high system priority. They must be willing to provide the resources needed to implement CL in the form of training opportunities and materials. If possible, the CEO should participate in administrative training sessions. The CEO must provide the leadership in order to create an environment supportive of CL.

Policy 2: Teachers must be involved from the start in planning for CL and throughout the process of implementing CL in their classes. Even though the initial impetus must come from the top levels of administration, the development work must be done by the teachers and department level administrators to guarantee its effectiveness.

Policy 3: Funding must be adequate to provide for training workshops, conferences, teacher presentations at conferences and in-house, release time for initial preparation, on-campus activities, materials for use in class and continuous training.

Policy 4: Textbook manufacturers must be involved in the conversion to CL by providing supplemental materials in the form of worksheets, handouts describing group activities, and faculty training materials. Eventually professors will develop materials unique to their courses; however, this process will take several years and an interim approach is needed. Publisher materials will also help model CL handouts for teachers who are just beginning to develop their own materials.

Policy 5: A support group mechanism must be developed and encouraged to involve teachers in the initial development process and in the initial training activities. Meeting times and facilities must be provided along with mentors to help the new groups function.

Policy 6: Teachers need to be encouraged to adopt CL in a risk-free environment. The teacher evaluation process must be modified to take into account the different teaching methods used, and student testing through standardized tests must be re-evaluated. Alternative forms of assessment will have to be introduced and accepted in order to provide an accurate assessment of the outcomes of CL.

Policy 7: CL should be modelled in institutional decision making. Meetings should be facilitated in a CL manner. Few leaders appear willing to delegate the power to teachers needed to implement institutional change. If we desire teachers to delegate power to their students and give up the control afforded by lectures, then administrators must be willing to make the same changes.

Policy 8: Administrators and supervisors should be trained in CL and group dynamics (Cohen 1986) in order to be able to evaluate it and model it for the teachers. This goal can be accomplished through seminars, by observing experienced teachers, by taking courses in CL and through inservice training (Noddings 1989).

Policy 9: A CL library should be established within the institution, and materials provided by teachers should be archived for use by other teachers. Funding must be provided for training materials, books, videotapes, journals, etc.

Policy 10: Students should be involved in the process through a student council, advisory group or college committee assignments. The student leaders should receive training in CL also via workshops and in-school activities.

Policy 11: The general student population should receive training in conflict resolution, group dynamics and proper social behavior. Teachers need to be trained in these techniques also. An institutional philosophy of cooperation and conflict resolution must to be established.

Policy 12: Teacher training colleges and universities must emphasize CL as the primary teaching paradigm and hire professors who can teach using CL methodology. Teachers will follow the same model they were taught by, which explains why the lecture method is predominant. CL must be modeled in every college class in order to establish this method in teachers' minds.

Policy 13: Colleges must adopt CL as the primary learning method in order to encourage secondary and primary teachers to follow suit. Secondary teachers use the lecture format because they feel they must train their students to succeed at the college level.

Policy 14: CL must be implemented at all grade levels. College professors bemoan the fact that students weren't trained in CL at the secondary level, high school teachers criticize junior high teachers, who in turn suggest that primary teachers need to start the process. We can't wait 12 years for the first class to start using cooperative learning when they reach college.

Policy 15: Absolute grading instead of grading on a curve must be adopted by the institution, and alternate forms of assessment (such as group grades and portfolios) must be encouraged. The bell curve grading system by its very nature fosters competition, restricts collaboration, and leads to anxiety among students.

Policy 16: Curriculum planning and instruction must go hand in hand. "When a curricula is created, instruction must be considered, and when instruction is planned, curriculum materials must be appropriate for the mode of instruction" (Noddings 1989).

Policy 17: Facilities must be provided which are conducive to CL. Lecture halls with fixed ampitheater type seating makes student interaction difficult. Rows of desks neatly lined up are an anathema to CL. Moveable chairs and/or tables where students can work together must be provided. Tables large enough to seat 5 people would be ideal. Classrooms must be large enough to enable the professor to move easily about the room when interacting with the groups.

Policy 18: Teachers who are just beginning to use CL must be placed in an environment which will foster success, remove anxiety producing environments and encourage a major change in teaching style. In order to accomplish this financing must be provided to maintain small class sizes.

In conclusion, as we enter the 21st century professors will be forced to change from the comfortable and familiar lecture style of teaching to a student-centered cooperative mode if they wish to remain relevant. Technologies undreamed of even 10 years ago will usurp the factual, mechanical information delivery systems exemplified by the lecture, making the "talking head" professor obsolete. However, machines will never be able to replace what makes up the heart of the education experience: the development of seasoned, critical reasoning and thinking skills, obtained through face to face discussion, disputation, and deliberation with other living human beings. The cooperative classroom is ideal for fostering these types of experiences.

From the Editor: Your responses are invited, either as an independent article or as a letter to the editor to be published in Issue #24.

REFERENCES

Ames, G.J., & Murray, F.B., (1982) "When two wrongs make a right: promoting cognitive change by social conflict." *Developmental Psychology* **18**, p. 894-897.

Aronson, E., Blaney, N., Stephan, C., Sikes, J., Snapp, M. (1978) *The Jigsaw Classroom.* Beverly Hills, CA: Sage Publications.

Bean, John (1996) *Engaging Ideas: The Professor's Guide to Integrating Writing, Critical Thinking, and Active Learning in the Classroom.* San Francisco, CA: Jossey-Bass.

Brown, A., & Palinscar, A. (1989) "Guided, cooperative learning and individual knowledge acquisition," in L. Resnick (Ed.) *Knowing, Learning, and Instruction*. pp. 393-451, Hillsdale, NJ: Erlbaum.

Brufee, K., (1993) *Collaborative Learning: Higher Education, Interdependence and the Authority of Knowledge*. Baltimore, MD: Johns Hopkins University Press.

Burnstein, E., McRae, A. (1962) "Some effects of shared threat and prejudice in racially mixed groups." *Journal of Abnormal Social Psychology* **64,** pp. 257-263.

Cohen, E.G., (1986) *Designing Group Work*. NY, NY: Teachers College Press.

Cooper, J., Prescott, S., Cook, L, Smith, L., Mueck, R., Cuseo, J., (1984) *Cooperative Learning and College Instruction—Effective use of Student Learning Teams*. California State University Foundation Publication, pp. 41-65.

Cross, P.K., Angelo, T., (1988, 1993 2nd ed.) *Classroom Assessment Techniques: A Handbook for Faculty.* San Francisco, CA: Josey-Bass.

Dansereau, D.F., (1985) "Learning strategy research" in Chipman & Glaser (Eds.) *Thinking & Learning Skills: Relating Instruction to Basic Research Vol. 1*. Hillsdale, NJ: Erlbaum.

Felder, R.M., (1997). e-mail communication from feldergeos.ncsu.edu, WWW page http://ww2.ncsu.edu/unity/lockers/users/f/felder/public/rmf.html

Fogarty, R., & Bellanca, J., (1992), "The new school "lecture:" Cooperative interactions that engage student thinking," in Davidson and Worsham (Eds.), *Enhancing Thinking Through Cooperative Learning*. p. 84-100, NY, NY: Teachers College Press.

Hertz-Lazarowitz, R., Kirkus, V., Miller, N., (1992) "An overview of

the theoretical anatomy of cooperation in the classroom," in Hertz-Lazarowitz (Ed.), *Interaction in Cooperative Groups: The Theoretical Anatomy of Group Learning.* p. 3-4, NY,NY: Cambridge University Press.

Hooper, S., Hannafin, M.J. (1988) "Cooperative CBI: The effects of heterogeneous vs homogeneous grouping on the learning of progressively complex concepts." *Journal of Educational Computing Research* **4**, p413-424.

Johnson, D. W. (1971) "Effectiveness of role reversal: actor or listener." *Psychological Reports* **28** pp. 275-282.

__, (1973) "Communication in conflict situations: A critical review of the research." *International Journal of Group Tensions* **3**, pp. 46-67.

___, (1974) "Communication and the inducement of cooperative behavior in conflicts: A critical review." *Speech Monographs* **41**, pp. 64-78.

Johnson, R.T., Johnson, D.W., (1972) "The effects of other's actions, attitude similarity, and race on attraction toward others." *Human Relations* **25(2)**.

__, (1979) "Conflict in the classroom: Controversy and learning." *Review of Educational Research* **49**, p. 51-70.

__, (1989) *Cooperation and Competition: Theory and Research*. Edina, MN: Interaction Book.

Johnson, D.W., Johnson, R.T., Holubec, E.J., (1990) *Circles of Learning*. Edina, MN: Interaction Book Co.

Johnson, D.W., Johnson, R.T., Smith, K.A., (1998) "Problems with lecturing." *Cooperative Learning and College Teaching* **9(1)**, p. 23.

Johnson, D.W., Johnson, R.T.., Roy, P., Zaidman, B., (1985) "Oral interaction in cooperative learning groups: Speaking, listening and the nature of statements made by high, medium and low-achieving students." *Journal of Psychology* **119**, pp. 303-321.

Kagan, S. (1986) "Cooperative learning and sociological factors in schooling" in *Beyond Language: Social and Cultural Factors in Schooling Language Minority Students*. Los Angeles, CA: California State University Evaluation, Dissemination and Assessment Center.

Kenney, S., (1998) *Reinventing Undergraduate Education: A Blueprint for America's Research Universities*. Stony Brook. NY: The Boyer Commission funded by the Carnegie Foundation for the Advancement of Teaching.

Kort, M.S., (1992) "Down from the podium" in Cohen, A.M. & Brawer, F.B. (Eds.), *New Directions for Community Colleges*. San Francisco, CA: Josey-Bass.

Margolis, M. (1997) "Brave New Universities." Firstmonday, Issue 3(5) pp. 1-7. http:/www.firstmonday.org/issues/issue3-5/margolis/index.html

Marzano, R.J., (1992) "The many faces of cooperation across the dimensions of learning" in Davidson & Worsham (Eds.), *Enhancing Thinking Through Cooperative Learning*. NY, NY: Teachers College Press.

McCarthey, S.J., & McMahon, S., (1992) "From convention to invention: Three approaches to peer interactions during writing" in Hertz-Lazarowitz & Miller (Eds.) *Interaction In Cooperative Groups*. NY,NY: Cambridge University Press

McDonald, B.A., Larson, C.O., Dansereau, D.F., & Spurlin, J.E., (1985) "Cooperative learning: Impact on acquitition of knowledge and skills." *Contemporary Educational Psychology* **10**, pp. 369-377.

Meier, M., & Panitz, T., (1996) "Ending on a high note: Better endings for classes and courses." *College Teaching* Fall 1996.

Midkiff, R.B., Thomasson, R.D. (1993) *A Practical Approach to Using Learning Styles in Math Instruction*. Springfield, IL: Charles Thomas Pub.

Nelson-LeGall, S., (1992) "Children's instrumental help-seeking: Its role in the social acquisition and construction of knowledge", in Hertz-Lazarowitz (Ed.) *Interaction in Cooperative groups: Theoretical Anatomy of Group Learning*. p. 120-141, NY,NY: Cambridge University Press.

Noble, D.F., (1997) "Digital Diploma Mills, Part I."
http://communication.ucsd.edu/dl/ddm.html, *Firstmonday*. **3(5)**.

O'Donnell, A.M., Dansereau, D.F., Rocklin, T.R., Lambiotte, I.G., Hythecker, V.I., Larson, C.O., (1985) "Cooperative writing: Direct effects and transfer." *Written Communication* **2**, pp. 307-315.

O'Donnell, A.M., Dansereau, D.F., Hythecker, V.I., Hall, R.H., Skaggs, L.P., Lambiotte, J.G., Young, M.D., (1988) "Coopera-

tive procedural learning: The effects of pre- vs distributed learning activities." *Journal of Educational Psychology* **8O**, pp. 161-171.

Panitz, T (1996) "Getting students ready for cooperative learning." *Cooperative Learning and College Teaching* **6(2)**, Winter 1996.

Panitz, T (1997) "Collaborative Versus Cooperative Learning: Comparing the Two Definitions Helps Understand the Nature of Interactive Learning" *Cooperative Learning and College Teaching*, **8(2)**, Winter 1997.

Panitz, T. (1999) "Benefits of Cooperative Learning in Relation to Student Motivation" in New Directions for Teaching and Learning Series #78-*Motivation in Teaching and Learning*, Michael Theall (Ed.), Josey-Bass publishers: San Francisco, CA.

Panitz, T, & Panitz, P., (1996) "Assessing students and yourself by observing students working cooperatively and using the One Minute Paper." *Cooperative Learning and College Teaching* **6(3)**, Spring 1996.

Panitz, T., Panitz, P. (1998) "Encouraging the use of collaborative learning in higher education" in *Issues Facing International Education,* J.J. Forest (Ed.), Boston, MA: Garland Pub.

Peterson, P., Swing, S., (1985) "Students' cognitions as mediators of the effectiveness of small-group learning." *Journal of Educational Psychology* **77(3)** pp. 299-312.

Rosenshine, B., R. Stevens, "Teaching Functions" in *Handbook of Research on Teaching,* Wittrock (Ed.) 3rd ed. pp. 376-391, NY, NY: Macmillan Publishing.

Sharan,Y., Sharan, C., (1976) *Small Group Teaching*. Englewood Cliffs, NJ: Prentice Hall.

Slavin, R.E., (1980) "Cooperative learning." *Review of Educational Research* **50**, pp. 315-342.

__, (1987) *Cooperative Learning: Student Teams*. 2nd Ed. Washington, DC: National Education Association.

___, (1990), *Cooperative Learning-Theory, Research and Practice*. Englewood Cliffs, NJ: Prentice Hall.

__, (1991) *Educational Psychology: Theory Into Practice*. 3rd ed. Englewood Cliffs, NJ: Prentice Hall.

Slavin, R.E., & Tanner, A.M., (1979) "Effects of cooperative reward structures and individual accountability in productivity and learning." *Journal of Educational Research* **72(5)**, p. 294-298.

Swing, S., Peterson, P., (1982) "The relationship of student ability and small group interaction to student achievement." *American Educational Research Journal* **19**, pp. 259-274.

Tannenberg, Josh, (1995) "Using Cooperative Learning in the Undergraduate Computer Science Classroom." Proceedings of the Midwest Small College Computing Conference, 1995, Available on the internet at http://phoenix.isub.edu/josh/coop/papers/mwscc95.html

Webb, N.M. (1982) "Group composition, group interaction and achievement in small groups." *Journal of Educational Psychology* **74(4)**, pp. 475-484.

Yager, S., Johnson, D.W., Johnson, R., (1985) "Oral discussion groups-to-individual transfer and achievement in cooperative learning groups." *Journal of Educational Psychology* **77(1)**, pp. 60-66.

"There comes a time when the mind takes a higher plane of knowledge but can never prove how it got there."
--Albert Einstein

The Story of a Service-Learning Project: Mathematics in the Park

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SUMMARY
The following is a description of the development and implementation of a service learning project carried out at Portland State University. Besides describing the mechanics of the project, the description includes excerpts from student journals, reflecting their growth in the service learning process.

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We often lead dual lives: the academic life full of intellectual discussions, classes, and textbooks, and a personal life focusing on hobbies, sports, and children. My "two lives" had become increasingly divergent with esoteric research and graduate classes at Portland State University competing with gang-impacted foster children and spiritual quests at home. My decision to devote my sabbatical year (1994-95) to access issues brought these two lives together in a unique way.

It all started with a conversation with a friend a year before my sabbatical was to begin. I had been thinking about ways to promote the popularization of math during my sabbatical, and she was relating her plans for Saturday classes for children. When she mentioned the importance of whole-body movement, I had it! Outdoor mathematics-based games—the ideas started pouring in!

Mathematics research was the foundation: searching for the essence of a mathematics concept. An active game built from the essence of a mathematics concept allows an internalization of that concept (examples follow). Shortly after receiving this inspiration, the Corporation for National and Community Service announced the Learn and Serve America funding. There was my key to making the ideas reality—PSU students could enroll for credit in a service-learning class in which they would create and deliver games. One problem: at the time, such credit did not satisfy any requirements other than elective credits—would anyone enroll? In learning about other service-learning programs, I was introduced to the concept of two-tiered service-learning, in which a small group trains a larger "second tier" group to provide the service. Anticipating low PSU student enrollment, I located a teacher at Jefferson High School (an inner-city high school) willing to loan me two of her school-to-work classes once a week. The university students worked with the high school students to create games and then provided backup support when the high school students taught the games to students at nearby middle schools.

I was fortunate to be awarded funding from PSU's Faculty Development funds as well as some of PSU's matching funds to our Learn and Serve America award. Northwest EQUALS (Peggy Noone in particular) played a significant role in developing the Math in the Park piece of PSU's Learn and Serve America proposal; Northwest EQUALS also provided essential community networking.

Before the school year began, I met with the high school teacher, Susan Schenk, to come to an understanding of our partnership. Fortunately, she had already planned to begin these two classes with exercises to develop creativity. Since PSU's quarter began several weeks after the beginning of the high school's semester, our project would begin immediately after the classes completed the creativity exercises.

A week before we were to meet with her classes for the first time, I met with Susan during her preparation period in her classroom. Climbing the stairs to Susan's third-floor classroom, I found myself follow-

ing a pair of young men sporting six inches of boxers above their sagging pants tops and carrying on a conversation in Spanish. Oh, I thought, fortified by my life with foster children, what an experience this will be for the PSU students!

In the initial quarter of the Math in the Park project, I recruited only two PSU students: my work-study assistant Connie Johnson (joining the project as her work-study employment) and a young woman majoring in business. Our first meeting with the high school students occurred on a sunny fall day, and we taught them some mathematics-based outdoor games to give them the flavor of the project. To our surprise, they had strong objections to going outside: the grass was wet and they would have to clean it off their shoes, playing with a basketball might soil their clothes! Well, they grudgingly cooperated but spent 20 minutes afterward getting cleaned up. The positive side was that they were confident that they could design better games than the lousy ones we'd taught them!

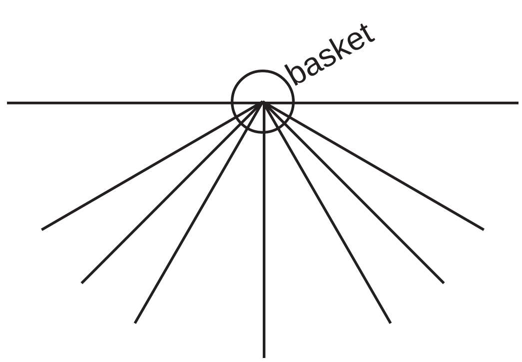
We divided the classes into design groups and set them to their task. Some groups came up with ideas immediately and some bogged down. Susan, with her enthusiasm and encouragement, kept everyone on task and suggested approaches. By the end of the first design session, each group had at least the outline of a game, and some had the game materials prepared and ready to go.

The first games created are among my favorites: Angleball and String Design.

Angleball

Tape is placed on a basketball court marking the angles 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180° relative to the edge of the court.

Divide the players into two teams. When the game leader calls out an angle, one player from each team races to the end of the line segment marking that angle. The first one to arrive wins the privilege of shooting a basket.



Besides reinforcing the concept of "angle," Angleball teaches players to visualize these nine angles that arise frequently in other contexts.

String Design

A group of players is handed a list of instructions, a ball of string, and a roll of masking tape. Following the instructions, they lay out the string, securing it with the tape. The result will be an outline of an object. For example, the following instructions result in the outline of a house:

Go 10 feet
Turn 90° to the right and go 3 feet
Turn 90° to the right and go 2 feet
Turn 90° to the left and go 2 feet
Turn 90° to the left and go 2 feet
Turn 90° to the right and go 3 feet
Turn 90° to the right and go 10 feet
Turn 30° to the right and go 8 feet
Turn 60° to the right and go 8 feet
("Feet" can be measured with a measuring device or by using one person's foot.)

Players get a chance to use the angles they learned in Angleball, as well as to internalize principles of measurement. In creating their own design instructions, they work through geometry concepts.

In the course of this design process, both PSU and high school students deepened their understanding of math concepts. In the design of Angleball, we watched one high school student teaching another how to use a protractor. They touched on radian measurement when they suggested marking the angles in fractions of a semicircle. In trying to design a probability game, the business major and her high school group struggled with the question of contexts in which probability is applicable.

After trying out the games on each other and on other high school classes, they made adjustments to the games and were ready to teach them to the middle school students. Well, we thought they were ready—actually, behind all the gang neighborhood bravado,

they were terrified! The day Susan's second period class was scheduled for their first middle school visit, only one student came to class. Fortunately, Susan had anticipated the problem and had recruited two honors students to assist with Math in the Park as their honors project. Eventually, more of the high school students joined us on outings to the middle schools, and we watched their self-esteem and confidence grow with each field trip. Their efforts to keep active sixth graders on task also gave them a new appreciation for the difficulty of a teacher's job!

Continuing the project into winter quarter, Connie and I were able to recruit six PSU students. The downside: most of them had schedules that didn't allow them to continue with Susan's classes. Consequently, we could only meet with Susan's second period class and had to find an afternoon class to work with. Jefferson High School's mathematics department connected us with Larry Pattee's general math class. Unlike the students of mixed ability in Susan's school-to-work classes, these students were homogeneously innumerate and undisciplined...but very creative!

The location and time of Larry's class required the PSU team to navigate Jefferson High School's halls at the end of the lunch hour. At that time, the floor is smeared with food and the halls are full of action: minor scuffles, major flirting, and insults filling the air. From PSU students' journals:

> "Walking into Jefferson for the first time was like marching upstairs into the twilight zone. My 7th grader's school is much brighter and less hostile looking."

> "I just observed how the kids interacted with one another. Gang hand signs seem to be a part of this community's language whether or not they are in a gang."

> "I was shocked and dismayed at the amount of disrespect that goes on in the classroom. I found the students to be disruptive and rude."

> "The kids were in complete and total control of the classroom. Not once did I even hear the teacher direct a statement to the class as a whole, and he left the room with a few students to go to the auditorium."

> "Many awkward silences occurred while trying to get to know the two girls in my group. They were the only ones that showed up in my group today. Noralee was absorbed in a fashion magazine, and Shaleen was quietly sitting there waiting for me to do something."

And so we persevered through winter quarter, refining our strategies as we went. The "learning" piece of this service-learning project (beyond the obvious) was a weekly reflection meeting. Several of the students were taking education classes at the same time and were eager to relate their field work to their class work. Lively discussions ensued, based on politics affecting schools, school policies like tracking and desegregation, and societal attitudes about children from low income neighborhoods:

> "Technology is changing our life at a pace never seen before, placing an ever increasing demand on our education system. To meet these demands, our education system must be flexible and designed to make changes as needed. This is not the current situation; bureaucracy has burdened down the present system, so that it can't adapt to current changes within a reasonable time span."

> "Too many students are shuffled along the school system, failing to learn not because they are incapable, but they are not expected to learn."

> "In just this one day at the school, I picked up more flaws in the school system than when I was a full-time high school student. To hear that teenage mothers are deprived of free day care to help them to achieve their diplomas saddens me. That was my first eye to eye interaction with budget cuts."

> "Last quarter my thoughts revolved around the social structure that the Jefferson students were trapped in. A dark, dismal school surrounded by small, poorly maintained houses. The students seem to have given up. Why expend the energy to think when it didn't do much for their role models? What I didn't know is that the students last quarter had not given up. They just didn't have enough people

in their lives to tell them that dreams are attainable with work and time."

"However, I realized one thing: in working with Jefferson High School kids, we need to treat them the same way as if they are of the same level. I feel they have more psychological problem than mathematical. That's why if we can change their thinking and low esteem by treating them equally in terms of academic abilities, maybe they can improve their school as well as personal lives."

"After a few slow rounds, I realized that there was a lack of self confidence around revealing an answer when a player wasn't sure of an answer. Outwardly, the students called each other stupid in what seemed to be in jest. But inside, I had the feeling that those students had been called stupid by some influential peers."

"How can I expect our education system to provide an equal education to all students when it is nested in a society that is already anything but equal? Some people believe that our current system is hindered by a 'hidden curriculum' that reproduces the existing social and economic class structure. In my utopian education system this would not be the case; students would be treated equally regardless of economic or social class. This would require a revamping of our current system where the privileged few have the advantage, and the poor are constantly oppressed. Herein lies the real paradox of education reform—if we break down the hidden agenda, we break down the very society we intended to serve."

In the spring quarter, Susan's classes were in the "work" phase of their school-to-work program and no longer available. Continuing with Larry's class, we added Dave Dampke's geometry class (our first high school class devoid of discipline problems) and ended the year with a collection of over 30 games, most of them having been field-tested.

Math in the Park continued as a service-leaming project through the fall quarter of 1995, expanding in new dimensions. Connie, my work-study student, wrote the instructions for 27 activities, and they were reproduced for dissemination. (These instructions are not in a very "polished" form yet, but you can order a copy from me by email.) During spring quarter, we included PSU students from Freshman Inquiry classes who had a service requirement, and they taught games in some after-school programs. Our predominant after-school partner was MESA (Mathematics, Engineering, Science Achievement), a program targeting traditionally under-represented middle school and high school students. The MESA director, Joan Kurowski, was very helpful in matching us with programs in several middle schools and in paving the way for us.

Based on my first attempt at community-based learning, I would emphasize two key components for success: a commitment by the community partners, and regular reflection meetings with the "service learners" (university students and high school students). Before the project started, I met with the first Jefferson High School teacher we worked with and discussed expectations. As schedules shifted and the project was "passed around" to other teachers at the high school, I was not able to set that groundwork, and the project did not proceed as smoothly in those classes. The university students' weekly reflection meetings allowed them to sort through issues that arose in their work with the high school students and to examine the politics of public education in this country.

As I had hoped, the university students had formative experiences:

"I am still working on what approach I feel I should take with these kids. My first instinct is authoritarian, but as the kids start to warm up to me (and they are), my instinct changes. I think they just need to know that someone cares. And I think that I do."

"I am a little bit confused with my emotions...I have never talked to a black person for more than 5 minutes. In my native country, The Republic of Georgia, we do not have any black people. And this was really an experience. I am surprised, though, that in the class of "Math in the Park" there was not more than one white kid! There was only one white girl...It's funny, but I feel pleased that I went there. I think my last five-minute talk influ-

> enced this emotion of mine. I talked with John, one of the students at Jefferson High School, and he showed his intellect as well as his heart."

> "I really have enjoyed this class overall, and it has really opened my eyes to what public school is like these days. Am I really cut out for this?"

Although racially Caucasian (with one exception), my PSU students were intriguingly diverse: majors in math, education, history, business, and engineering; the single mother raising a child with chronic health problems, the auto mechanic returning to college to make a mid-life career change, the recent immigrant from the former Soviet Union; childhood experiences ranging from extreme poverty in inner-city Portland to wealthy suburban environments. In the midst of this diversity lay a common commitment to creating exciting educational experiences for young people.

> "I had a *wonderful* time playing with kids, I have to admit I did *not* think that it would be so much fun. Kids were really into it. I loved it. I even forgot that I was freezing."

> "I haven't spent much time with people of different cultural backgrounds, and I was kind of afraid that I would say the wrong thing. Not to mention that I had no experience teaching students in a refresher class; the only tutoring that I had done had been with 'gifted' students who just needed a push in the correct direction. Thank heavens I was sadly mistaken; these students were great. I don't know what I was so nervous about. They were curious and very interested in learning new things. It made me realize that it didn't matter what level a student was at, as long as you presented a fun and interesting idea to them, they were more than willing to learn."

> "Jordan seemed to do a good job explaining the game while Shaleen did an outstanding job making sure everyone was doing the math correctly. Things really seemed to go well when students that were cutting other classes and hanging out with their buddies in our class wanted a score card so they could play. Believe it or not!"

For the past four years. Math in the Park as a service-learning project has been tabled as I have been repeatedly recruited to develop other exciting curricula. But it has a place in the future of PSU now that our new general education requirements are in place. I anticipate a revival of Math in the Park as a senior capstone project, which is now a requirement for all PSU students. Meanwhile, Math in the Park activities continue to be conducted by PSU education majors in their practica and by the after-school program AWSEM (Advocates for Women in Science, Engineering, and Mathematics).

What will I do during my next sabbatical? It will be hard to top the amazing experiences that occurred in developing Mathematics in the Park!

"The secret to creativity is knowing how to hide your sources."

--Albert Einstein

Training Elementary Teachers for the New Millennium

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The elementary teacher of the future will work with children who are surrounded by numerous technologies and confronted with an explosion of information. The teachers and the children will be unable to learn all the new technologies and all the information, because it will be impossible to keep abreast of all the new knowledge that will be created. These teachers of the future will need to be able to be lifelong learners and will need to help children become more independent learners.

With the support of the Exxon Education Foundation, we are working with small cohorts of elementary teachers and preservice teachers to change the students' world view of mathematics from a subject that is to be "taught" to one that is to be "uncovered." The basic idea of the PRIME (Partnership for Reform in Mathematics Education) Project is to allow the students to construct knowledge concerning new mathematics concepts and pedagogy. The students take college mathematics classes with this focus, and are mentored in elementary classrooms by teachers in the project.

In this article we will describe the classes taken by the project members and the student field experiences. We will share our successes along with lessons which will guide us as the project continues.

A PERIOD OF DISCOVERY

The project started during the summer of 1997 with two classes for the mentoring elementary teachers, one based on applications of elementary mathematics and the other based on current trends in mathematics pedagogy. The pedagogy class included some training in mentoring of preservice students. We selected the teachers primarily from Title I schools, based on principals' recommendations. We asked the principals to choose teachers who felt comfortable with mathematics and whom they thought would be good mentors for our students.

At the end of the first summer institute, the project evaluator interviewed the teachers and read their journals. Two themes which emerged from the journals were how the teachers were exploring theories of learning and how they were thinking about how mathematics can be taught. All teachers stated that they were intrigued by the constructivist approach to teaching, where teachers create classrooms to assist students in understanding mathematics. They were concerned about how they would succeed, but expressed willingness to try the approach. The teachers began to question some of their own practices in light of the theory and research presented during the first week of the institute.

The teachers had to come to terms with their own mathematical proficiencies. They had to learn to recognize where weaknesses existed and to take action in these areas. These teachers recognized that there was a dimension of "knowing" mathematics that they had not acquired. They needed to rethink their existing belief systems, question their own practices and knowledge of mathematics, and then work collegially to learn new skills.

The teachers commented favorably on the opportunity to read professional literature, to learn strategies for incorporating technology, and to find meaningful applications for children. The teachers considered their having time to read professional literature as a plus of the institute. Likewise, the participants enjoyed being able to work with calculators and software and to be able to see how they could use these technologies in their classrooms.

Finally, the teachers told how they enjoyed both a field trip to the Exxon refinery where they learned how the workers used mathematics in their jobs, and a discussion by a Montana teacher who spoke about using literature to teach mathematics.

The teachers benefited from becoming learners and working in group situations. One teacher commented:

> I need to build a classroom environment that allows children to feel safe. The students need

> to feel confident with taking risks. They also need to learn, as I am, that we don't always have to go the same route to come to the answer.

Teachers identified that their confidence in mathematics improved due to the ability to feel comfortable in a group situation. The recognition that it was possible to learn from others in a group has transferred to their own classrooms.

The teachers appreciated being given the time to create meaningful instructional units. The project evaluator used her Instructional Evaluation Guide (1997) to analyze the units. She found that the teachers demonstrated the ability (1) to create contexts that foster problem-solving skills; (2) to encourage students to communicate mathematically; (3) to explore issues or topics that would be interesting to young people; and (4) to encourage the use of mathematical tools to pursue these investigations in greater detail. The teachers needed to develop greater understanding of the mathematics being studied and its applications and the improvement of student thinking.

We recruited the elementary preservice teachers from the required mathematics content classes. The two of us, together with one other instructor, teach these classes. We looked for students who had some talent in mathematics and in explaining the subject to other students. Students who were capable in mathematics but were lacking ability to communicate mathematics to others were discouraged from applying to join the project.

The first class for the elementary preservice teachers was taught in the fall of 1997. The 23 students were about evenly divided between sophomores, juniors and seniors. The focus of the course was applications of mathematics in the elementary and middle school, since elementary certification in our state includes grades K-8. Topics included geometry, measurement, business applications, social studies applications, probability, and number theory. All of these topics built upon those that the students had studied in the two required semesters of mathematics content. This course was planned and taught by us, although the students had minor input into what they would be learning. Pairs of students prepared units on the topics we had discussed in the class.

The project evaluator analyzed the units designed by the students. She found that a large percentage (87.5%) created units that would be interesting to young people. Additionally, they demonstrated the ability to design problem solving investigations that were relevant and encouraged students to communicate about mathematics. As with the elementary teachers, the students need to increase their understanding of core concepts and their ability to develop and extend the critical thinking skills of their students.

Eight of the students were placed with mentor teachers during the fall of 1997. The students all experienced growth in their ability to conduct elementary classroom activities. One of the teachers was so pleased with the work of her student, Jennifer, that she asked her to help with a presentation at the state teachers' meeting. When we contacted the teachers during the semester, some shared that they did not feel fully prepared to mentor preservice teachers. We agreed to provide additional training the following summer. We were able to visit about eight of the teachers' classrooms to observe lessons they were teaching. We noted that about half of them were using excellent pedagogy and had created communities of mathematics learners. The children were responsible for their learning, and the teachers did not give the children all the answers but had them evaluate their own thinking. The other teachers still maintained more formal classroom environments and offered their students little input into their learning. We planned to try to change these approaches with the next two classes for the teachers.

CREATION OF A COMMUNITY OF LEARNERS

In the spring of 1998 we taught a class to the mentor teachers which featured the Curriculum and Evaluation Standards (NCTM, 1989). In this class we coordinated the activities about 50 percent of the time and the teachers led the instruction the rest of the time. Each pair of teachers was responsible for designing activities illustrating one NCTM curriculum standard. The mentor teachers really enjoyed being able to share their ideas with their peers. In this way, we realized our goal of the teachers interacting as a community of mathematics learners. All of us involved in this learning situation demonstrated growth in becoming more constructivist.

In the summer of 1998 the class featured the Profes-

sional Teaching Standards (NCTM, 1991) along with more mentoring training. In this class we also allowed the teachers to have more input into their learning, and they were able to suggest some of the activities that would take place in the class. The teachers appreciated being able to read about and gain understanding of these standards. The mentoring training was more meaningful, since a number of the teachers had already worked with preservice students.

Four preservice teachers were placed with PRIME teachers during the Spring 1998 semester. These students worked in the classrooms two days a week. Most of the placements were productive for both students and teachers. Six students were placed with PRIME teachers during the fall of 1998; one student teacher did such an excellent job that the PRIME teacher wanted to use her as a long term substitute when she went on maternity leave.

That fall the students took a class on the Curriculum and Evaluation Standards (NCTM, 1989) modeled after the one that the teachers had taken. We were pleased with the growth that the students had made in mathematics knowledge and also in their ability to present mathematics to others. Five pairs of students were chosen by their peers to present the activities they had designed at a regional NCTM meeting.

During the fall 1998 class, the students also dealt briefly with the Professional Teaching Standards (NCTM, 1991). They were asked to develop six major topics to be covered by middle school students. All groups gave very sophisticated answers to this assignment. One example included beginning algebra, and helping students move to more abstract thinking. Other examples were fractions and decimals with money and cooking applications; basic geometry emphasizing that shapes are everywhere; problem solving to help build reasoning skills; statistics and probability with real life applications; and number sense.

During the spring of 1999, the students presented their activities at an NCTM regional conference and received positive evaluations from their audience. Two of the elementary teachers also presented and asked that one of the students who had worked in their classrooms be allowed to assist them. We noted that the students' presentation abilities had improved not only at professional meetings but also in their other college classes. One group of the students presented a lesson in the geometry class, and their lesson was outstanding compared to the work of the other students.

During the spring of 1999, the students and professors also met with the project evaluator. She asked them to share their impressions of their experiences with their mentor teachers. Several of the students had worked with the same mentor teacher and were all pleased with their opportunities to work in her classroom. She was one of the teachers we judged to be modeling the pedagogy that we hoped all the mentor teachers would exhibit. Students praised other teachers and said they exemplified what we were teaching. The students had seen the Annenberg K-4 mathematics video tapes. One student said her mentor was just like the teachers on the tapes. However, students also commented that other teachers had made little progress in changing their classroom procedures.

MATHEMATICS LEARNING FOR ALL STUDENTS

Our final class for the mentor teachers was conducted in the summer of 1999. Since we had deliberately recruited most of the students from Title I schools, the topic chosen was mathematics learning for all students. We had the teachers formulate plans for working with special needs students. One special part of the class was to have each of the teachers do a self-evaluation and to formulate a plan to improve his/her teaching. The mentoring part of the class involved having the teachers plan specific tasks for the preservice teachers who would be in their classrooms. Finally, we introduced the teachers to working with cognitively guided instruction (Carpenter et al, 1999) by allowing them to investigate cognitive levels of children and strategies for their use to enhance children's learning.

OVERVIEW OF THE PROJECT

The student portion of the PRIME Project has been made sustainable by having a consistent recruiting plan: as each group of approximately eight seniors graduates, they are replaced by an incoming group of sophomores who have been recruited. The PRIME courses, along with existing mathematics courses in technology, geometry, history of mathematics, statistics, and finite mathematics, make up a mathematics concentration that reflects the spirit of PRIME reform.

LESSONS WE HAVE LEARNED

We learned that we should have given the mentor teachers more support during the school year while they had students in their classrooms so that their concerns could be addressed in a timely fashion. It might have been helpful to "audition" the teachers in the first summer classes just as we did the students. There were several teachers whom we found would not be good mentors for our students. Some had no understanding of their peers who did not have mathematics backgrounds as strong as theirs, and felt the same way about the college students.

We should have started giving the preservice teachers more responsibility for selecting topics and approaches in the first class on applications of mathematics. We found that they showed greater improvement when we allowed them to give more input in their learning.

CONCLUSION

We believe that the partnership among the classroom teachers, college students, and professors has created a beneficial learning experience for all of us. Each group complements the others and increases the learning opportunity of both the college students and the elementary children. As the college students graduate, they are being employed as elementary teachers. As they gain experience, we expect them to become peer leaders in the area of mathematics. Moreover, we look forward to the prospect of their becoming mentors for future generations of college students in the project.

REFERENCES

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics—Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Leppien, J. H. (1997). *Instructional materials evaluation guide*. Unpublished scoring guide. University of Great Falls, Great Falls, MT.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: NCTM.

Schifter, D. (1996). *What's happening in math class*? New York: Teachers College Press.

True Prime

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In the dark invisible crevises of the night
a prime number sleuthed
Searching the lifeless coordinates for clues
to the unsolved, unresolved equation.

Greater than or equal to the task
the prime number trudged on,
Dodging villainous vectors, untangling taloned tangents,
and uncovering unnacountable sets.

Into both the positive and negative bounds of the infinite,
the prime number prodded forth.
While exponents powered, derivatives differentiated,
and integrals disintegrated all about.

But as the first foggy rays of sunrise broke through,
the prime number proved potent.
Unveiling the sinister and silent silhouette
of the ever-vacuous empty set.

Alas in the tabloids and periodicals of the dewy morning news
the prime number was sainted
For discovering and recovering the imaginative
and illusively vacant solution set.

The School/Home Communication Project: A Study of the Effect of More Frequent Grade Reporting on the Achievement of High School Mathematics Students

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INTRODUCTION

This study in empowering the family explored the impact of an increased flow of information on student achievement and productivity from the school to the family. The investigation into the effect of family management of student learning on the student's achievement was empirically grounded in an experiment that provided support for the managerial role of family by manipulating the periodicity and quantity of information on student progress that teachers sent to families. Findings support the general thesis that increases in both frequency and detail of individual student progress reporting to the family are positively related to student achievement.

The study was conducted in eight mathematics classes at a comprehensive high school in eastern Los Angeles County. Designated the School/Home Communication Project, the experiment was designed to inform those responsible for student learning, the families and students themselves, by transferring information collected as part of the teaching process from the classroom to the home quickly, frequently, and with as much of the detail available to the classroom teacher as possible. Parallel treatment and control classes for each participating teacher, identical as to content, were identified and selected. After a baseline data collection period ending at the first grade reporting of the fall semester, information usually available only to the teacher was sent each week to the families of students in treatment classes while students in control classes received only standard grade cards issued at six to seven week intervals called triads.

BACKGROUND

The investigation rested on a basic assumption that the family is responsible for the management of the student's learning. Confounding the acceptability of that role by the family are institutional practices, legal precedents, and societal norms which support a traditional view of the classroom teacher as the manager of student learning.

The power of parents to influence their children has been the subject of inquiry for thousands of years. Eby and Arrowood (1940) noted that "The family in Athens controlled the education of the child, and family influence has always been recognized as an individualizing force" (p.197). In recognition of the importance of family support in positively affecting learning, schools have employed involvement strategies in an effort to control and manipulate that support. Family members have been asked to tutor, volunteer in the classroom, raise funds, serve on committees, select textbooks, and participate in the development of the curriculum. Myriad reasons have been given by families for not participating. Whether single, working, committed to a belief that the school has total responsibility for educating students, or disinterested, the lack of family involvement in the educational process has been vehemently noted by professional educators. Cleveland (1929) even went so far as to suggest that "The community was beginning to awaken to the need of training for parenthood" (p. 49).

This perennial argument over the amount of time a family must donate to the education of its children has produced an accusatory dialogue which has done little to remedy the failure of students to learn. Teachers accuse the family of shirking a responsibility. Families pursue legal strategies as redress for their perception that schools have failed to educate their children.

D'Evelyn in 1945 presented the popular view that "The home and school have joint responsibility for a child's development" (p.1). However, that conceptualization of school/home responsibilities has not been upheld by the courts. Schools have not been held accountable by the courts for failure of a student to learn. The home is ultimately responsible for the educational

outcomes of its child or children (Collis, 1990; Schimmel and Fischer, 1977).

The courts have distinguished between the school's responsibility to provide the facilities and means for learning and the student's responsibility and his or her family's responsibility to accept and use those facilities and means (Collis, 1990; Schimmel and Fischer, 1977). Failure of one student to learn has not been accepted by trial courts or appellate courts as evidence of a failure to teach. If the school is to provide facilities and means for learning, such as curricula and instruction, and the student and his or her family are responsible for using those facilities and means, then the managerial responsibility for learning should be placed at the point of responsibility—in the home. And, if the family and the student are ultimately responsible for learning, then information gathered by the teacher useful in making classroom management decisions should be made available to those individuals who are able to exert influence and control over student learning.

If the locus of control lies within the family, then it is the family which needs information gathered by the school for assessing and reporting on student behavior and progress. Teachers capture much of the detail about student academic production and achievement which parents could use to monitor the educational progress of their child. This information, collected weekly, if not daily, is generated through assignments, quizzes and tests, condensed into a letter grade, and sent to parents every six or nine weeks and at the end of the semester or school year. To what end this information benefits the family may be proportionally related to its availability and dissemination.

METHODOLOGY

An experimental design using treatment and control groups was chosen as a means of determining the effect of increasing the flow of information about student productivity and achievement from the teacher to the student's family. The experiment was conducted at a Southern California comprehensive high school with a student population of about 1450. Ethnic distribution at the school was approximately 67 percent Hispanic, 26 percent White, and 7 percent other. District sponsorship of the study included authorization to use district letterhead for cover letters mailed to the family contacts.

At this school each semester comprised three standard grading periods called triads. Data was collected over four standard grading periods, three triads of the fall semester, 1991, and the first triad of the spring semester, 1992. The first triad in the fall was used to gather baseline data on both treatment and control groups. Treatment was conducted during the following two triads of the first semester. Treatment class students received weekly progress reports during the middle two triads of the study. These progress reports were generated by the teacher using a computerized grading program. Students were expected to take the reports home each Monday. The family contact was to sign a "Communication Receipt" as evidence of receiving the report and send it back with the student the next day. The intent was to maintain an expectation of regularly scheduled communication between the teacher and family. If circumstances prevented the teacher from sending the scheduled progress reports, a form notifying families of the delay and naming a date when the report should arrive was to be sent in place of the report. District grade reports were made available to all students at the end of each triad.

Four mathematics teachers were selected to participate in the study on the basis of their interest in the project and familiarity with and interest in using a computerized grade program. Each teacher had at least two classes with identical content, one of which was identified as the treatment class in which the students received weekly progress reports during the two treatment triads, and the other section which served as the control class that received only the regular district progress reports at the end of each triad.

Families of students in classes selected for treatment were invited to participate in the high school School/Home Communication Project. Every student in each of the treatment classes was given a progress report each Monday during the two treatment triads. Solicitation of a family contact was a device intended to foster commitment on the part of the respondent. The decision to include all treatment class students in the process was intended to preclude a sense of voluntary participation by any student that might lead to a refusal to participate. Students were used as couriers, as is the custom in this school district, to carry communications between school and home, but no student was required to participate and no action was taken against any student for refusing to carry papers

between school and home or home and school.

Printouts of teachers' gradebooks were collected weekly. This practice provided an opportunity for the project director to monitor each teacher's adherence to project requirements, answer questions related to procedural problems, and discuss possible solutions for anticipated problems. Data collected at the end of each triad were labeled by Week 7, 13, 19, or 26 and used for quantitative analyses.

Every effort was made to ensure as little difference as possible between paired classes. Each teacher's treatment and control classes received the same lessons, assignments, quizzes, and tests. Time of day was considered in the selection of treatment and control classes to reduce any effect on students that might result from taking morning or afternoon classes. No teachers other than the four involved in the project used a computerized grade program and no student in the school who was not in a treatment class received a special grade report during the four triads of the study.

Family contact interviews were conducted at the end of the study. The specific interest of each interview was to determine the value to the family of the teacher reports on student progress. Secondary considerations for these interviews centered on the relationship between the family contact and the child, how the child's time was structured by adults in the home, and conflict resolution strategies of both family members and the child.

QUANTITATIVE FINDINGS

Quantitative data were collected weekly, but only accumulated data from those weeks coinciding with the end of triads were used for analyses. These were the weeks (7, 13, 19, and 26) when all student grades were submitted to data processing. From those data the district compiled grade cards for each student. Report cards were usually available to families within two weeks of the end of the triad. Since these were the only reports received by control group students and their families, the decision was made to compare the groups at district grade reporting intervals. To provide for a fair statistical comparison of the effect of family knowledge on student academic productivity and achievement, the decision was made to confine data analyses to standardized grade reporting intervals correlative with the school district reporting requirement.

Prior to treatment, data were collected during the first seven weeks of the study to establish a baseline against which later findings could be compared over time. The t-test findings for baseline data comparing treatment and control groups revealed no observable differences between the groups. Variations noted for each of the variables and constructs at Week 7 were attributable to chance.

The construct *Grade Percentage* was comprised of two variables, *Assignment Points Earned* and *Quiz/Test Points*. As separate variables, neither reached significance, although a trend toward significance persisted over the course of the study. However, in combination, as measured by the *Grade Percentage* construct at Week 26, the two variables did mark a statistical difference ($F=5.249$, $p<.025$) between treatment and control groups. Comparison of graphed means for the *Assignment Points Earned* variable and the *Quiz/Test Points* variable indicate that differences favored the treatment group students (Figures 1 and 2).

The variable *Assignment Points Earned* is closely linked to the variable *Assignments Turned In*. There is a noticeable relationship between the number of assignments a student turned in and the total points earned for those assignments (Figures 1 and 3). Four factors were considered which might affect the number of assignments that a student turns in. First, the student's familiarity with the concepts and skills presented in the assignment affect his or her disposition to attempt and/or complete the assignment. Second, the student's personal belief in the relationship between course grade and assignments turned in might be a factor in not turning in assignments that have been completed. This phenomenon is not unusual among students and has never been explained by any student to the satisfaction of the writer. Third, the student might fail to do the assignment because of a lack of enabling skills and knowledge. Such students often hide their lack of knowledge, and, it seems, would prefer to appear disinterested in the course than have anyone discover their lack of knowledge. Conversations with these students over the years led the writer to attribute this behavior to a belief in predetermined intelligence quotients and innate capabilities that are probably thought to be inherited from family members who gave comfort by relating their own school

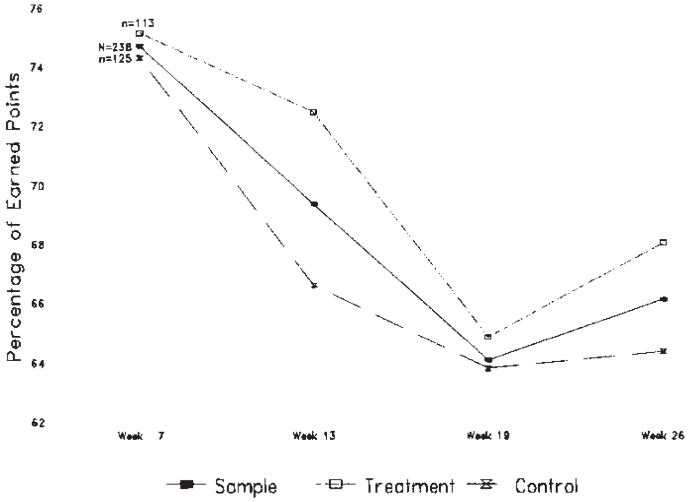

Figure 1
Assignment Points Earned
Group Means by Data Collection Interval

failure. Finally, it is possible that parents do not see a relation between assignments and achievement. Many families hold a view of seatwork as a classroom management device to promote a quiet, work-like environment with little, if any, relation to student learning. Any or all of these factors may have contributed to the denigration of the productivity means over time.

As shown in Figures 1 and 3, means for treatment and control groups were very high at Week 7, slid to lows at Week 19, the end of the semester, and rose at Week 26. That pattern would correlate well for Weeks 7

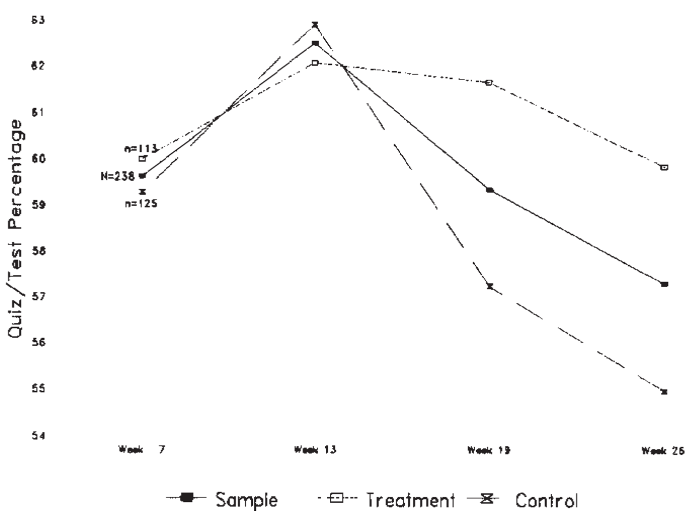

Figure 2
Quiz/Test Points
Group Means by Data Collection Interval

through 19 because traditional textbook and teacher instructional strategies place familiar material at the beginning of the course and gradually ease into new material. An upslope identified between Weeks 19 and 26 does not fit this pattern. The material presented during these weeks was new. This evidence that students turn in their assignments at the beginning of a semester may be indicative of a resolve to keep pace with instruction that many of the students are unable to sustain over the course of a semester.

Another factor that may have affected the *Assignments Turned In* variable was the acceptance of late assignments. Teachers in the study accepted students' late

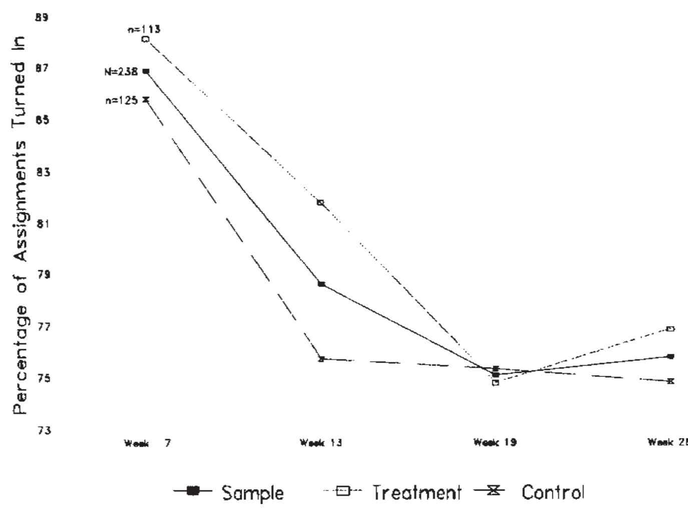

Figure 3
Assignments Turned In
Group Means by Data Collection Interval

work up to the last day of each triad. Work from a previous triad could not be turned in once grades had been formally recorded, but up to the last moment before teachers were to turn in grade collection sheets to the district, students were allowed to submit missing assignments. In effect, this practice produced very high means for both treatment and control groups and confounded any instrumental assessment that might have produced a credible differentiation between group means.

A graphic presentation of percentile distribution findings for *Assignments Turned In* at Week 13 (Figure 4) complemented analysis of variance results ($F=4.135$, $p<.05$). At Week 13 the greatest distance between group means was found at the 25th and 50th percen-

tiles. An analysis of variance at this interval disclosed a significant difference ($p<.05$) between groups. This effect disappeared by the end of the semester, at Week 19 ($F=.921$, $p=.338$), and by Week 26, the F ratio was .012 with a probability of .913.

The 50th percentile medians at Week 7 for the treatment group were 94.54 and 91.33 for the control group. Nineteen weeks later, at the end of the study, the median for the treatment group was 82.50 and 80.14 for the control group. At the 75th percentile, the spread between group medians at Week 26 was 2.81 points.

Had it not been for the practice of accepting late work, a more definitive analysis of the *Assignments Turned In* variable might have been possible. The practice of accepting late work inflated rankings at the 50th and 75th percentiles to such an extent that there was no room in the curve tails for the analysis of variance test to distinguish between groups. The validity of this variable as a measure of student productivity was not affected by the teachers' acceptance of late work. The variable was a valid measure of assignments turned in. It did not, however, provide a measure of the effect of the treatment. To use this variable successfully would require the use of a deadline for turning in assignments.

Assignment Points Earned naturally paralleled the results of *Assignments Turned In*. Figure 5 shows greater distances between treatment and control groups than evident for *Assignments Turned In* percentile distributions. However, the link between these two variables as a measure of student productivity was not directly correlated. The number of points earned on an assignment reflected the effort of the student to learn the material and complete the assignments. As a measure of student productivity, the *Assignment Points Earned* variable proved to be as robust as the *Assignments Turned In* variable.

Analysis of variance findings for the *Assignment Points Earned* variable at Week 13 showed an F ratio of 6.444, $p=.012$, which was comparable to the *Assignments Turned In* finding. At Week 19, both variables tended away from significance. However, during the last data collection interval these variables tended in opposite directions, with the *Assignment Points Earned* variable shifting toward significance with an F ratio of 1.687, $p=.195$ and, as stated previously, the *Assignments*

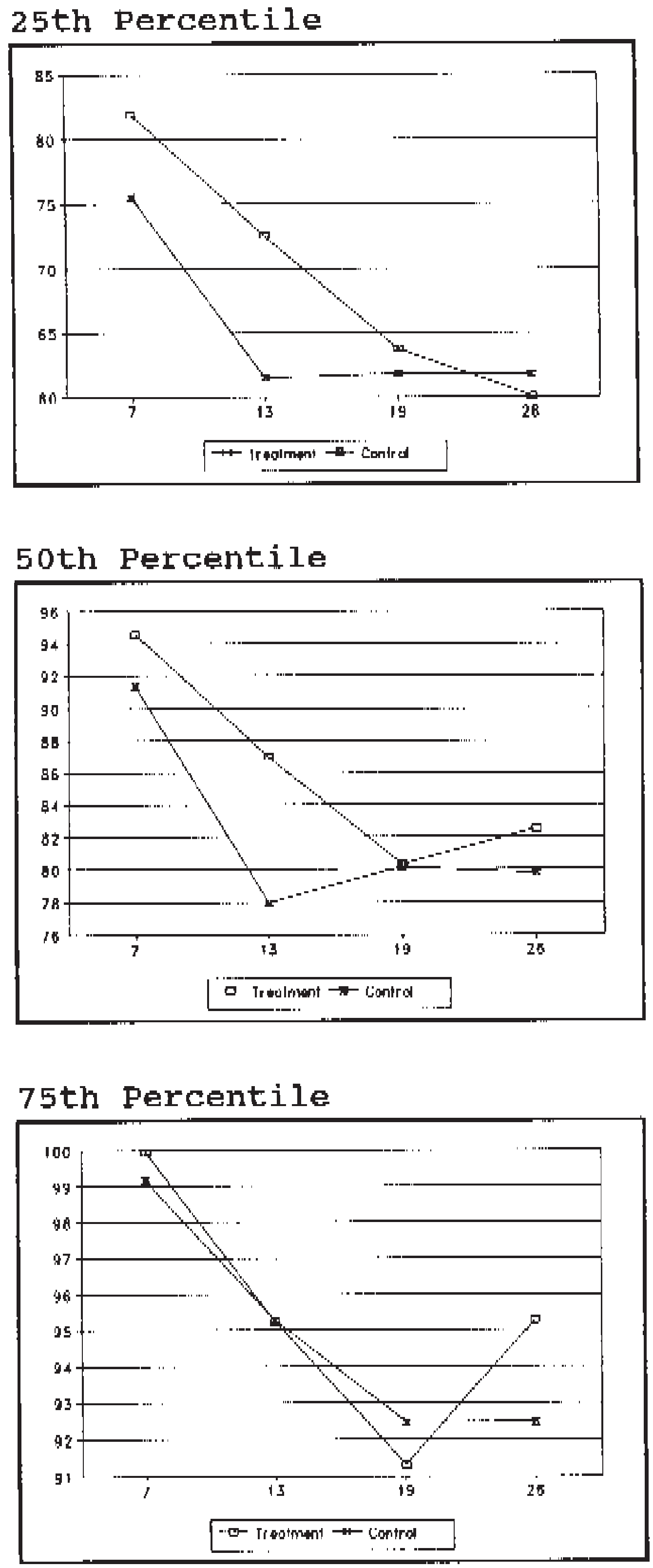

Figure 4
Assignments Turned In
Percentile Distributions

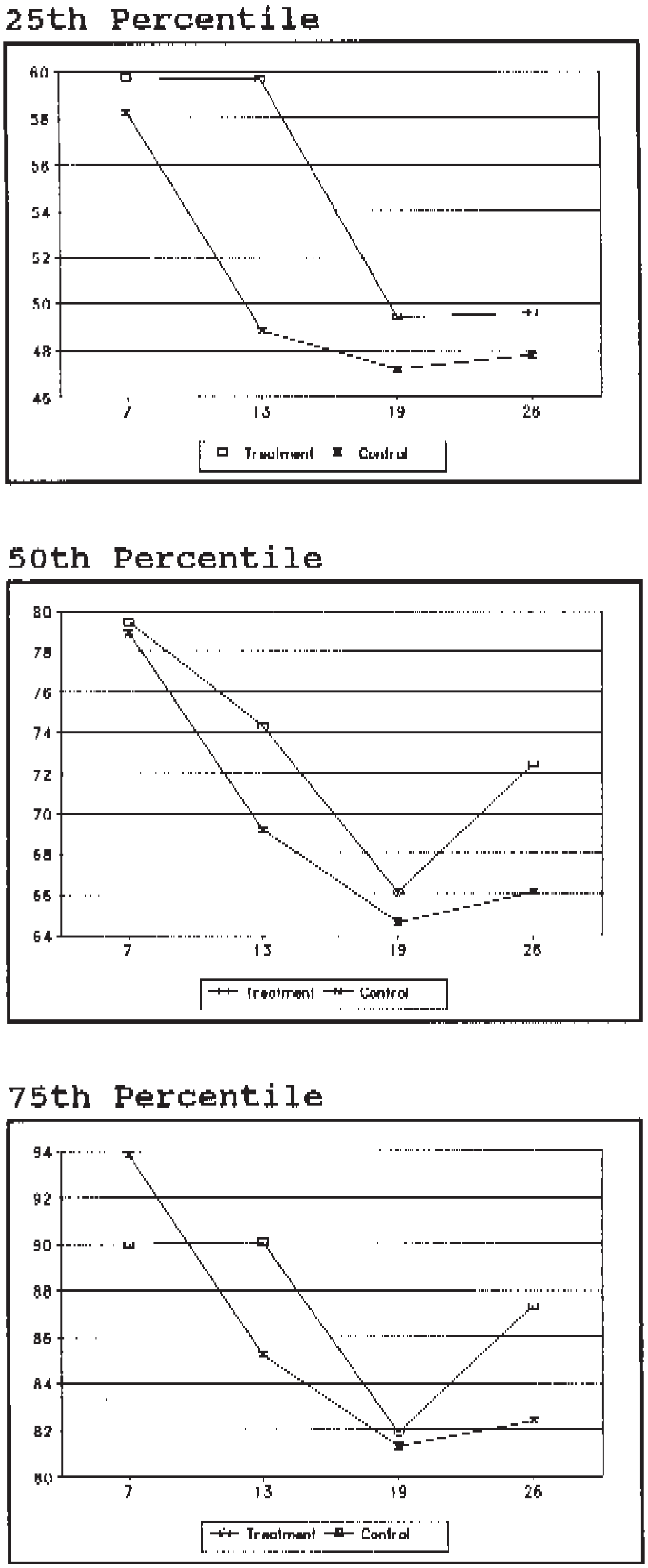

Figure 5
Assignments Points Earned
Percentile Distributions

Turned In variable reaching an extreme F-ratio of .012, p =.913.

Again, the practice of accepting late work may have had some affect on the *Assignment Points Earned* variable. It is, however, interesting to note the spread between groups at the 50th and 75th percentiles (Figure 4). Better control over the acceptance of late work might have provided for a testable distinction between groups.

Ethical considerations attendant in human research designs precluded the use of any device or procedure that interfered with practices known to promote student learning when such interference would reduce the student's course grade. A procedure for tracking the promptness or lateness of assignments would probably suffice as a means of controlling for the effect of late work on the productivity variables. Such a procedure would result in a duplicate set of gradebooks, one for the classroom teacher and one for the researcher, and would bear no empirical relation to the students' actual academic performance. In that event, the findings would represent a contrived model of the effect of family management on student learning and not the empirical reality from which the model would have been extracted. Apparently, the measurement of student productivity is an issue that requires rethinking.

The *Quiz/Test Points* variable almost reached significance in both of the final two data collection intervals of the study. This trend toward significance did not meet the alpha level requirement of .05 chosen for the study, but from an exploratory perspective, the shift from an initial F-ratio of .306, p=.581 at Week 13 to an F-ratio of 3.138, p=.078 at Week 19 and an F-ratio of 3.312, p=.070 at Week 26 coupled with graphic evidence of treatment impact at higher percentiles (Figure 6) supports a need for further investigation of the effect that frequent and detailed information about student academic productivity and achievement have on the family management of student learning.

The construct Grade Percentage was significant at Week 26 (F=5.249, p<.025), controlled for Week 7 baseline covariate. Figure 7 presents graphic evidence of the ANOVA results. The percentile rankings presented in Figure 8 show the impact of treatment on students above the 25th percentile.

Examination of variable and construct percentile distributions reveals an increasing spread between treatment and control groups corresponding to increased percentile rank. ANOVAs were run for the grouped data only. The spread between treatment and control groups at the 75th percentile indicate a probable impact from the treatment on academically proficient students with supportive families.

QUALITATIVE ANALYSIS

Data gathered from conversations, written communications, notes, formal documents, and interviews confirmed the effectiveness of the conceptualization of the communications component of the hypothesis. The agreement between all parties to the study on the value of information on student productivity and academic progress was evident from the project's inception.

Administrative support from school district officials validated the study and sanctioned those procedures necessary to its successful completion. The participation of students as couriers was high, in part because of district approval for the use of School/Home Communication Project as a title and high school letterhead for project communications.

Site administrators showed some concern regarding the potential for divisiveness that might result between treatment and control group students. There was also some concern about the response of families to communications. A formal requirement of the principal centered on the right to preview any and all materials that would be sent to families and students prior to release. There was also a concern about using students to carry messages between school and home. None of these concerns impeded the project.

Students displayed a concern when initially informed of the project. Such concern is not unusual for students when their routine is disturbed. Some treatment group students neglected to take the envelopes containing information about the project and a letter requesting family participation. Telephone follow-ups resolved most of these problems. A replacement information packet was distributed at the family's request. No follow-up was done for a nonresponse to the second distribution.

The intrusive nature of the study design and the

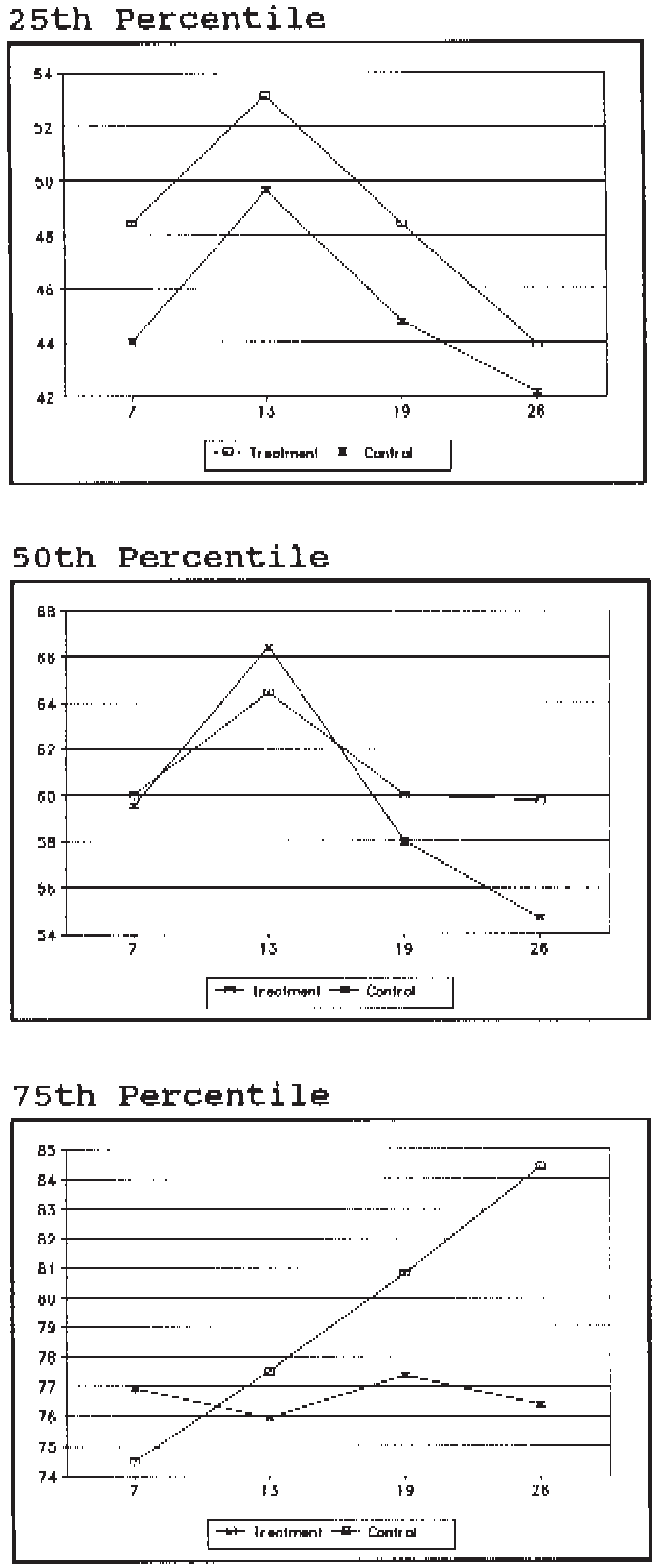

Figure 6
Quiz/Test Points
Percentile Distributions

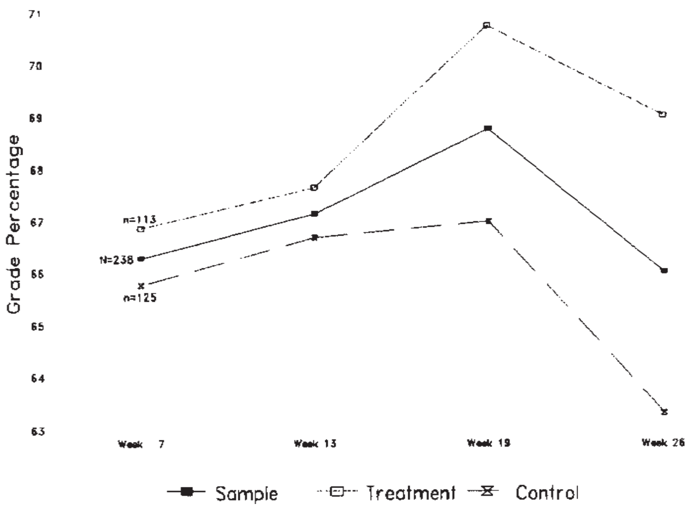

Figure 7
Grade Percentage
Group Means by Data Collection Interval

openly evident difference in the way students in treatment and control classes were apprised of their progress never appeared to present a problem. Students in control classes who inquired about receiving the progress reports were told that specific classes had been selected to receive the reports and because of the time involved in preparing the reports it was not possible to give them to all classes. Since there were no student or parent inquiries that went beyond the classroom teacher, it seemed that this answer satisfied the curiosity and concerns of students in the control group and in classes which were not participating in the study.

Teachers involved in the study were aware of the problems associated with grading papers, entering results into a grade program database, and printing out the student grades for family contacts each Monday of the treatment triads. The workload associated with the project was understood and agreed to by the three teachers who had consented to work with the project director who also participated in the study as a teacher. At the conclusion of the study, the participating teachers continued the practice of reporting on student progress to the families of all of their students independently of district requirements. A reporting cycle of alternating weeks with odd numbered period classed receiving reports one week and even numbered period classes the next satisfied the concern teachers had regarding an unmanageable drain on their time. In effect, this process almost matched the

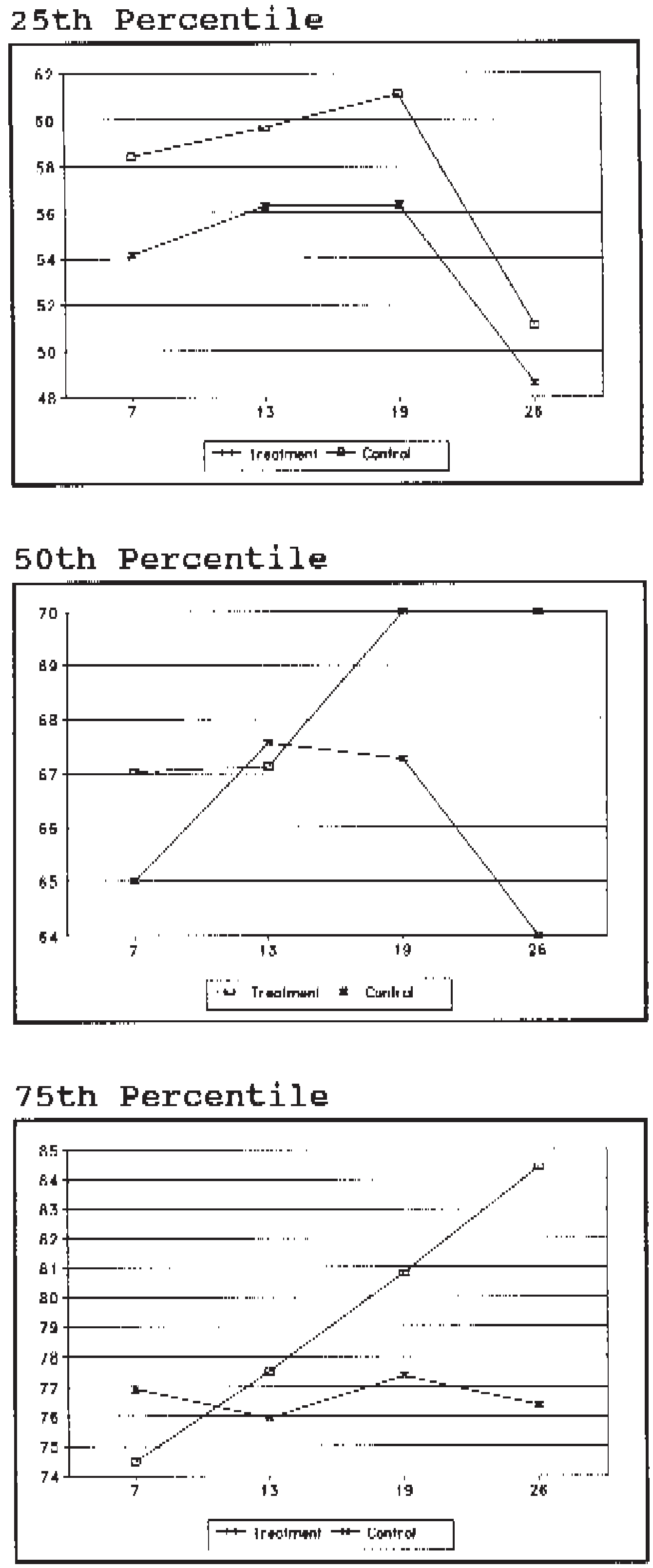

Figure 8
Grade Percentage
Percentile Distributions

time requirements of the study.

Teachers not involved in the project expressed interest in the amount of extra time required to enter grades into a computerized program and report to parents. Many of these teachers purchased the computerized gradebook program and began to use it as part of an instructional strategy. The compelling factor which appears to have prompted teachers to accept the challenge of computerizing their gradebook was their growing awareness that teachers who use these programs have ready availability to information upon requests by counselors, parents, and students. Teachers were impressed by the ability of computerized gradebook users to hand a parent a printout of a student's classroom conduct as indicated by number of assignments turned in. Coupled with the student's acceptance of responsibility for academic achievement, this method of recording and maintaining a gradebook was seen by teachers as a powerful tool in their efforts to affect student learning.

At the beginning of the study teachers expressed a concern that providing parents with detailed information about student progress would trigger numerous telephone conversations requesting explanations of grading practices and policies. Occasionally teachers were contacted, but in most instances the adult was seeking professional guidance to help the child become academically successful.

The intent of the study was to investigate the effect of frequent and detailed reporting of student progress to the student's family. The underlying procedural intent was to transfer the management of student learning from classroom to home. Other than the writer, who participated as one of the project teachers, none of the teachers in the study was aware of this design intention. It is interesting to note that the main effect which prompted teachers to adopt the practice of computerized grade reporting was the transfer of responsibility to the student. In effect, student learning became a parental or family responsibility since the students might fail to comprehend the importance of the deferred benefits of education.

Family contact response to the study was supportive and favorable from the outset. Unsolicited comments indicated the acceptance with which project intentions were greeted by the adults who obviously cared about what their children were doing in school. The only complaint was that frequent and detailed reports were not available from all teachers. That plea was heard most often from the families of students who were having academic difficulties in several classes.

Comments that arrived on Communication Receipts and notes and comments delivered in person revealed feelings of parental empowerment generated by an increase in information. This immediate reaction by the family to the benefit of receiving information about the student was welcomed by the teachers. Teachers discussed differences in parents' attitudes and in the responsiveness of treatment class students to the instructional requirements. This sense of goodwill may have been a result of teachers feeling positive about their involvement in the study. In that case, the responsiveness of parents may have been a result of the teachers' attitude. Whatever the cause, the reality of parental approval of the frequency and detail of the computerized progress reports was well documented in writing by many of the family respondents.

In the interviews conducted after the 26th week of the study, all of the respondents expressed their satisfaction with the project. Each of them appreciated the frequency of the reports, but, although aware of the detail, had not always known how to read them. Many were dependent upon their children to explain the meaning of the reports and the procedures and policies of the school and teacher. While family members did not always believe what their child told them, they seemed to be obliged to accept the explanation. Parents usually waited for face-to-face encounters with teachers to confirm or disconfirm their children's explanations.

IMPLICATIONS

Analysis of variance significance of the *Grade Percentage* construct at Week 26 ($F=5.249$, $p<.025$), controlled for Week 7 baseline covariate, and the graphic comparisons shown in Figures 7 and 8 present a compelling justification for continuation of research into the underlying conceptualization of this study that the family management of student learning affects student achievement. The *Grade Percentage* construct, comprised of achievement and productivity measures, illustrated the effect on student academic success of frequent and detailed progress reporting to the family. Quantitative and qualitative findings partially

supported the research question: Given equivalent course content and the same instructor, will more frequent, more detailed reporting of student progress to a single, responsible family contact result in statistically significant improvement in a student's 1) assignment completion ratio as a measure of productivity and 2) academic achievement as measured by course grade? A procedural problem resulting in a neglect to distinguish between assignments turned in on time and those submitted late diminished the value of the *Assignments Turned In* variable as a measure of student productivity.

The results of the study support a rethinking of the traditional view of the teacher as the manager of student learning. Teachers do gather pertinent and helpful information from student assessments. This study demonstrated that sending available information to the family frequently and in detail did have an effect on the student's grade. This experimental intervention demonstrated the value of detailed and frequent progress reporting to the families of students as compared to the traditional grade reporting practice of providing single letter grades for each subject two, three, or four times a year. Based on data from this study, it would seem that the benefit of providing information far exceeds its utility to teachers. There is no question that teachers need information upon which to base instructional decisions. The study illustrated the parallel that families need this data as well to inform decisions made in the home that are of consequence to the student's academic performance.

Contemporary research supports a reconsideration of the customary belief that nontraditional parenting has little effect on student achievement. What may negatively affect student achievement is the distancing between school and home resulting from an absence of timely and pertinent information that would allow the family to perform its managerial function in raising its children. The results of this study indicate that when families are aware of their child's academic performance in time, they accept their responsibility as managers of their children's learning and do affect the student's academic productivity and achievement.

Future research might concentrate on distinguishing between the student and family effect of progress reporting. Some consideration might be given to defining optimal reporting frequencies that teachers could comfortably meet. A better measure of achievement might be obtained with pre-and post-testing. Productivity might better be measured by initial assignment response rates as opposed to rates that reflect a teacher policy of accepting late work up to and including the last day of the triad. Interview inquiries might be made into the beliefs that both families and students have concerning the relationship between assignments and performance on quizzes and tests. No significant change was noted at and below the 25th percentile on any of the variables or the construct. Research specifically targeting the families of students in the 25th percentile might be more profitable in terms of developing an understanding of the relationship between family management of student learning and student academic success.

REFERENCES

Cleveland, E. (1929). *If Parents only knew.* New York: W.W. Norton.

Collis, J. (1990). *Educational malpractice: Liability of educators, school administrators, and school officials.* Charlottesville, VA: The Michie Company.

D'Evelyn, K.E. (1945). *Individual parent-teacher conferences: A manual for teachers of young children.* New York: Bureau of Publications, Teachers College, Columbia University.

Eby, F. and Arrowood, C.F. (1940). *The history and philosophy of education: Ancient and medieval.* New York: Prentice-Hall.

Schimmel, D. and Fischer, L. (1977). The rights of parents in the education of their children. Columbia, MD: National Committee for citizens in Education. In J. Rivera (1987). A phenomenological investigation of reporting practices at K-6 schools, Ph.D. dissertation. Claremont, CA: The Claremont Graduate School.

Book Review: *Experiencing School Mathematics* by Jo Boaler

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Book Review: *Experiencing School Mathematics* by Jo Boaler (1997) Buckingham: Open University Press, ISBN 0-335-19962-3

In her book, *Experiencing School Mathematics*, Jo Boaler provides a comparative study of the mathematics teaching, learning and attitudes at two United Kingdom schools. Boaler's work is a pertinent contribution to mathematics education. Firstly, she reports on a comprehensive research study comparing students' learning in a traditional mathematics classroom and a 'progressive' (Boaler's usage) mathematics classroom. The book captures some of the processes and results of the two very different approaches to mathematics teaching. Secondly, Boaler uses situated learning theory to analyze the nature of the learning and teaching in the school context. Previously, situated learning theory has been used to explain and characterize the mathematics used by people in various informal contexts (e.g. Saxe, 1991; Lave, 1988). Consequently, Boaler's work demonstrates new analytic possibilities for situated learning theory.

In this review I provide a brief description of the two schools. This is followed by a summary of Boaler's analysis of learning that occurred in the two contexts. Then I describe my primary concern with the nature of the claims and evidence provided in the book. Lastly, I raise a challenge to situated learning theory derived from the work presented in the book.

Boaler provides a detailed description of the contexts at the two schools. Both schools are set in middle income communities. Amber Hill teachers perceived their students as lower level mathematics achievers and consequently relied on teacher dominated explanations and concise textbooks. Consequently, students perceived mathematics as 'rules to follow.' To complete the curriculum the teachers maintained a fast pace set to the (perceived) ability of the average student in the group. The result was that many students were either bored or frustrated. In contrast, at Pheonix Park mathematics lessons were problem-based and characterized by an open learning environment, student independence, a good number of students off task and tasks that related, in varying degrees, to contexts outside of school. Students at Pheonix Park perceived mathematics in various ways and developed diverse attitudes towards the learning of mathematics. Changes in attitude and expectations for learning are useful, but how do these two teaching strategies influence students' mathematics learning?

Boaler provides compelling quantitative and qualitative evidence supporting a problem centered approach to mathematics teaching. To illustrate the compelling evidence presented by her, I will briefly summarize two of the sets of data used as evidence in the book. Firstly, she presented quantitative data from the GCSE (school leaving) mathematics exam results for both groups of 11th grade students. The results of the two schools were comparable. However, the Amber Hill students who took the exam were from the top two ability groupings. In contrast the Pheonix Park students were from mixed ability groups and completed the exam without receiving specific instruction directed at exam content. Secondly, Boaler provides a qualitative description of the performance of Pheonix Park and Amber Hill students in an 'architectural' activity. This activity required students to apply council regulations to their evaluation of a given house design. The Pheonix Park students were better able to make sense of the task and produced reasoned and reasonable responses. Amber Hill students tended to introduce irrelevant considerations and applied inappropriate mathematical procedures to the task. In summary, the problem-centered approach did not disadvantage the Pheonix Park students in the traditional test while clearly enhancing their ability to reason mathematically. The results presented in the book clearly indicate the benefits of 'progressive' mathematics education and are some of the first research results to so comprehensively support reform efforts in mathematics teaching.

In addition to the impressive results of the research, there are other features of the book that will appeal to the reader. Boaler references many theoretical positions in her discussion of the students, teachers and two schools. Her conclusions are primarily argued from a situated cognition perspective and more particularly Jean Lave's (1988, 1991) work. However, her reference to diverse social, education and psychological theories as she explains various events that occurred at the two schools provides the reader an intriguing meander through theory and practice. What impressed me the most about this book was Boaler's frequent use of the students' and teachers' voices, through verbatim transcripts, to illustrate her claims. The writing stays close to the classroom, making it an enjoyable read. The transcripts provide direct access to the data used to evidence her claims and allows the reader to draw his/her own conclusions from the given data.

In some cases, Boaler was prone to draw conclusions from limited evidence. For example, in chapter eight, she argues that the different contexts in which the students operated produced qualitatively different forms of knowledge. She claims that, "the communities of practice making up school and the real world were not inherently different" (p. 106). As evidence Boaler uses the lack of uniforms and teacher demands as well as the freedom students experienced in the organization of their work and mathematics learning. Firstly, more compelling evidence could have been obtained from discussions with students about their perceptions of their school. Secondly, there are many attributes of schooling that make it inherently different from the "real world," for example, forced attendance and timetables. More detail and evidence comparing the students' (and teachers) perceptions of the differences between school and the real world would have added substance to her claim.

Secondly, I wish to address a feature of Boaler's book that can be found in other writings on situated learning and cognition. Boaler provides a comprehensive analysis of the differences in mathematical knowledge between the two groups of students operating in a school context. In addition, she reports on the many students from Pheonix Park who suggested that they used mathematical techniques developed in school in contexts outside of school while students from Amber Hill saw little practical relevance in their school mathematics. I was, however, disappointed not to find evidence or examples of Pheonix Park students' use of their mathematics in everyday life. When Boaler introduces students' perceived uses of mathematics to out of school problems she creates an imperative to address the nature of the students' mathematical activity outside of school. That many students recognized possible applications of mathematics to experiences outside of formal schooling is a valuable contribution to mathematics education. It demonstrates how learning environments influence students' perceptions of the value of mathematics in other contexts or communities of practice. However, what remains unclear is if, when, and how they actually choose to employ their mathematics learning to these contexts.

This leads me to a question that needs to be addressed more thoroughly by researchers in the field of situated cognition. A central aspect of understanding the mathematical practices of various communities of practice is providing evidence or examples of their practice. Besides Saxe (1991), research in situated learning, particularly as it relates to the role of formal school mathematics education applied to other contexts, tends to evaluate the participants' ability to make sense of mathematics in contexts significant to the researcher rather than describing the participants' use of mathematics in contexts that are significant to them. In the case of this book, Boaler provides quotes from students that demonstrate they were able to perceive uses for mathematics outside the classroom. However significant it is for students to recognize the usefulness of mathematics outside the classroom, it is vital that as a mathematics education research community we begin to understand how they use their mathematics in contexts that are significant to them. This is a difficult but necessary task. Therefore, we must ask, what school mathematics do students apply to contexts that are significant to their lived experience?

REFERENCES

Lave, J. (1988). *Cognition In Practice: Mind, Mathematics And Culture In Everyday Life*. Cambridge: Cambridge University Press.

Lave, J. and Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. Cambridge: Cambridge University Press.

Saxe, G.B. (1991). *Culture And Cognitive Development: Studies In Mathematical Understanding*. Hillsdale, N.J.: Lawrence Erlbaum Publishers.

Ode to the Square Root: A Historical Journey

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SUMMARY
The author gives a personal history of experiences in finding the square root of a number by the "do it thus" method—from algorithm to table to calculator. Why each procedure works is elucidated, making liberal use of the history of mathematics.

ODE TO THE SQUARE ROOT: A HISTORICAL JOURNEY
Just as the scribe Ahmes in 1650 B.C. would direct the reader of the Rhind Papyrus to "Do it thus"[1] in solving a problem, so would my teachers instruct me to find the square root of a number in the secondary schools of the 1940's. It was an elaborate, laborious procedure, performed by rote, one mysterious step after the other.

In college we abandoned that square root algorithm and turned to tables. I still own my copy of "Mathematical Tables from the Handbook of Chemistry and Physics,"[2] which also contained trigonometric and logarithmic tables, tables of squares, cubes, cube roots, reciprocals and factorials, interest tables and pages of all kinds of mathematical formulas.

Fresh out of college in the late 40's, and wanting to work in the "real world" (as opposed to the academic world), I became a junior mathematician for a company that manufactured an early analogue computer. I was assigned to calculate the numerical solution of a differential equation describing the motion of a guided missile. To find the value of a trigonometric function correct to ten places, I used the giant books of tables prepared by mathematicians hired by the Works Progress Administration (WPA) during the depression. But to find the square root of a number correct to ten places I was directed to use Newton's Method. The directions given were in the style of the Rind Papyrus: "Do it thus." No reference was given to Newton's iterative formula. Only the algorithm, sometimes called the divide-and-average method,[3] was prescribed. Fortunately, I had at my disposal large electromechanical desk calculators (Frieden, Marchant, Monroe) capable of performing division, as well as multiplication, addition and subtraction.

What a relief it was in the 60's to have access to the electronic handheld scientific calculator to perform these arithmetic operations and soon after to just press a key to get the square root of any positive real number.

Now I am old and gray and have access to the graphing calculator, to the computer, and I can surf the Internet. To find the square root of a number, or its cube root or any root, is a trivial procedure—and I'm happy about it.

HOW AND WHY THE SQUARE ROOT ALGORITHM WORKS
The square root algorithm taught in the 40's was taught in Victorian times.[4] More than two thousand years ago the Greeks used a similar method. Basic to both methods is Proposition 4 in Book II of Euclid's Elements: "If a straight line be cut at random, the square on the whole is equal to the squares on the segments and twice the rectangle contained by the segments: (See Fig. 1).[5] Since this proposition, like all fourteen propositions in Book II, can be interpreted algebraically, Euclid's diagram has been given an algebraic interpretation, the identity $(a+x)^2 = a^2 + 2ax + x^2$.

To find the square root of n we use a trial and error process. Let a represent the first digit in the square root of n, where a is in the place held by the highest power of ten in the square root. Now we use the identity to find x, by dividing $n-a^2$ by $2a$, yielding x as a quotient, and at the same time ascertaining that $2ax+x^2$ be less than $n-a^2$. Suppose the highest possible value of x satisfying the condition is b, then $2ab+b^2$ would be subtracted from the first remainder $n-a^2$ and from the second remainder left a third digit in the square root would be found in the same way.[6]

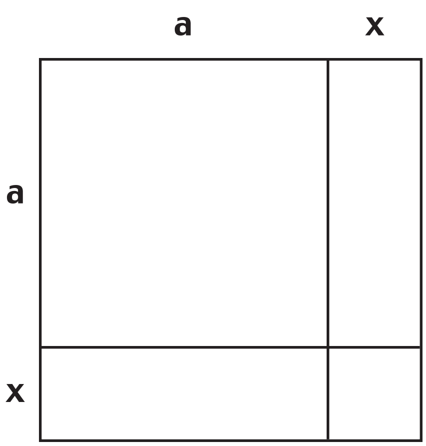

Figure 1

Suppose n = 1225. Guess a = 3, so 3•10 is our first guess of the square root of 1225. If $(3\bullet10)^2(30)^2$ is subtracted from 1225 we get 325, which must contain not only twice the product of 30 and the next digit in the square root, but also the square of the next digit. Now twice 30 is 60, and dividing 325 by 60 suggests 5 as the next digit in the square root. This happens to be exactly what we need, since $(2\bullet30\bullet5)+5^2 = 325$. See Fig. 2.

In a typical Victorian text[4], the algorithm is given without a geometric explanation:

1. Designate in the given number n "periods" of two digits each, counting from the decimal point toward the left and the right.
2. Find the greatest square number in the most left-hand period, and write its square root for the first digit in the square root of n. Subtract the square number from the left-hand period, and to the remainder bring down the next period providing a dividend.
3. At the left of the dividend write twice the first digit in the square root of n, for a trial divisor. Divide the dividend, exclusive of its right-hand digit, by the trial divisor, and write the quotient for the next trial digit in the square root of n.
4. Annex the trial digit of the square root of n to the trial divisor for a complete divisor. Multiply the complete divisor by the trial digit in the square root of n, subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.
5. So far there are two digits in the square root of n. Double this number and use as the next trial divisor, and proceed as before.

As an example, find the square root of 540577.8576 . See Fig. 3.

USING TABLES OF SQUARE ROOTS

The square root table (from the "Handbook of Chemistry and Physics") lists the square roots of a positive integer n from 1 to 1000, correct to seven significant figures. Since the square roots of $10n$ are also given in the table, values of the square roots of numbers from 1 to 10,000 can be found directly.

For the square roots of numbers above and below this range, a simple adjustment can be made. For example,

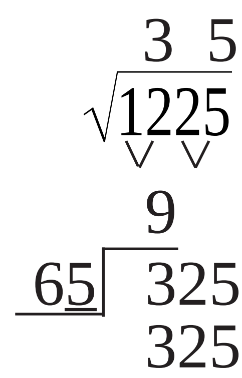

Figure 2

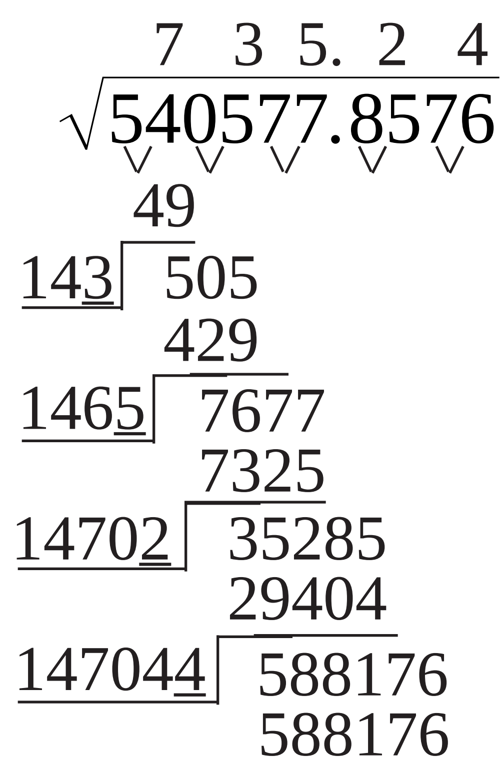

Figure 3

$\sqrt{10.268} = \frac{1}{100}\sqrt{10 \bullet 268}$. The tabular value for the square root of $10n$, for $n = 268$, is 51.76872, so the desired root is .5176872.

How and Why Newton's Method Works

The divide-and-average method, alias Newton's Method, is a common sense algorithm. Let's say we must find the square root of 125. Make a guess; say it's 11.1. Divide 125 by 11.1 and get a quotient 11.26126126. Take the average of 11.1 and 11.26126126, which yields 11.18063063 and let this be the next trial divisor. Now 125 divided by 11.18063063 is 11.18004915. Take the average and let this be the next trial divisor. Continue in this manner until the quotient is equal to the divisor, which is the square root of 125, correct to ten significant figures, 11.18033989.

Newton's Method generally is an iterative procedure used to approximate a solution of an equation $f(x) = 0$. It makes use of a corollary to the Intermediate Value Theorem in differential calculus: "If $f(a)$ denotes a function continuous on a closed interval $[a,b]$ and if $f(a)$ and $f(b)$ have opposite algebraic signs, then there exists some value of x between a and b for which $f(x) = 0$."[7] This means that there is at least one solution of $f(x) = 0$ in the interval (a,b).

Suppose f is differentiable and suppose r represents a solution of $f(x) = 0$. Then the graph of f crosses the x-axis at $x = r$ (See Fig. 4). Examining the graph, we approximate r. Our first guess is x_0. If $f(x_0) = 0$, then usually a better approximation to r can be made by moving along the tangent line to $y = f(x)$ at $x = x_0$, to where the tangent line crosses the x-axis at $x = x_1$.

Slope of line = $f'(x_0) = f(x_0)/(x_0 - x_1)$.
Solving for x_1, we get $x_1 = x_0 - f(x_0)/f'(x_0)$.

Repeating the procedure at the point $(x_1, f(x_1)$ and observing where the second tangent line crosses the x-axis, yields $f'(x_1) = f(x_1)/(x_1 - x_2)$.
Solving for x_2, we get $x_2 = x_1 - f(x_1)/f'(x_1)$.

If we continue in this manner, in the usual course of events, we get better and better approximations of r: $x_0, x_1, x_2, \ldots$, where $x_{n+1} = x_n - f(x_n)/f'(x_n)$. Of course, the method is not foolproof. Sometimes $f'(x_t) = 0$ so that x_{t+1} can't be calculated because there is division by 0. Sometimes the approximations $x_0, x_1, x_2, \ldots$ do not converge to the solution r.

Let's see how the divide-and-average method is really Newton's method. We are solving $x^2 - 125 = 0$, So $f(x) = x^2 - 125$. $f'(x) = 2x$.

Let $x_0 = 11.1$, then $x_1 =$ 11.1-(f(11.1)/f'(11.1)) = 11.1-((123.21 -125)/22.2) = 11.18063063. Now x_2 =11.18063063-((125.0065013-125)/22.36126126) = 11.18033989. Then x_3 turns out also to be 11.18033989, so we have the square root of 125.

Last Thoughts

I'm not sorry that we no longer must do hideous calculations to find the square root of a number. Looking back at past history makes us more informed and appreciative, too.

REFERENCES

1. Chace, Arnold Buffurn. *The Rhind Mathematical Papyrus*. Reston, VA: The National Council of Teachers of Mathematics, 1979.
2. Hodgman, Charles D. (Compiler). *Mathematical Tables from Handbook of Chemistry and Physics* (7th Ed.). Cleveland OH: Chemical Rubber Publishing Co., 1941.
3. Dugdale, Sharon. "Newton's Method for Square Root: A Spreadsheet Investigation and Extension into Chaos." *Mathematics Teacher* 91 (October 1998): 576-585.
4. Fish, Daniel W. (Ed). *The Progressive Higher Arithmetic for Schools, Academies, and Mercantile Colleges*. New York and Chicago: Ivison, Blakeman, Taylor & Co., 1875.
5. Fauvel, John and Jeremy Gray. (Eds). *The History of Mathematics: A Reader*. London: Macmillan Press, 1987.
6. Heath, Sir Thomas L. *A Manual of Greek Mathematics*. New York: Dover Publications, 1963.
7. Zill, Dennis G. *Calculus* (3rd Ed.). Boston: PWS-Kent Publishing Co., 1992.

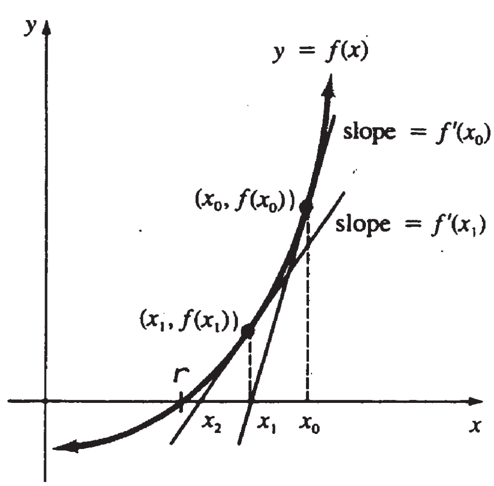

Figure 4

Book Review: *Knowing and Teaching Elementary Mathematics* by Liping Ma

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Knowing and Teaching Elementary Mathematics: Teachers'Understanding of Fundamental Mathematics in China and the United States. Liping Ma. Lawrence Erlbaurn Associates, Inc., 1999.
Cloth, $45.00, ISBN 0-8058-2908-3
Softcover, $19.95, ISBN 0-8058-2909-1

Note: The reviewer will refer to the book as KTEM.

For all who are concerned with mathematics education (a set which should include nearly everyone receiving the *Notices*), KTEM is an important book. For those who are skeptical that mathematics education research can say much of value, it can serve as a counterexample. For those interested in improving precollege mathematics education in the U.S., it provides important clues to the nature of the problem. An added bonus is that, despite the somewhat forbidding educationese of its title, the book is quite readable. (You should be getting the idea that I recommend this book!)

Since the publication in 1989 of the Curriculum and Evaluation Standards by the National Council of Teachers of Mathematics [NCTM], there has been a steady increase in discussion and debate about reforming mathematics education in the U.S., including increased attention from university mathematicians (cf. Ho]). Many mathematicians who take time to consider precollege education form an intuition that it would help the situation if teachers knew more mathematics. If these mathematicians get more involved in mathematics education, they are likely to be surprised by how little this intuition seems to affect the agenda in mathematics education reform.

Partly this noninterest in mathematical expertise reflects an attitude widespread among educators [Hi] that "facts," and indeed all subject matter, are secondary in importance to a generalized, subject-independent teaching skill and the development of "higher-order thinking." Concerning mathematics in particular, the study [Be] is often cited as evidence for the irrelevance of subject matter knowledge. For this study, college mathematics training, as measured by courses taken, was used as a proxy for a teacher's mathematical knowledge. The correlation of this with student achievement was found to be slightly negative. A similar but less specific method was used in the recent huge Third International Mathematics and Science Study (TIMSS) of comparative mathematics achievement in forty-odd countries. For TIMSS, U.S. students demonstrated adequate (in fourth grade) to poor (in twelfth grade) mathematics achievement [DoEd1-3]. To analyze whether teacher knowledge might help explain TIMSS outcomes, data on teacher training was gathered. In terms of college study, U.S. teachers appear to be comparable to their counterparts in other countries [DoEd1-3].

How can this intuition—that better grasp of mathematics would produce better teaching—appear to be so wrong? KTEM suggests an answer. It seems that successful completion of college course work is not evidence of thorough understanding of elementary mathematics. Most university mathematicians see much of advanced mathematics as a deepening and broadening, a refinement and clarification, an extension and fulfillment of elementary mathematics. However, it seems that it is possible to take and pass advanced courses without understanding how they illuminate more elementary material, particularly if one's understanding of that material is superficial. Over the past ten years or so, Deborah Ball and others [B1-3] have interviewed many teachers and prospective teachers, probing their grasp of the principles behind school mathematics. KTEM extends this work to a transnational context. The picture that emerges is highly instructive—and sobering. Mathematicians can

be pleased to have at last powerful evidence that mathematical knowledge of teachers does play a vital role in mathematics learning. However, it seems also that the kind of kind of knowledge that is needed is different from what most U.S. teacher preparation schemes provide, and we have currently hardly any institutional structures for fostering the appropriate kind of understanding.

The main body of KTEM (Chapters 1-4) presents the results of interviews with elementary school teachers from the U.S. (23 in all) and China (72 in all). The U.S. teachers were roughly evenly split between experienced teachers and beginners. Ma judged the group as a whole to be "above average." In particular, although "math anxiety" is rampant among elementary school teachers, this group had positive attitudes about mathematics: they overwhelmingly felt that they could handle basic mathematics and that they could learn advanced mathematics. The Chinese teachers were from schools chosen to represent the range of Chinese teaching experience and expertise: urban schools and rural, stronger schools and weaker.

The teachers' grasp of mathematics was probed in interviews organized around four questions. in summary form, the questions were as follows:

1) How would you teach subtraction of two-digit numbers when "borrowing" or "regrouping" is needed?

2) In a multiplication problem such as 123 x 645, how would you explain what is wrong to a student who performs the calculation as follows?

$$\begin{array}{r} 123 \\ \times\underline{645} \\ 615 \\ 492 \\ \underline{738} \\ 1845 \end{array}$$

(The student has correctly formed the partial products of 123 with the digits of 645, but has not "shifted them to the left," as required to get a correct answer.)

3) Compute $\dfrac{1\frac{3}{4}}{\frac{1}{2}}$. Then make up a story problem which models this computation, that is, for which this computation provides the answer.

4) Suppose you have been studying perimeter and area and a student comes to you excited by a new "theory:" area increases with perimeter. As justification the student provides the example of a 4 x 4 square changing to a 4 x 8 rectangle: perimeter increases from 16 to 24, while area increases from 16 to 32. How would you respond to this student?

These questions are in order of increasing depth. The first two involve basic issues of place-value decimal notation. The third involves rational numbers and also involves division, the most difficult of the arithmetic operations. It further requires "modeling" or "representation"—connecting a calculation with a "real-world" situation. The last problem, which was originally stated in terms of perimeter and area of a "closed figure," potentially involves very deep issues. Even if one replaces "closed figure" with "rectangle," as all the teachers did, one must still compare the behavior of two functions of two real variables.

On sheepskin the American teachers seemed decidedly superior to the Chinese: they all were college graduates, and several had MAs. The Chinese teachers had nine years of regular schooling, and then three years of normal school for teachers—in terms of study time, a high school degree. However, measured in terms of mastery of elementary school mathematics, the Chinese teachers came out better.

The rough summary of the results of the interviews is: the Chinese teachers responded more or less as one would hope that a mathematics teacher would, while the American teachers revealed disturbing deficiencies. In more detail, on the first two problems, all teachers could perform the calculations correctly and could explain how to do them, that is, describe the correct procedure. However, even on the first problem, fewer than 20% of the U.S. teachers had a conceptual grasp of the regrouping process—decomposing one 10 into 10 ones. By contrast, the Chinese teachers overwhelmingly (86%) understood and could explain this decomposition procedure. On the second problem, about 40% of the U.S. teachers could explain the reason for the correct method of aligning the partial products, while over 90% of the Chinese teachers showed a firm grasp of the place value considerations that prescribe

the alignment procedure.

On the third problem, a gap appeared even at the computational level: well under half of the American teachers performed the indicated calculation correctly. Only one came up with a technically acceptable story problem. Even this one was pedagogically questionable, since the units for the answer ($3\frac{1}{2}$) was persons, which children might expect to come in whole numbers. The Chinese teachers again all did the calculation correctly, and 90% of them could make up a correct story problem. Some suggested multiple problems, illustrating different interpretations of division.

On the fourth problem, the U.S. teachers did exhibit some good teaching instincts, and most, though not all, could state the formulas for area and perimeter of rectangles. However, when it came to analyzing the mathematics, they were lost at sea. Although most wanted to see more examples, over 90% were inclined to believe that the student's claim was valid. Some proposed to look something up in a book. Only three attempted a mathematical investigation of the claim, and again a lone one found a counterexample. The Chinese teachers also found this problem challenging, and most had to think about it for some time. After consideration, 70% of them arrived at a correct understanding, with valid counterexamples. Of the 30% who did not find the answer, most did think mathematically about the problem, though not sufficiently rigorously to find the defect in the student's proposal.

The contrast between the performances of the two groups of teachers was even more dramatic than this surnmary reveals. Some Chinese teachers gave responses that more than answered the question. They sometimes offered multiple solution methods. In the integer arithmetic problems, some indicated that, if the student was having trouble here, it meant that something more fundamental had not been learned properly. These comments point to a deeper layer of teaching culture that simply does not exist in the U.S. For example, American teaching of two-digit subtraction is usually based on "subtraction facts," the results of subtracting a one-digit number from a one- or two-digit number to get a one-digit number. These are simply to be learned by rote. The Chinese base subtraction on these same facts, but they refer to this topic as "subtraction within 20" and treat it as one to be understood thoroughly, since they regard it as the link between the computational and the conceptual basis for multidigit subtraction. In answering question 3, some Chinese teachers suggested that the given problem was too easy and offered harder ones. Also, the Chinese teachers were comfortable with the algebra that is implicitly involved in performing arithmetic with our standard decimal notation—for example, many explicitly invoked the distributive law when discussing multidigit multiplication. No such awareness of the algebraic backbone of arithmetic was shown by the American teachers.

In these first four chapters, KTEM also discusses issues of teaching methods. Without going into detail about this, I will report that the same limitations that teachers showed in giving a conventional explanation of a topic also prevented them from getting to the conceptual heart of the issue when using teaching aids such as manipulatives.

Thus, KTEM suggests that Chinese teachers have a much better grasp of the mathematics they teach than do American teachers. The hard-nosed might ask for evidence that this extra expertise actually produces better learning. Since Ma's work did not extend to a simultaneous study of the students of the teachers, KTEM cannot address this question. However, the substantial studies of Stevenson and Stigler [SS] do document superior mathematics achievement in China. (The Stevenson-Stigler project provided part of the motivation for Ma's work.) KTEM itself also provides some evidence of superior learning in China and of a sort directly related to the knowledge of teachers, as indicated in the interviews. The four interview questions were presented to a group of Chinese ninth-grade students from an unremarkable school in Shanghai. They all (with one quite minor lapse) could do all the calculations correctly and knew the perimeter and area formulas for rectangles. Over 60% found a counterexample to the student's claim about area and perimeter, and over 40% could make up a story problem for the division of fractions in question 3. These Chinese ninth-grade students demonstrated better understanding of the interview problems than did the American teachers.

One should also entertain the possibility that Ma was overly optimistic in judging her group of American

teachers to be "above average." However, this rating is broadly consistent with evidence from a much larger set of interviews conducted by Deborah Ball [B1-3] and also with the study [PHBL] of over two hundred teachers in the Midwest. In that study, for example, only slightly over half the subjects could provide an example of a number between 3.1 and 3.11. The portion of satisfactory responses to questions testing pedagogical competence was considerably smaller. The results of KTEM are also consistent with massive informal testimony from serious workers in professional development for teachers. The remarkable thing is that this problem—the failure of our system to produce teachers with strong subject matter knowledge and the negative impact of this failure—is not more explicitly recognized. Furthermore, solving this problem is not a major focus of mathematical education research and of education policy. I hope that KTEM will provide impetus for making it so.

> **"*The remarkable thing is that this problem—the failure of our system to produce teachers with strong subject matter knowledge and the negative impact of this failure—is not more explicitly recognized.*"**

KTEM gives us new perspectives on the problems involved in improving mathematics education in the U.S. For example, it strongly suggests that without a radical change in the state of mathematical preparedness of the American teaching corps, calls for teaching with or for "understanding," such as those contained in the NCTM Standards, are simply doomed. To the extent that they divert attention from the crucial factor of teacher preparedness, they may well be counterproductive. KTEM also indicates that claims that the traditional curriculum failed are misdirected. The traditional curriculum allowed millions of people to be taught reliable procedures for finding correct answers to important problems, without either the teachers or the students having to understand why the procedures worked. At the same time, students with high mathematical aptitude could learn substantially more mathematics, enough to support various technical or academic careers. This has to be counted a major success.

However, times have changed. The success of the traditional curriculum has fostered a mathematically based technology, which in turn has created conditions in which that curriculum is no longer appropriate. There are at least two reasons for this. First, we have cheap calculators that will do (at least approximately) any calculation of the elementary curriculum (and much more) with the push of a couple of buttons. These machines are typically much faster and more reliable than we are in doing these calculations. We also have "computer algebra" systems that will do more kinds of calculations than any single human knows how to do. It has always been one of the strengths of mathematics to seek reliable and systematic methods of computation, which has often meant creating algorithms. Anything that has been algorithmized can be done by a computer. Automation of calculation means that actually performing a calculation is no longer a problem working people usually have to worry about.

At the same time, it means that calculation is much more prevalent than before. Hence, people have to spend more time determining what calculation to do. That is the second reason that mathematics education needs to change. My daughter was a solid mathematics student but had no enthusiasm for the subject and did not expect to use it in whatever career she might choose. Now she works in management consulting, and she finds that her high school algebra comes in handy in creating spreadsheets. Simply learning computational procedures without understanding them will not develop the ability to reason about what sort of calculations are needed. In short, to function at work, people now need more understanding and less procedural virtuosity than they did a generation ago. (Who knows what they will need in another generation!)

The good news from KTEM is that there is no serious conflict between procedural knowledge and conceptual knowledge: Chinese teachers seem to be able to develop both in their students. (This is another intuition of most mathematicians I know who have been studying educational issues: it should be the case that procedural ability and conceptual understanding support each other. The Chinese teachers had a traditional saying to describe this learning goal: "Know how, and also know why.") The bad news is that our current

teaching corps is not capable of delivering this kind of double understanding: we can only reasonably ask them for procedural facility. Let us be clear that this is not a matter of teachers lacking certification or teaching outside their specialty, which are both frequent problems that aggravate the situation. The certification procedures, the teaching methods courses, most college mathematics courses, the recruitment processes, the conditions of employment, most current teacher development—none of these is geared to ensuring that U.S. mathematics teachers have themselves the understanding needed to teach for understanding. In short, virtually the whole American K-12 mathematics education enterprise is out of date.

How might the U.S. create a teaching corps with capabilities more like those of the Chinese teachers? To begin to answer, we should try to be precise as to what the differences are between the two groups. From the evidence of KTEM, I would list three salient differences:

1. Chinese teachers receive better early training—good training produces good trainers, in a virtuous cycle.

2. Chinese mathematics teachers are specialists. Making mathematics teaching a specialty can be expected to increase the mathematical aptitude of the teaching corps in two ways: it reduces the manpower requirements for mathematics education by concentrating it in the hands of the mathematically most qualified teachers, and it raises the incentives for mathematically inclined people to become teachers. Beyond its recruitment implications, it means that Chinese teachers have more time and motivation for developing their understanding of mathematics. This self-improvement is amplified by a social effect: specialization creates a corps of colleagues who can work together to deepen the common teaching culture in mathematics. Thus, making mathematics teaching a specialty works in multiple ways to increase the quality of mathematics education.

3. Chinese teachers have working conditions which favor maturation of understanding. U.S. teachers spend virtually their whole day in front of a class, while the Chinese teachers have time during the school day to study their teaching materials, to work with students who need or merit special attention, and to interact with colleagues. New teachers can learn from more experienced ones. All can study together the key aspects of individual lessons, an activity they engage in systematically. They can also sharpen their skills by discussing mathematical problems. Stevenson and Stigler [SS] have observed that time for self-development is a general feature of mathematics education in East Asia, which, to go by TIMSS [DoEd 1-3] as well as [SS], has the most successful systems of mathematics education in the world today.

The combination of training, recruitment, and job conditions that prevails in China helps produce a level of teaching excellence that Ma calls PUFM, "profound understanding of fundamental mathematics." PUFM and how it is attained is the concern of Chapters 5 and 6. It is important to understand that PUFM involves more than subject matter expertise, vital as that is; it also involves how to communicate that subject matter to students. Education involves two fundamental ingredients: subject matter and students. Teaching is the art of getting the students to learn the subject matter. Doing this successfully requires excellent understanding of both. As simple and obvious as this proposition may seem, it is often forgotten in discussions of mathematics education in the U.S., and one of the two core ingredients is emphasized over the other. In K-12 education the tendency is to emphasize knowing students over knowing subject matter, while at the university level the emphasis is frequently the opposite. (This cultural difference may well be part of the reason some university mathematicians have reacted negatively to the NCTM Standards. The emphasis on teaching methods over subject matter is prominent in the recommendations and "vignettes" of this document.) Both these views of teaching are incomplete.

What educational policies in the U.S might promote the development of a teaching corps in which PUFM were, if not commonplace, at least not extremely rare? This question is discussed in Chapter 7, the final chapter of KTEM. I would like to add my own perspective on the issue. The differences (1), (2), and (3) listed above suggest part of the answer.

Differences (2) and (3) are primarily matters of edu-

GETTING THE MATHEMATICS TO THE STUDENTS

Ma's notion of "profound understanding of fundamental mathematics (PUFM)," involves both expertise in mathematics and an understanding of how to communicate with students. Teacher Mao, one of the teachers Ma identified as possessing PUFM, eloquently expressed the need for both types of understanding:

> I always spend more time on preparing a class than on teaching, sometimes three, even four times the latter. I spend the time in studying the teaching materials; what is it that I am going to teach in this lesson? How should I introduce the topic? What concepts or skills have the students learned that I should draw on? Is it a key piece on which other pieces of knowledge will build, or is it built on other knowledge? If it is a key piece of knowledge, how can I teach it so students grasp it solidly enough to support their later learning? If it is not a key piece, what is the concept or the procedure it is built on? How am I going to pull out that knowledge and make sure my students are aware of it and the relation between the old knowledge and the new topic? What kind of review will my students need? How should I present the topic step-by-step? How will students respond after I raise a certain question? Where should I explain it at length, and where should I leave it to students to learn it by themselves? What are the topics that the students will learn which are built directly or indirectly on this topic? How can my lesson set a basis for their learning of the next topic, and for related topics that they will learn in their future? What do I expect the advanced students to learn from this lesson? What do I expect the slow students to learn? How can I reach these goals? etc. In a word, one thing is to study whom you are teaching, the other thing is to study the knowledge you are teaching. If you can interweave the two things together nicely, you will succeed. We think about these two things over and over in studying teaching materials. Believe me, it seems to be simple when I talk about it, but when you really do it, it is very complicated, subtle, and takes a lot of time. It is easy to be an elementary school teacher, but it is difficult to be a good elementary school teacher.

I would like to highlight the concern in Teacher Wang's statement for the connectedness of mathematics, the desire to make sure that students see mathematics as a coherent whole. This is certainly how mathematicians see it, and to us it is one of the major attractions of the field: mathematics makes sense and helps us make sense of the world. For me, perhaps the most discouraging aspect of working on K-12 educational issues has been confronting the fact that most Americans see mathematics as an arbitrary set of rules with no relation to one another or to other parts of life. Many teachers share this view. A teacher who is blind to the coherence of mathematics cannot help students see it.

—R.H.

cational policy. No revolution in American habits is required to create mathematics specialists or to give them opportunity for study and collegial interaction. What is mainly required is political will.

Regarding difference (2), the manpower considerations which favor mathematics pecialists beginning in the early grades are much stronger in the U.S. than in China. The U.S information society has much higher demand for mathematically able people than does the predominantly rural economy of China. Hence, schools face much heavier competition for mathematically competent personnel, and every policy that could lower their manpower requirements or improve their competitive position would benefit mathematics education. The difference in technological level also makes the need for coherent mathematics education greater in the U.S. than in China. Simply partitioning the present cadre of elementary teachers into math specialists and nonmath would already offer the average child a better-qualified (elementary) math teacher while relieving many others of what is now an onerous duty, all without raising overall personnel requirements. Some educators have for some time been call-

ing for mathematics specialists even in the elementary grades [US]. Perhaps the evidence from KTEM that having teachers who understand mathematics can make a difference already in the second grade (the usual time for two-digit subtraction) can convince education policyrnakers to heed this call.

Regarding difference (3), testimony from interviews of teachers with PUFM indicates that having time for study and collegial interaction is an important factor in developing PUFM. Such time would be most productive in the context of mathematics specialists—both study and discussion would be more focused on mathematics. Scheduling this time might be more controversial than creating specialists because it requires resources. In fact, in East Asia classes are larger than here, so a given teacher there handles about the same number of students as does a teacher in the U.S. [SS]. The improvement in lessons promoted by study and interaction with colleagues seems to more than make up for larger class size. There is currently in the U.S. a call to reduce class size. On the evidence of KTEM and [SS], I believe that the resources required for such a change would be better spent in eliminating difference (3).

What will be hardest is eliminating difference (1), that is, establishing in the U.S. the virtuous cycle, in which students would already graduate from ninth grade or from high school with a solid conceptual understanding of mathematics, a strong base on which to build teaching excellence. I expect that movement in that direction will, at least at the start, require massive intervention from higher education. New professional development programs, both preservice and inservice, that focus sharply on fostering deep understanding of elementary mathematics in a teaching context will need to be created on a large scale. Current university mathematics courses will not serve; as KTEM makes clear, the needs of teachers at present are of a completely different nature from the needs of professional mathematicians or technical users of mathematics, for whom almost all current offerings were designed.

I would recommend that these programs be joint efforts of education departments and mathematics departments to guarantee that the two poles of teaching, the subject matter and the pedagogy, both get emphasized. These departments have rather different cultures, and developing productive working relationships will not be a simple task, but with sufficient backing from policymakers who understand the current purposes and needs of mathematics education and the shortfall between current capabilities and these needs, some beneficial programs should emerge.

While the greatest need for improvement is probably at the elementary level, middle school and secondary teachers should not be neglected in the new professional development programs. Undoubtedly they know more mathematics than the typical elementary school teacher, but they too must have suffered from the lack of attention to understanding during their early education. Moreover, they need to deal with a larger body of material than do elementary teachers.

There is also the issue of texts. The Chinese teachers have materials, texts, and teaching guides that support their self-study. American texts tend to be lavishly produced but disjointed in presentation [Sc, DoEd1-3], and the teacher's guides do not help much either. Thus, the intervention programs should also work to create materials which will help teachers both learn and transmit a coherent view of mathematics. Eventually, these might be the basis for new texts.

At least at the start, these programs should be multiyear in scope, both so that teachers who do not have the favorable working conditions of Chnese teachers can nevertheless refresh and progressively improve their understanding of mathematics and so that those teachers who do obtain such working conditions can get to the level where self-directed study can be a reliable mode of improvement. One of the most outmoded ideas in education is that a teacher can reasonably be expected to know all that he or she needs to know, subject matter or teaching, at the start of work. Continued study, especially of subject matter, since teaching itself will provide plenty of opportunities for learning about children, should becorne the norm. If a program of this sort is implemented successfully, it should gradually become less necessary. The step-by-step improvement in education provided by teachers with better understanding and the gradual deepening of teaching culture by teachers interacting collegially among themselves should allow elaborate development programs to shrink and eventually disappear or to shift to study of more sophisticated topics, becoming, in subject matter at least,

more like standard college mathematics courses. This would constitute truly satisfying progress in our system of mathematics education. However, it will require great effort and resolve to achieve.

In summary, KTEM has lessons for all educational policymakers. Legislators, departments of education, and school boards need to understand the potential value in creating a corps of elementary-grade mathematics specialists who have scheduled time for study and collegial interaction. University educators need to understand teacher training in mathematics as a distinct activity, different from but of comparable value to training scientists, engineers, or generalist teachers. I believe that these mutually supportive changes would give us a fighting chance for successful mathematics education reform.

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REFERENCES

[B1] D. Ball, *The Subject Matter Preparation of Prospective Teachers: Challenging the Myths*, National Center for Research in Teacher Education, East Lansing, MI, 1988.

[B2] ____, *Teaching Mathematics for Understanding: What Do Teachers Need to Know about the Subject Matter?*, National Center for Research in Teacher Education, East Lansing, MI, 1989.

[B3] __, Prospective elementary and secondary teachers' understanding of division, *J. Res. Math. Ed*., 21 (1990), 132-144.

[Be] E. Begle, *Critical variables in mathematics education: Findings from a survey of empirical literature*, MAA and NCTM, Washington, DC, 1979.

[DoEd1] U.S. Department of Education, *Pursuing excellence: A study of U.S. fourth-grade mathematics and science achievement in an international context*, U.S. Government Printing Office, Washington, D.C., 1997.

[DoEd2] U.S. Department of Education. *Pursuing excellence: A study of U.S. eighth-grade mathematics and science teaching, learning, curriculum, and achievement in an international context*, U.S. Government Printing Office, Washington, D.C., 1996.

[DoEd3] U.S. Department of Education, *Pursuing excellence: A study of U.S. twelfth-grade mathematics and science achievement in an international context*, U.S. Government Printing Office, Washington, D.C., 1998.

[Hi] E. D. Hirsch, *The Schools We Need and Why We Don't Have Them*, Doubleday, New York, 1996.

[Ho] R. Howe, The AMS and mathematics education: The revision of the "NCTM Standards," *Notices of the AMS* **45** (1998), 243-247.

[NCTM] *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1989.

[PHBL] T. Post, G. Harel, M. Behr, and R. Lesh, Intermediate teachers' knowledge of rational number concepts, E. Fennema, T. Carpenter, and S. Lamon (eds.), *Integrating Research on Teaching and Learning Mathematics*, SUNY, Albany, NY, 1991, pp. 177-198.

[SC] W. Schmidt et al., *A Summary of Facing the Consequences: Using TIMSS for a Closer Look at United States Mathematics and Science Education*, Kluwer Academic Publishers, Dordrecht, 1998.

[SS] H. Stevenson and J. Stigler, *The Learning Gap, Why Our Schools Are Failing and What We Can Learn from Japanese and Chinese Education*, Paperback Reprint edition, Touchstone Books, January 1994.

[US] Z. Usiskin, The beliefs underlying UCSMP, UCSMP Newsletter, no. 2 (Winter 1988).

Senior Seminar: A Capstone Course in the Computer and Mathematical Sciences

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SUMMARY
This paper describes the evolution of a course developed to tie together many strands of activity encountered by students in the computer and mathematical sciences (**CMS**). The senior level course is required of all majors in our computer science, applied mathematics and statistics undergraduate degree programs. One of the primary purposes of the course is to refine writing and presentation skills needed for those who will later pursue individual research projects. Writing projects are organized around the theme of "Ethical Decision Making in the Computer and Mathematical Sciences". Numerous case studies are investigated. Additional topics in the course include designing resumes, starting a placement file, and a general introduction to the CMS culture. A course outline is given and various projects are discussed.

This paper is based on a talk given by the authors at the joint national meetings of MAA/AMS in San Antonio on January 14, 1999.

INTRODUCTION
Since 1984, all students at UH/D, regardless of major, must fulfill writing and speech requirements to graduate. These include at least one writing course given in their major department. In addition, since 1995, the university has required all graduating seniors to successfully complete some course material on ethics as it relates to their major.

The CMS department has developed a general approach to meet these requirements, utilizing its preexisting Senior Seminar (**SS**) course, which had been introduced in 1987. Originally the ethics requirement was satisfied by a separate course, but the material was added to the seminar in 1997 by increasing the credit hours from one to two.

In addition to the usual CMS courses, each major is required to successfully complete a speech course (SPCH 1304) and a technical writing course (ENG 3302), offered outside the department. These are prerequisites for our senior seminar course (CS/MATH 4294). Based on student performance in the seminar and other factors, the student follows up with either an individual research project (CS/MATH 4395), directed by a faculty member and/or an outside mentor, or the student selects one of our senior level writing courses (**W**) in the department to fulfill his/her writing requirement. W-courses, cross-listed, include Math Modeling and Computer Simulation (CS/MATH 4306), History of Mathematics and Computer Science (CS/MATH 4312), Parallel Programming (CS/MATH 4328), and Advanced Numerical Methods (CS/MATH 4301).

SENIOR SEMINAR—AN EVOLVING COURSE
While the original purpose of the SS course was to refine the writing and presentation skills needed to successfully complete later senior research projects, the course has now evolved to address the changing needs and college requirements.

To address the college-wide **ethics** requirement (noted earlier), the SS has developed a focus on ethical issues in some of the written and oral projects assigned. Because of the large number of required courses in our degree programs, we did not have the luxury of offering an entire course in ethics, in our department, as some other colleges have done [7].

EARLIEST VERSIONS OF SENIOR SEMINAR COURSE
Originally this course consisted of a number of readings from somewhat "popular" books oriented towards the computer and mathematical sciences, and then the assignment of short written and oral reports on the results of those readings. Among the books assigned were *Godel, Escher, Bach: An Eternal Golden Braid* [3], *Mathematics—The Loss of Certainty* [5], *The*

Dream of Reason [6] and *A Brief History of Time* [2].

In addition, each student prepared a one page vita/resume. Students discussed each others resumes and we discussed how to tailor different forms of a vita to different audiences. Students also critiqued each others' oral presentations and quickly learned to be more diplomatic in their criticisms as they discovered the truth of the adage that "What goes round, comes round."

The final written and oral projects in this SS course were the individual student Senior Project Research proposals. In the earlier years of our CMS programs, **all** our majors were required to follow the SS course with individual Senior Projects. However, as the number of majors increased dramatically, it became clear that other alternatives to individual Senior Projects were needed. Also, as noted earlier, in 1995, the college instituted an **ethics** component for all degree programs.

PRESENT FORM OF SENIOR SEMINAR COURSE

Course outline of topics:

a. Our present course begins with material from our text on *Ethical Decision Making and Information Technology—An Introduction with Cases* [4]. For ethical viewpoints focused on statistics see articles in the column Ethical Statistics, by Janice Derr, especially [1] in *AMSTAT NEWS*. We begin with definitions and draw out the distinctions among ethical, moral and legal actions and activities. We cover the first three chapters of the text, which deals with why ethics is important, how to make defensible decisions with a series of informal and formal guidelines, and different approaches to developing ethical principles, via various ethical paradigms, that can be used to support ethical decisions.

   We also consider special ethical challenges in the computer and mathematical sciences and how to respond to them. In addition, we consider material on how to solve ethical dilemmas by using sample case studies and applying the techniques covered earlier.

   In our course, each student is required to write short papers on two ethical problems recently in the news (CMS related)—either from the print media or from the WEB. In each paper, the student first summarizes the ethical issues involved, and then attempts suggestions for a resolution , using the text guidelines. In addition, each student selects one of the 19 case studies included in part II of the text and "works it out" in written and oral presentations.

   These case studies have generated considerable student discussion, and it is important to point out that, in general, there does not always exist a "best solution" to an ethical dilemma. What may work in one environment, may not succeed in another. These discussions have often caused students to re-evaluate various viewpoints as they apply ethical guidelines to the "real world". Some of our students, who are already working full time in CMS professions, often point out to the other students how other "side" issues, office politics, etc…may affect the success of various approaches to solving ethical dilemmas. Discussions help to underscore the value of constructive criticism and teamwork to make better decisions.

   We believe that this added **ethics** component in our course has been extremely valuable, not only in introducing and developing ethical principles (as they relate to CMS), but also in introducing our students to the CMS cultural paradigm that they are becoming a part of.

b. In addition to writing projects, students have discussed ethical problems and issues that they perceive in the CMS department, the university and/or in their fields. Such discussions can have meaningful impact on the true importance of ethics. Past discussions have led to student proposals for written ethical guidelines for the use of departmental computer facilities and labs to augment the written ethical guidelines already in place for the academic computing center. Some of our math education students have also pointed out ethical problems related to the manipulation of public school student assessment statistics on state mandated test, and also noted problems with the abuse of emergency staffing guidelines in public school math and science hiring.

c. In addition to the ethical components of our SS

course, each student is required to make an oral presentation on some topic in his/her field, that is not required, but that he/she believes should be understood by all graduates in the field. Some of the past topics discussed include:

1. How to do basic business math calculations such as amortization schedules for house and car payments, annuities, IRAs, credit cards and lotteries;
2. How to navigate the WEB;
3. Use of graphical/programmable calculators;
4. Understanding basics of computer hardware—e.g. take the back off your PC, how disc drives, etc…actually work;
5. How to use spreadsheets for taxes, investing, record keeping…as well as for math/stat analysis.
6. Logic
7. Graph Theory
8. Current computer languages and innovations

From these and similar suggestions, we occasionally have added short discussions on some of these "slighted" topics. Among other topics added for brief discussions are cardinality, methods of proof, and the problems of completeness and consistency in mathematics.

In these discussions, students have expressed a serious interest in contributing to the continued development of our CMS programs.

d. Each student constructs a one-page vita/resume. In introducing this subject, the class is first introduced to the student placement center facilities. They are encouraged to develop a student placement file which will include copies of letters of recommendation and transcripts as well as a copy of their vita. The idea of tailoring different vitas for different audiences is noted. Each student constructs a first draft vita, then copies are provided to all students, and they discuss which features they want to add/or delete. After these discussions, each student constructs a "final" vita. = It is emphasized that there is no "right" or "wrong" format. If it works, go with it. These discussions can lead to such issues as whether foreign students should mention foreign language abilities, green card status…; as well as how much and how should they include in the vita on their areas of specialization or special competence.

e. Students are required to attend the weekly CMS Colloquium Series. These talks expose them to the CMS culture, with speakers from industry and government as well as academia. In addition, some of the talks in Bio-mathematics, bio-statistics and other areas have generated student senior projects as well as graduate studies by some of our graduates.

f. The capstone of this course is the writing and oral presentation of potential Senior Project proposals. The merit of these proposals is considered when deciding which students will continue with individual Senior Projects, in the following semester, and which students will take W-courses to complete their final writing requirements in CMS. A few of the over 150 individual Senior Projects are listed below.

Note that many of our projects were developed jointly with NASA, industry or other non-CMS co-sponsors including grants and the large Biomedical community in the Houston Medical Center.

1. Investigations of Certification Issues for Secondary Teachers
2. Bio-mathematical Modeling of the RNA Cell Cycle
3. Data Gathering and Analysis of Student Usage of the Math Lab Tutoring Center
4. An Investigation into the Feasibility of Incorporating a Fuzzy Logic Control System into a Spider Robot
5. Neural Networks and Fuzzy Logic to Control an Unmanned Reconnaissance Vehicle
6. Economic Cost Models of the NASA Space Station
7. Decision Tree Analysis of Gas Pipeline Options—A Feasibility Study
8. Reliability Investigations Using Fault Tree Analysis
9. A Comparison of Inner City, Suburban and Rural High Schools to Determine Differences (if any) in Qualifications, Background and Experience of Math Teachers
10. Development of a 2-Dimensional Video Game

11. Statistical Survey of Math Students at the University of Houston-Downtown and at Houston Community College
12. Mathematical Simulations for a Megaplex Theater
13. Applications of VRML and Java in Simulation
14. Job Scheduling on Heterogeneous Networks Using Mobile Agents

SOME COMMENTS ON THE PRESENTATION OF SENIOR SEMINAR

Since incorporating **ethics** and the other additional topics described above, three different faculty members have offered the modified course. The course is offered as a one semester, 2 credit hour course meeting on two days a week for 50 minutes each class. All instructors have included basically the same material as outlined above. Each supplies the students with a detailed syllabus including due dates for the various projects. While all weight the written and oral presentations at about 60 percent of the course grade and the resume and class participation the same (10 percent each), the instructors vary on the relative allocations for the Senior Project proposal and the final exam.

It is important to stress the need for student attendance and participation in the classes and for attendance at the Colloquium. When students enter their senior year, many demands are made on their time, and if the instructor is not careful, students may tend to skip classes to work on their projects and other course work. At least one of the SS instructors reduces the final grade for missing classes. Similarly, students need to be strongly encouraged not to miss project deadlines. In the real world, missing days of work and failing to meet deadlines can have a real impact on your career (or lack of one). Since this seminar is to help the students prepare for the real world that CMS practitioners will live in, the instructors should settle for no less than industry, government and the rest of the real world will demand.

STUDENT FEEDBACK AND RESPONSES TO THE COURSE

In addition to observations by the students in classes and on course evaluation forms, the final exams included questions that asked the students to (1) briefly describe an ethical problem that you perceive in the university or in your job and explain how you believe the problem could be addressed; and (2) briefly describe what you liked or disliked about this course and topics you believe should have been added, changed or excluded.

Note: The student breakdown in the course is approximately 60% computer science majors and 40% in applied mathematics and/or statistics.

Student comments on ethical problems, they perceive at the university, included the following:

1. "One of the ethical problems I see in the university is the loopholes around prerequisites for certain classes…it shouldn't be so easy for students to circumvent this. The telephone registration system has been updated, so it may not allow it now, but before you could sign up for any class you wanted, regardless of whether you had the prerequisites or not…I think the teachers shouldn't be able to give final approval about whether the prerequisites should be waived; the final approval should come from the department (chair). They need to know how many people are getting around prerequisites and track how well those who are waived do in those courses.

   Similar comments in the classroom generated a considerable amount of discussion and in the course of this discussion, it became clear why one of our courses offered in the previous semester had an extreme bimodal grade distribution. Incidentally, the telephone registration did not solve this problem! These and similar comments were helpful in addressing issues of which some of the faculty were unaware.

2. "Some instructors use the same assignments and tests every semester. Students who have access to this have an unfair advantage."

   Suggestions to address this issue included: "Students should have to explain their solutions." and "Can solve the problem by asking instructors to collect all tests and assignments they plan to reuse or by assigning different projects/tests for every semester."

3. "In classes where there is a computer based assignment, both math and computer science courses, students routinely have other fellow stu-

dents do/complete (or overly help) them with their computer based assignments. Since most of these types of assignments are done outside of the actual classroom, the instructor has no way of knowing who actually completed the work. To solve this, since more and more computer classrooms are available, more in-class time needs to be devoted to mini program-based quizzes to force students who routinely rely on others to do their own work in order to receive a passing grade."

With the increasing integration of computer projects into math courses, this is becoming a serious issue in courses in the applied mathematical sciences and statistics, as well as in computer science courses. The calculus sequence and linear algebra courses at our university require considerable computer activity as do the numerous statistics courses that are heavily dependent on statistical software packages.

4. "One of the ethical problems…Allowing too much freedom to students in the labs. More often than not, I find students chatting, surfing the net or doing other things that are not school related. The problem is that they are using computer resources and university resources for personal pleasure. The computers that we have should be used for school related projects only."

   "The problem can be addressed as such: Only allow students to use the computers during lab hours and enforce that…Allow the assistant to monitor activity of the students working in the lab and allow them to report what they find. Make people sign in and explicitly detail why they are there (make this mandatory to use the labs).

   In this regard in earlier classroom discussions, one student employed by one of the major computer companies noted that in an unannounced inspection 60 workers were terminated for unacceptable employee use of the WEB during office hours.

5. Other real or imagined ethical problems were noted. Many of these provide interesting material for use in the next semester.

SOME STUDENT COMMENTS ON THE COURSE

1. "I liked the ethics portion; it made me think seriously about some practices that I have seen or even considered practicing myself and ask, is this right? Is it ethical?"—This student mentioned that at his job one employee complained of sexual harassment, and before it ended, a number of employees were terminated!

2. "I liked this course because you were allowed and encouraged to openly discuss problems of and solutions for the CMS department with a reasonable amount of discretion. This tells the students that the university is concerned about the type of education they are providing."

3. "It was easy to talk in a small group of people." Another noted that "I liked the open discussions we had in class about different ethical issues…relaxed environment."

4. A number were concerned that the 50 minute class period didn't give enough time to get into particular discussions. Some comments included: "More time should be included for more discussions" and "Change time of offering, since once discussion begins, it is hard to stop."

5. "A type of exam based on the material from the first and second chapters of the ethics book is needed." At the present time the course consists of only a series of projects (described earlier) and a final exam.

SUMMARY AND CONCLUSIONS

From the student comments and class participation, we believe that our present Senior Seminar course has been very successful in exposing the students to the ethical dimensions of the CMS professions and providing them with tools to make good ethical choices. We believe that the written and oral projects have helped them to more effectively communicate their own CMS related ideas and activities. The high quality of the various student discussions of issues indicates that they have a serious stake and interest in being a part of the CMS community. Their many thoughtful comments on how to solve ethical problems as well as suggestions to improve our academic community are greatly appreciated. We believe that this course has helped not only our students to become more active and involved members of our profession, but it has also helped our faculty to become

more aware of some ways we can act to improve our departmental programs.

REFERENCES

1. Derr, Janice, "Teaching Ethical Practice in Statistics," *AMSTAT News*, May 1995, pp.13-14.

2. Hawking, Stephen W., *A Brief History of Time From the Big Bang to Black Holes,* Bantam Books, New York, 1988.

3. Hofstadter, Douglas R., *Goedel, Escher Bach: An Eternal Golden Braid,* Vintage Books, New York, 1980.

4. Kallman, Ernest A., and Grillo, John P., *Ethical Decision Making and Information Technology—An Introduction with Cases,* 2nd ed., McGraw Hill, New York, 1996.

5. Kline, Morris, *Mathematics—The Loss of Certainty,* Oxford University Press, New York, 1980.

6. Pagels, Heinz R., *The Dreams of Reason—The Computer and the Rise of the Sciences of Complexity,* Bantam Books, New York, 1988.

7. Webber, Robert P., "A Course in Mathematical Ethics", *Humanistic Mathematics Network Journal*, #16, Claremont, Ca. 91711.

Weizmann Day's "Math Night" Brings Parents to their Knees

Jewish Community News
Covina, CA

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It didn't approach the scale of the Atlanta train yard scene in *Gone with the Wind*, but a visitor entering the long assembly hall a Weizmann Day School in Pasadena on the evening of February 1 would have witnessed the kneeling, sitting or reclining bodies of dozens of children and their parents scattered across the carpeted floor.

It was Weizmann's first annual "Math Night" and everyone was busy, very, very busy counting, measuring, estimating, spinning, stacking, building to the hum of voices and delighted laughter. Weizmann parents were receiving a hands-on demonstration of the activities generated by the school's new math program, and everyone had something to do. "This isn't math as I remember it," one parent remarked as his daughter carefully slipped a tape measure around his head. "It's not just math, Daddy. It's fun." The smiling faces around the room offered this assessment their clear and enthusiastic support.

Approached as a sequenced series of games and involving tasks, each appropriate to the grade level engaged, Weizmann's new math curriculum draws students into the pleasures and challenges of quantitative analysis. This innovative program, pursued in progressive steps from kindergarten through sixth grade, employs the student's natural curiosity and delight with problem solving to generate the enthusiasm so central to successful learning. In kindergarten, toothpicks and marshmallows become a creative gateway to geometric form. In third grade the length of the big toes and the circumference of the neck give insight into measurement and proportion. In fifth grade estimating games, such as "How many beans in the jar?" and "How many rubber ducks in the frog pond?" illustrate new approaches to volume and spatial relationships.

Progressing from simple activities in the early grades to the more complex skills required by "Top It," a uniquely designed series of card games in the higher grades, students are invited into a process geared toward making "math anxiety" an oddity of the distant past.

On "Math Night" parents were invited to the school to join their children in this process not simply to involve the parents in their children's education—Weizmann parents are well-known for their eager and very active support of the school's mission—but also to give everyone a fun night out learning new ways of looking at traditional studies. By all estimates, from the 80% attendance record to the 100% consumption of the cookies and ice cream provided for dessert, the event was a un-compromised success no matter how you add it up.

Book Review: *Geometry from Africa: Mathematical and Educational Explorations* by Paulus Gerdes

Claudia Zaslavsky
New York, NY

Geometry from Africa: Mathematical and Educational Explorations. Paulus Gerdes. Washington, DC: The Mathematical Association of America, 1999.

Books by Paulus Gerdes fill a complete shelf of my bookcase—and I don't have all of them. A professor of mathematics and former rector of the Pedagogical University in Mozambique, he writes in Portuguese, French, English, and German, and has published over a hundred journal articles and books, several of which have won awards. In this 1999 MAA publication, he has gathered into one compendium many of the ideas from his previous works. The volume is beautifully illustrated by hundreds of his own masterful computer graphics, as well as by photographs of the art objects discussed in the text.

In his insightful Foreword, Arthur B. Powell (Rutgers University) writes: "Through [Paulus Gerdes], we learn of the diversity, richness, and pleasure of mathematical ideas found in Sub-Saharan Africa. From a careful reading and working through this delightful book, one will find a fresh approach to mathematical inquiry as well as encounter a subtle challenge to Eurocentric discourses concerning the when, where, who, and why of mathematics" (p. *v*).

Gerdes is a major contributor to the emerging field of ethnomathematics. He and his research team have been investigating the geometrical ideas encoded in African cultural products, and have brought to light the mathematical concepts "hidden" or "frozen" in these artifacts and ornaments.

The book is divided into four sections, each concluding with a voluminous bibliography. The first section, "On geometric ideas in Africa south of the Sahara," opens with a discussion and reproductions of ancient rock paintings and petroglyphs, pots and gravegoods from various parts of the continent, some dating back several millennia. Gerdes invites the reader to analyze the symmetries in thousand-year-old textiles from Mali, interwoven in a variety of patterns of white and indigo-dyed strands. Many pages are devoted to the stunning productions of the Kuba artists of Congo/Zaire, all adorned with geometric designs and repeated patterns—objects of copper and iron, woven baskets and mats, carved wooden cups and drums, beaded hats, pottery, and much more. Photographs and carefully drawn computer renderings encourage the reader to apply the concepts of transformational geometry to the patterns.

The kente cloth of the Ashanti (Asante) of Ghana has become popular in the United States in its many derivative forms. Less familiar are the repeated patterns stamped on adinkra cloth and the designs on brass weights used to weigh gold dust in past centuries.

Included in this section are examples of various arts in many areas of the continent—patterns in body painting and hair styles, mural decoration, basket weaving, leather work, calabash engraving, wood carving, and decorative metal work. Gerdes concludes with the statement: "May the examples given in this chapter convey to the reader an idea for how women and men all over Africa south of the Sahara, in diverse historical and cultural contexts, traditionally have been geometrizing" (p. 50).

In chapter two, "From African designs to discovering the Pythagorean Theorem," Gerdes demonstrates how "African ornaments and artifacts may be used to create an attractive educational context for the discovery of the Pythagorean Theorem and for finding proofs of it" (p. 55). He begins with a description of button-making in southern Mozambique, using two strips of a palm leaf. With some abstraction and manipulation of the process, one arrives at a diagram showing that the sum of the areas of the squares on the two legs of a right triangle is equal to the area of the square on the hypotenuse. From this humble example, he extends his exploration to a variety of traditional African designs having four-fold rotational symmetry.

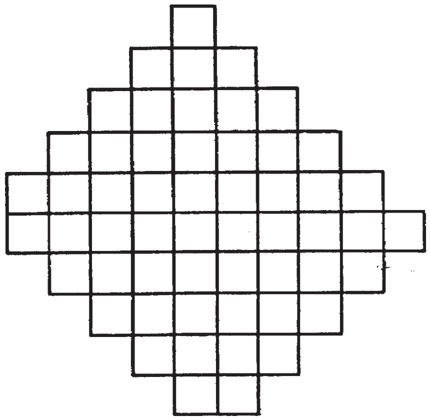

Figure 1
Kuba Network

From several different patterns based on squares, Gerdes derives "an infinity of proofs" of the right-triangle theorem, as well as formulas for the sum of the first (n - 1) natural numbers and the first n odd integers. A different embodiment of squares in a woven mat leads, again, to the theorem, as well as to Latin squares, magic squares, and arithmetic modulo n.

The third section, "Geometrical ideas in crafts and possibilities for their educational exploration," deals with such topics as symmetry of strip patterns and plane patterns, areas and volumes of various shapes, and, surprisingly, the connections between the geometry underlying a hexagonal basket-weaving technique and that of models of certain carbon molecules. The Nobel prize in chemistry was awarded in 1996 for the discovery of these molecules, named buckminsterfullerenes.

I have a special affection for chapter four, "The 'sona' sand drawing tradition and possibilities for its educational use." Long before I had met Paulus Gerdes and learned about his work, I included in my book *Africa Counts: Number and Pattern in African Cultures* (1973, 1999) a brief description of network patterns of the Kuba and Chokwe (Tchokwe, Jokwe) peoples. Kuba children drew designs in the sand in imitation of adult fishing nets (Fig. 1). The children challenged a visiting Hungarian ethnologist, Emil Torday, to draw each of these figures in one sweep, without lifting his finger or retracing a line segment, something he declared to be an "impossible task." Yet these children were doing it. The Chokwe of Angola, as well as neighboring peoples, had a tradition of drawing *sona* in the sand to accompany stories, proverbs, and riddles, a way of transmitting knowledge to the younger generation (Fig. 2).

Soon after the publication of *Africa Counts*, I collaborated with several middle and secondary classes to adapt these networks, an aspect of graph theory, for classroom use (Zaslavsky 1981, 1991, 1996, in press). Students found it difficult to believe that these "fun" activities were really math! As one mathematically advanced ninth-grader commented, with his algebra and geometry classes in mind, "They are nice for recreation, but they are not *real* math." On the other hand, an African-American sixth-grade student wrote: "I feel very strongly and am in deep thrust [sic] with my black people, and the math has made me feel better."

Gerdes opens the chapter with information about the Chokwe and neighboring peoples, and the role of *sona* in their culture. Mathematically, he goes far beyond the content of the lessons I had prepared. Among the concepts he applies to these *sona* are symmetry, monolinearity, chain rules for joining several *sona*, arithmetic progressions, and various numerical patterns. From his analysis of these *sona*, Gerdes derived a new class of geometric activities that he named Lunda-designs and Lunda-patterns, from the Lunda region inhabited by the Chokwe. Applying the symmetries of certain *sona*, one can derive sets of fascinat-

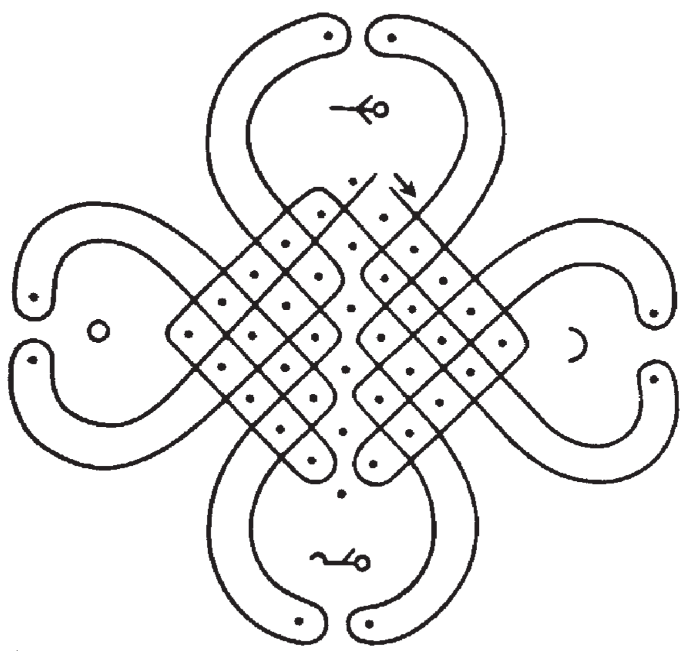

Figure 2
Chokwe Network

ing geometric designs, leading to the building of fractals, matrix addition, polyhedra designs, and polyomino activities appropriate for children (and adults) of any age.

In his Foreword, Arthur Powell quotes Gerdes on his research methodology:

> We looked to the geometrical forms and patterns of traditional objects...and posed the question: why do these material products possess the form they have? In order to answer this question, we learned the usual production techniques and tried to vary the forms. It came out that the form of these objects is almost never arbitrary, but generally represents many practical advantages and is, quite a lot of times, the only possible or optimal solution of a production problem...Applying this method, we discovered quite a lot of 'hidden' or 'frozen' mathematics (p.*vii*).

Mathematicians, students of mathematics, anthropologists, and the lay public can learn a great deal of mathematics from this book, while gaining an appreciation of the beauty and intricacies of African art and culture.

REFERENCES

Zaslavsky, Claudia. 1973, revised 1999. *Africa Counts: Number and Pattern in African Cultures*. Chicago: Lawrence Hill Books.

———. 1981. "Networks: New York subways, a piece of string, and African traditions." *Arithmetic Teacher* 29 (October):36-43.

———. 1991. "Multicultural mathematics education for the middle grades." *Arithmetic Teacher* 38 (February):8-13.

———. 1996. *The Multicultural Math Classroom: Bringing In the World*. Portsmouth, NH: Heinemann.

——— In press. "African Networks and African American Students." In Marilyn Strutchens, ed. *Changing the Faces of Mathematics: Perspectives on African Americans*. Reston, VA: National Council of Teachers of Mathematics.

Qualitative Quantities

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Numbers, we thought, were empty holes
for things in multiples and rows:
spaces for lineal progressions,
holders for nominal abstractions—
empty, vacuous, estranged
from colors and emotive names.
    But now we find that numbers hold
    associations—fragrant, bold—
    innoculants against the cold.

An oily seven, rounded five
are bouncy, quirky, half alive.
Eight is rough and ten is smooth,
and nine an incipient burst balloon.
Numbers with personality
are the ultimate irrationality
    in a field renowned for its perfection:
    math as queen, as Number One
    in platonic space, ad infinitum.

Only humans would make a religion
from wholes, square roots, and fractions,
and greet with unfathomable horror
the realization of the zero;
and find in numerology
a qualitative alchemy—
    to work with abacus and crucifix
    in efforts hardly apolitical
    to fix the names of enemies to 666.

There is more between heaven and earth, Horatio,
than square and cube and ratio;
between stone tool and complex widget
more than the abstract humble digit.
Embodying secret codes of hope and fate,
justice, transcendental love and hate,
    like Rorschachs, numbers are used to carry
    sums of self—the qualitative territory
    of the perfect imaginary.

Number, Infinity, and Truth: Reflections on the Spiritual in Mathematics

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1. INTRODUCTION

Mathematics, as a whole, has a reputation for precision, unambiguity, and cold reason. These are rarely considered spiritual properties which, in contrast, tend towards mystery, paradox, and transcendence. Nevertheless, mathematics has had a spiritual aspect throughout its history. Indeed, in many cultures there has been little to distinguish the mathematician from the priest.

In ancient Egypt, in Mesopotamia, and in the Maya kingdoms of Guatemala and Mexico, mathematical knowledge and spiritual knowledge converged and merged in a spectacular interplay. This mathematico-spiritual interplay existed in other cultures besides the great civilizations of antiquity and persists to the present day.

This essay looks at the nature of the interplay between mathematics and spirituality in some traditional and modern contexts.

2.0 TRADITIONS

The intersection between mathematics and spirituality appears in many religious traditions. In the western tradition, we have the case of the Pythagoreans (circa 500 B.C.E.) who assigned a definite number to everything material and spiritual. *One* was the number of reason, *four* of justice, and *five* of marriage (Burton, 1997:92). Numerology continues to attract devotees as any trip to a "new age" bookstore or a quick scan of the internet will reveal.

So numerous are the intersections of religion and number that to even mention all of the traditions in which they occur would require a book-length treatise. In this section I'll highlight a handful of examples of the interplay of mathematics and religion chosen for their diversity and ability to shine light on this interplay. The examples come from the Upanishads of India, the religious beliefs of the Oglala Sioux of North America, and the visions of a West African Dogon elder.

2.1 THE FIVE- AND SEVEN-FOLD CHANTS OF THE CHANDOGYA UPANISHAD

The intersection of mathematics and religion and spirituality in the realm of numbers is well known. One of the oldest of these intersection points is found in the Upanishads, Hindu religious texts written between 800–300 B.C.E., which may represent a tradition dating back to 4000 B.C.E. (Hume, 1977; Frawley, 1995).

In one of these texts, known as the Chandogya Upanishad, the Second Prapathaka (Chapter) is an extended discussion of the structure and meaning of the five-fold and seven-fold chants. The five-fold chant is characterized by the Hinkāra (preliminary vocalizing), the Prastāva (introductory praise), the Udgitha (loud chant), the Pratihāra (response), and the Nidhana (conclusion). An example is given in the way we may reverently understand the rainstorm:

> "In a rain-storm one should reverence the five-fold Sāman.
> The preceding wind is a Hinkāra.
> A cloud is formed—that is a Prastāva.
> It rains—that is an Udgitha.
> It lightens, it thunders—that is a Pratihāra.
> It lifts—that is a Nidhana."
> (Hume, 1977:191)

The seven-fold chant adds two new sections to the five-fold chant. The Adi (beginning) appears just before the Udgitha, and the Upadrava (approach to the end) precedes the Nidhana. The Upanishad tells us how the course of the sun through the sky parallels the seven-fold chant:

> "When it is before sunrise—that is a Hinkāra…
> Now, when it is just after sunrise—that is a Pratāva…

> Now, when it is the cowgathering time—that is an Adi…
> Now, when it is just at midday—that is an Udgitha…
> Now, when it is past midday and before the afternoon—that is a Pratihāra…
> Now, when it is past afternoon and before sunset—that is an Upadrava…
> Now, when it is just after sunset—that is the Nidhana…"
> (Hume, 1977:194)

Several other examples of these five- and seven-fold patterns are related in the Second Prapathaka of the Chandogya Upanishad, but the most striking mathematical passage is found in the Tenth Khanda (verse). This passage explains the mystical significance of the syllables in the categories of the seven-fold chant.

> "Now then, one should reverence the *Sāman* (chant), measured in itself, as leading to death.
>
> *hinkāra* has three syllables. *prastāva* has three syllables. That is the same.
>
> *adi* has two syllables, *pratihāra* has four syllables. One from there, here—that is the same.
>
> *udgitha* has three syllables. *upadrava* has four syllables. Three and three—that is the same, one syllable left over. Having three syllables—that is the same.
>
> *nidhana* has three syllables. That is the same, too. These are twenty-two syllables.
>
> With the twenty-one one obtains the sun. Verily, the sun is the twenty-first from here. With the twenty-two one wins what is beyond the sun. That is heaven."
> (Hume, 1977:194-5)

In the Tenth Khanda we see mathematics (arithmetic and algebra) at the service of spirituality. The Upanishadic calculation aims towards heaven using the syllables of the classificatory words of the chant as its data. We see two fundamental characteristics of mathematics at works here: balance and sum. The Khanda calculates the sum of the syllables to 22, while emphasizing a balance between terms based upon 3. The first paragraph affirms a balance between *hinkāra* and *prastāva*, 3 = 3. In the second paragraph, *adi* and *pratihāra* are brought into balance with some simple algebra "One from there, here." That is, take one syllable from *pratihāra* and add it to the syllables of *adi*. In modern notation, the Upanishad computes

$$4-1 = 2+1$$
$$3=3$$

Achieving balance between *udgitha* (3 syllables) and *upadrava* (4 syllables) caused the writer of this Khanda some difficulty. The writer's solution is to emphasize that both words have three syllables and that the fourth syllable of *upadrava* is "left over." Mathematically, this maneuver is suspect. Nonetheless, the writer clearly recognizes the problem and assures us that it is not a problem. Indeed, the extra syllable returns in the sum with powerful impact. We can summarize the mystical mathematics of the Tenth Khanda this way:

$$[3+3]+[(2+1)+(4-1)]+[3+3]+1+3=21+1$$

where the square brackets group balanced pairs. The writer of the Tenth Khanda employed mathematics to emphasize and perhaps justify the terms and relationships between terms of the seven-fold chant.

2.2 SACRED NUMBERS OF THE OGLALA SIOUX

The importance of numbers in the world-view of the indigenous peoples of the Americas is well documented. We find sacred or mystical numbers in the cultures of the Hopi (Young, 1988), the Maya (Freidel, et. al., 1995), the Inka (Urton, 1997), the Ojibwa (Closs, 1986), and the Oglala Sioux (Powers, 1977). I choose the Oglala for special examination here because the interplay between number and religion in their culture is representative of a large number of Indian cultures of North America. Indeed, the quadripartite view of the universe and the supernatural is prevalent in North America (Levi-Strauss, 1968; Bullchild, 1990; Young, 1988). The information about the Oglala presented in this section is from Powers (1977:48-51).

The numbers 4 and 7 are sacred to the Oglala. *Four* is of central importance because it is the number of world directions, the number of divisions of time (day, night, moon, year), and the number of periods of life (baby, child, adult, old age). The number four is cen-

tral to certain aspects of human physiology, including the important observation that four is the number of fingers on each hand, the number of toes on each foot, and the total number of big toes and thumbs on a person. We thus have the "human" compound equation:

$$5\text{-}1=5\text{-}1=5\text{-}1=5\text{-}1=1+1+1+1$$

In the supernatural world, the number four is central to the structure of *wakantanka*, an Oglala concept that embodies all supernatural beings and powers. The supernatural is classified into four aspects, each of which is subdivided into four more. This hexadecimal structure looks like this:

- *Wakan akanta* (Superior *wakan*)
  - Sun
  - Sky
  - Earth
  - Rock
- *Wakan kolaya* (Those who *wakan* call friends)
  - Moon
  - Wind
  - Falling Star
  - Thunder-being
- *Wakan kuya* (lower or lesser *wakan*)
  - Buffalo
  - Two-legged
  - Four winds
  - Whirlwind
- *Wakanlapi* (those similar to *wakan*)
  - Shade (Apparition)
  - Life (Breath)
  - Shadelike
  - Potency

It must be emphasized that this 4x4 structure is not the classificatory scheme of the European anthropologist, but the Oglala themselves emphasize the "fourness" of their cosmology. When a sweat lodge is constructed 16 willow saplings are needed, one for each aspect of *wakantanka*.

Seven, the other primary sacred number of the Oglala, is constructed from *four* arithmetically and spiritually. To the four directions (West, North, East, and South) are added the zenith, the nadir, and the center of the universe. Seven is explained as 4+2+1; the four directions plus sky and earth plus the universe. Prayers are smoked and sung to these seven directions when invoking the supernatural. The sociopolitical groups of the Sioux and the birth-order names of Oglala children also follow the 4+2+1 = 7 pattern.

Significantly, within the 4x4 structure of the *wakantanka* we also find the calculation 4+2+1 = 7. *Wakan akanta* and *wakan kolaya* are grouped together as *wakan kin* (the Sacred). *Wakan kuya* and *wakanlapi* form *taku wakan* (Sacred Things). The entire collection is *wakantanka*. Thus, the heptadic classification is seen simultaneously as monadic, dyadic, and tetradic.

Beyond the examples given above, all natural and cultural phenomena are classified by the Oglala into structures of 4 or 7 or 16 (4x4) or 28 (4x7). Powers (1977:51) quotes Black Elk on the importance of four and seven:

> "…the numbers four and seven are sacred; then if you add four sevens you get twenty-eight. Also, the moon lives twenty-eight days, and this is our month; each of these days of the month represents something sacred to us: two days represent the Great Spirit; two are for Mother Earth; four are for the four winds; one is for the Spotted Eagle; one for the sun; and one for the moon; one is for the Morning Star; and four for the four ages; seven are for our seven great rites; one is for the buffalo; one for fire; one for water; one for the rock; and finally one is for the two-legged people."

2.3 THE VISION OF OGOTEMMLI

Perhaps no vision of the universe is so wonderfully intertwined with numbers than that of the Dogon elder, Ogotemmli, as related by the French anthropologist Marcel Griaule (Giaule,1965). For Ogotemmli, everything in the universe has a number and the numbers themselves have qualities. The central number in Ogotemmli's vision is eight. There were eight sections to the structure of the world-system, eight primeval ancestors at the creation of the world, and eight families descended from them. There were eight seeds at the beginning of creation. There are eight joints in humans, eight *dougu* (covenant-stones) that are the repositories of the life-forces of the ancestors, eight locations of the council house pillars, eight crafts, and eight regions populated by people speaking eight languages.

For each number 1-8, Ogotemmli revealed correspondences. The correspondences expose the inherent number of the idea or thing as well as explaining the qualities of the number. As an example, consider the numbers 2 and 7. *Two* corresponds to the southwest pillar, white millet, the Toro language, the Ende region, the colors red and white, and the craft of tanning. The second ancestor was the leather-worker. *Seven* corresponds to the north pillar, rice, the Ireli language, the Ireli region, the color rose, and the crafts of weaving, music and language. The seventh ancestor was the "master of speech."

The number 8 also serves as a kind of culmination or unification of the preceding numbers. Ogotemmli says,

> "Seven is the rank of the Master of Speech; 1+7 = 8. The eight rank is that of Speech itself. Speech is separate from the one who teaches it, that is the seventh ancestor; it is the eighth ancestor. The eighth ancestor is the foundation of the speech which all the other ancestors used and which the seventh taught." (Griaule, 1965:48)

The number *eight* corresponds to the lingua franca of the Dogon area, all regions, and agriculture, the art that encompasses all the arts and crafts of the Dogon.

2.4 WHY NUMBERS?

In the preceding sketches of number and arithmetic in religious belief, we see that the mathematics is more than an overlay or afterthought. Indeed, the mathematics provides a framework for the organization of the spiritual world. Number and arithmetic provide a bit of precision and definiteness to concepts, beliefs, and ideas that are imprecise and indefinite. S.N. Pandey, a proponent of a school of mathematics that ties all of mathematics to the Vedas, expresses this notion succinctly.

> "Knowledge gains perfection and unambiguity and clarity if it is expressed in terms of numbers." (Pandey, 1991:103)

Number also seems to lend power to religious practice. The author of the Chandogya Upanishad feels compelled to provide an analysis of the number of syllables in the names of the parts of the seven-fold chant. Is it to emphasize that these terms are not arbitrary, but instead incorporate some essential nature of reality?

Numbers provide unity. The pervasiveness of 4 and 7 in the natural and supernatural world provide evidence for the Oglala that the universe is not random, that there is a divine presence and a sacred unity to everything.

> "
> ***Numbers are a vehicle for beginning to understand what is perhaps not understandable.***

Numbers are a vehicle for beginning to understand what is perhaps not understandable. Ogotemmli's correspondences help the Dogon to see how things and ideas are related, interrelated, and connected to the whole. Ogotemmli 's equation 7+1 = 8 is a powerful notion of increase, culmination, and inclusiveness.

When we begin to look at mathematics beyond numbers and simple arithmetic, we can see more opportunities for its use in mystical understanding of the supernatural. In the next section, I'll examine some of the ways in which mathematical notions begin to blend with the mystical.

3.0 MATHEMATICAL NOTIONS & MYSTICAL VISION

How are mathematical notions mystical? Consider infinity. Mathematical infinity is a preponderance of seemingly contradictory and paradoxical notions. The modern study of infinity, begun by Georg Cantor in the 1870's, has shown us that there are levels of infinities, that some infinities are infinitely greater than others, that parts can be the same size as wholes, and other wonders. Here I'll only examine one of these wonders. (For a detailed non-technical discussion of the whole matter of infinity see Pickover (1995), Rucker (1995), or Vilenkin (1995). A complete mathematical discussion can be found in Fraenkel (1961), Monk (1969), or Suppes (1972).)

Cantor asks that we consider the counting numbers 1, 2, 3, 4, etc. not as a sequence of numbers, but rather as a totality, an infinite collection. It is a consideration that requires some mental wrangling and a certain amount of faith. We must behold a sequence that has no end all at once. In effect, we must see the sequence,

as it were, from "outside," from a supernatural vantage point. This viewpoint, although perhaps unusual to a non-mathematician is not unique to modern mathematics. Artists wrestled with the problem geometrically in the Renaissance (Fields, 1997) and Mimica (1988) has shown that viewing the counting numbers this way is an important aspect of the culture of the Iqwaye people of Papua New Guinea.

Holding the infinite collection of counting numbers in our minds, now let us consider those that are even (2, 4, 6, …). Cantor now asks us to compare the sizes of the two collections. One way of comparing the sizes of two collections is by matching. This is a particularly efficient means of comparison if the collections are large. For example, if we want to compare the number of people with the number of seats in a large auditorium we may simply invite each person to take a seat. If we see anyone standing after everyone tries to sit down, then we know there are more people than seats. On the other hand, if everyone is seated and there are empty seats, then we know there are more seats than people. If every person has a seat and every seat has a person then we know that there were the same number of people as seats.

Now let's try to match the counting numbers with the even counting numbers. Because we are viewing the collections separately, it will be useful to distinguish numbers selected from the counting number collection from those selected from the even number collection. If a number is taken to be from the even numbers I will write it with **bold** print and if it is taken from the counting numbers I will write it in *italics*. Now we can construct a matching between the two collections by matching each counting number with its double, like this:

counting numbers:	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11 …*
even numbers:	**2**	**4**	**6**	**8**	**10**	**12**	**14**	**16**	**18**	**20**	**22 …**

Notice that every counting number is matched with an even number. Indeed, whatever counting number we pick, we can easily determine which even number it is matched with by doubling the number we pick. So, *1999* is matched with **3998**, *150,000,000* is matched with **300,000,000**, and so on. Furthermore, every even number is matched with a counting number. If we pick an even number we can find the counting number it is matched with by dividing the chosen even number by 2. Thus, **24** is matched with *12*, **1492** is matched with *746*, and so on.

What does the result of this matching process suggest? There are no unmatched numbers from either collection, therefore the collections are the same size! Yet, clearly the odd numbers are not included in the collection of even numbers. So, shouldn't the collection of counting numbers be greater than a collection which is part of it? Normal intuitions fail when we consider the infinite.

So then, we may ask, are all infinite collections ultimately the same size? To this Cantor's surprising answer is "no." To explain his answer, I'll need the convenient mathematical notions of a set and subset. A set of things is simply a collection of those things. A subset of a set is a collection containing items from the set. Sets and subsets are denoted mathematically with braces surrounding the names or descriptions of the items in the set. The subset of the counting numbers that contains the numbers 2, 4, and 6 is written {2, 4, 6}. Thus, for the set of counting numbers we have many subsets, including

{2},
{1,2,3,4,5},
{1492,1776,1812,1864,1969,1998}
{2,4,6,8,...} the even numbers,
{1,3,5,7} the odd numbers,
{2,3,5,7,11,13,...} the prime numbers,
{4,7} the sacred numbers of the Oglala,
{1,2,3,4,5,6,7,...} the counting numbers,
etc.

There are an infinite number of subsets of the set of counting numbers. This is easily seen because each counting number can be form a one-item set {1}, {2}, {3}, and so on. Is it possible to match the collection of subsets of the counting numbers with the counting numbers? Cantor says no, and here's why.

Suppose that some very bright person came up with what he or she thought was a matching scheme between the subsets of the counting numbers and the counting numbers. The matching would look like this:

$1 \rightarrow S_1$
$2 \rightarrow S_2$

$3 \rightarrow S_3$
…
$1200 \rightarrow S_{1200}$
$1201 \rightarrow S_{1201}$
…

The subscripted S indicates the set that is matched with that number. So, S_{24} is the set that is matched with counting number 24. We don't know what S_{24} is, but our bright person should be able to tell us if we ask. Similarly, if we ask our bright person what number the set {4,7} is matched with, he or she should be able to tell us.

Now we'll identify a particular subset of the counting numbers and ask what number with which it is matched. In the matching, some sets will contain the number they are matched with and some will not. That is, 24 may be an element of S_{24} or it may not. Consider the set containing only those numbers which are not elements of the set with which they are matched. Call this set C. Now if 24 is not in S_{24}, then 24 is in C. On the other hand, if 24 is in S_{24} then 24 is not in C. Our bright person's matching claims to account for all the subsets of the counting numbers, so C ought to be matched with some number, call it *k*. Thus, $C = S_k$.

Is *k* in C? If *k* is in C, then *k* is not in S_k. But S_k=C, so *k* is not in C. On the other hand, if *k* isn't in C, then *k* is in S_k which means *k* is in C. Either possibility is contradictory. Logically, then, some earlier assumption in the argument must be false. C is a reasonably defined set. Thus, we are left with the conclusion that C must not be matched with any counting number. The set of subsets of the counting numbers is larger than the set of counting numbers. We have a collection that is infinite, yet is larger than infinity. Cantor went on to show that there are infinite levels of infinitely larger infinities.

Mathematical infinity demands considerable contemplation and is arguably similar to paradoxical mystical language. Cantor, of course, did not suggest that his infinities were a path to the mystical understanding that everything is interconnected and has a single source. Nor did he offer them as triggers of mystical experience. Nevertheless, contemplation of Cantor's infinite hierarchy of infinities, each of which is infinitely greater than its predecessor, can affect one in a fashion reminiscent of the Zen *koans*.

Borchert, in describing the mystical experience, may come close to describing the experience one has when contemplating the paradoxes of infinity:

> "These opposites seem to exclude one another; to destroy—to recreate; miserly—openhanded; terrifying—attractive. Because of the tension between opposites, a narrow opening comes into existence, and it is through this channel that the mystic sees something which cannot be set down in *one* word. What it is cannot be expressed, but can only be suggested." (Borchert, 1994:19)

Infinity is only one of many mathematical notions that suggest mystical visions. The mathematical field of complex dynamical systems brings before us the infinitely self-replicating patterns of the Mandelbrot set and engaging fractal portraits (Peitgen & Richter, 1986). Computer generated fractal images seem to be not far removed from the meditative art forms of mandalas or yantras used in Eastern mysticism. And, in the mathematics of quantum computing we see parallel universes, timelessness, and a host of other metaphysical constructs. (Deutsch, 1997).

What all of these mathematical notions share is their ability to articulate with some precision notions that transcend everyday experience. The mathematics allows us to peer into aspects of reality that go beyond our senses and often beyond our commonsense notions of rationality. And this transcendence leads us to questions of truth and reality.

4.0 MATHEMATICAL TRUTH AND SPIRITUAL TRUTH

A reasonable question concerning this mind-bending infinity of infinities is that of its connection to what we commonly consider to be reality. We might wonder if these arguments are no more than mental recreation. Have we learned a truth about something? If so, what? The philosopher Michael Resnick (Resnick, 1998) offers some notions of truth that I think are useful in assessing the truth of Cantor's visions. Resnick sees at least two aspects of mathematical truth: immanent truth and transcendent truth.

Immanent truth applies only to statements within its own language. The truth of the statements about infinity that I have offered is established entirely within the realm of the infinities of sets. Sets, numbers, and

logic provide the basis for the truth of the hierarchy of infinities. These truths do not rely on things, relationships, or observations outside of the realm of mathematics. As a mathematician I am prepared to learns truths about mathematical objects because I have been trained to understand claims about mathematical objects.

Immanent mathematical truth may be contrasted to transcendent mathematical truth, which seeks support in reference to physical objects or correspondence between mathematical objects and non-mathematical objects. I learn transcendent mathematical truth through experiment as well as through proof.

For example, as a transcendent mathematical truth, 2 + 1 = 3 makes claims about the number of people in my car after my son and I meet and pick up my wife. The truth of the equation is confirmed by its correspondence to my experienced world. The same equation is viewed as an immanent truth when I am persuaded by this mathematical proof about sets:

> “
> ***Debates about the existence of numbers and other mathematical objects parallel those about the existence of gods and spirits.***

1. The set { } is a number. Call it 0. (Definition of a number.)
2. If x is a number, then S_x = { x, {x}}. (Definition of S_x)
3. If x is a number, then S_x is a number. (Definition of number.)
4. Only things satisfying Statements 1 or 3 are numbers. (Definition of number)
5. Let 1 denote S_O = {0, {0}}. (Shorthand notation for S_0)
6. Let 2 denote S_1 = {1, {1}} = {{0,{0}}, {{0,{0}}}}. (Shorthand notation for S_1)
7. Let 3 denote S_2 = { {{0,{0}}, {{0,{0}}}}, { {{0,{0}}, {{0,{0}}}} } }. (Shorthand notation for S_2)
8. If x is a number, then either x=0 or there is a number y such that S_y = x. (Follows from Statement 4.)
9. If x is a number, then x + 1 = S_x. (Definition of + 1)
10. 2 + 1 = S_2 (Replace x by 2 in statement 9)
11. S_2 = 3 (Statement 7)
12. Therefore, 2 + 1 = 3. (From statements 10 & 11)

I have learned that 2 + 1 = 3 is true in this case because I understand the notion of a set and some simple principles of logic. It is a truth in the world of sets and logic and will be true for anyone prepared to receive it.

I think that spiritual truths also have this immanent/transcendent Janus face. On the one hand they are truths within their own language. If one is taught to understand the spiritual concepts, then one can learn truths about them. Further, it is legitimate to say that without adequate preparation, one may well be incapable of understanding spiritual truths. On the other hand, many spiritual truths aspire to transcend their own language and claim the status of truth in other realms.

The Islamic teacher Ayatollah Khalkhalli said that “Reality will always prevail.” (Naipaul 1998:210) From this we are to understand that reality means truth and that truth stands against falsity. For Khalkhalli, the spiritual truths of his faith have transcended the language of Islam to become truths of all languages, everywhere.

The parallel between mathematical truth and spiritual truth is important because of the success each has had in transcending its own language. Mathematics has provided numerous truths to science and spirituality has done the same for human culture.

Debates about the existence of numbers and other mathematical objects parallel those about the existence of gods and spirits. How are these entities to be identified? Can they be discovered in any objective way? Can only those who believe really understand? Mathematical and spiritual truths have survived outside of their home realms despite, or perhaps because, they are truths about objects with ambiguous existential status.

5.0 CONCLUSION

I doubt that mathematics will ever become a theology (although stranger things *have* happened), but its value in understanding reality is undeniable, and it has the power to bring its practitioner to a meditative state. Indeed, early Islamic scholars saw mathematics as religiously legitimate and as a way to Holy knowl-

edge (Høyrup, 1994:111). Whether we experience God or the Tao or the ground of being when deeply involved in mathematical thought I will leave to the theologians. Nevertheless, the realms of mathematics and spirituality do intersect.

That intersection may be beautifully summarized by the following passage from Chapter 21 of the *Tao Te Ching*.

"The Tao is elusive and intangible.
Oh, it is intangible and elusive, and yet within is image.
Oh, it is elusive and intangible, and yet within is form.
Oh, it is dim and dark, and yet within is essence.
This essence is very real, and therein lies faith."
(Feng & English, 1974)

REFERENCES

BORCHERT, BRUNO, 1994. *Mysticism: Its History and Challenge*. York Beach, Maine: Samuel Weiser, Inc.

BULLCHILD, PERCY, 1990. *The Sun Came Down: The History of the World as My Blackfeet Elders Told It*. San Francisco: Harper & Row.

BURTON, DAVID M., 1997. *The History of Mathematics: An Introduction (3rd edition).* New York: McGraw-Hill.

CLOSS, MICHAEL, 1986. Tallies and the ritual use of number in Ojibway pictography. In Michael Closs (ed.), *Native American Mathematics*, 181-211. Austin: University of Texas Press.

DEUTSCH, DAVID, 1997. *The Fabric of Reality*. New York: Allan Lane, The Penguin Press.

FIELD, J.V., 1997. *The Invention of Infinity: Mathematics and Art in the Renaissance*. Oxford: Oxford University Press.

FRAENKEL, ABRAHAM A., 1961. *Abstract Set Theory*. Amsterdam: North-Holland.

FRAWLEY, DAVID, 1995. *The Myth of the Aryan Invasion of India*. New Delhi: Voice of India.

FREIDEL, DAVID, LINDA SCHELE, and JOY PARKER, 1995. *Maya Cosmos: Three Thousand Years in the Shaman's Path.* New York: William Morrow & Co.

GRIAULE, MARCEL, 1965. *Conversations with Ogotemmli: An Introduction to Dogon Religious Ideas*. London: Oxford University Press.

HUME, ROBERT ERNEST (trans.), 1977. *The Thirteen Principal Upanishads*. London: Oxford University Press.

HØYRUP, JENS, 1994. *In Measure, Number, and Weight: Studies in Mathematics and Culture*. Albany, N.Y.: SUNY Press.

KHARE, H.C. (ed.), 1991. *Issues in Vedic Mathematics*. New Delhi: Rashtriya Veda Vidya Pratishthan.

LEVI-STRAUSS, CLAUDE, 1968. *The Origin of Table Manners: Introduction to a Science of Mythology, 3*. New York: Harper & Row.

LAO TSU, 1974. *Tao Te Ching*. Translated by Gia-fu Feng and Jane English. New York: Alfred A. Knopf.

MIMICA, JADRAN, 1992. *Intimations of Infinity: The Cultural Meaning of the Iqwaye Counting and Number System*. Oxford: Berg Publishers, Inc.

MONK, J. DONALD, 1969. *Introduction to Set Theory*. New York: McGraw-Hill.

MORRIS, GREG, 1995. Developing the spiritual in mathematics. *Mathematics Teaching* 153:36-37.

NAIPAUL, V.S. 1998. *Beyond Belief: Islamic Excursions Among the Converted Peoples*. New York: Random House.

PANDEY, S.N., 1991. Vedic mathematics: an appraisal from the perspective of modern mathematics. In H.C. Khare (ed.), *Issues in Vedic Mathematics*, 103-104. New Delhi: Rashtriya Veda Vidya Pratishthan.

PEITGEN, H.-O, and P.H. RICHTER, 1986. *The Beauty of Fractals: Images of Complex Dynamical Systems.* Berlin: Springer-Verlag.

PICKOVER, CLIFFORD A., 1995. *Keys to Infinity*. New York: John Wiley & Sons, Inc.

POWERS, WILLIAM K., 1977. *Oglala Religion*. Lincoln: University of Nebraska Press.

RESNICK, MICHAEL, 1998. Proof as a source of truth. In Tymoczko, Thomas, ed., *New Directions in the Philosophy of Mathematics.* Princeton, NJ: Princeton University Press, pp.317-336.

RUCKER, RUDY, 1995. *Infinity and the Mind*. Princeton, N.J.: Princeton University Press.

SUPPES, PATRICK, 1972. *Axiomatic Set Theory*. New York: Dover.

URTON, GARY, 1997. *The Social Life of Numbers: A Quechua Ontology of Numbers and Philosophy of Arithmetic*. Austin: University of Texas Press.

VILENKIN, N. Ya., 1995. *In Search of Infinity.* Boston: Birkhuser.

YOUNG, M. JANE, 1988. *Signs from the Ancestors: Zuni Cultural Symbolism and Perceptions of Rock Art*. Albuquerque: University of New Mexico Press.

Skolem's Paradox and Contradictory Popular Songs

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Skolem's paradox (named after the logician Thoralf Skolem) essentially points out that logic is relative: it depends on where you sit. More specifically, it is a paradox in set theory. It states that set theory has a countable model, which nevertheless contains uncountable sets. Formal set theory implies that there exists a set which is infinite, but no function exists which will map this set one-to-one onto the natural numbers: it is uncountable. Hence any model of set theory mirrors this "uncountable" set. But, according to the well-known Lowenheim-Skolem theorem, set theory has a model with only a countable number of objects in it. How can this be? The answer often given is "it depends on where you put the emphasis:" Do you emphasize the metamathematical countability or the formalized uncountability?

So now we turn to contradictory popular songs. Whether they are contradictory or not depends on where you put the emphasis. With some mental effort they might even be consistent. In these love songs we are supposed to imagine hopeful lovers: clearly, the emphasis is on "yes" rather than "no."

1. LET'S CALL THE WHOLE THING OFF

In this song the lovers are debating whether or not to call off their relationship (or a planned rendezvous). It seems that they disagree on the pronunciation of words such as "oyster," "pajamas," "either" and such (I would like to add "quark"). The debate continues until the last two lines, which are "so let's call the calling off off' and "let's call the whole thing off." These last two lines contradict each other, and I for one do not know whether it was called off or not.

2. BEGIN THE BEGUINE

According to the literature the Beguine was said by Cole Porter to be a romantic dance among certain natives, but he denied it later. Apparently the issue is whether or not to begin this memorable love dance or song. In one line you hear, "So don't let them begin the Beguine!...Let the love that was once a fire remain an ember," to be soon followed by "Oh yes, let them begin the Beguine, make them play!" This contradictory behavior can be understood by allowing for the emotional state of the singer. It seems to me that the emphasis is on the "yes" here, rather than the "no." Artie Shaw circumvented having to make the decision by producing a strictly instrumental version of the song (which is presently in top place on a popular radio station).

3. I'M IN THE MOOD FOR LOVE

The song begins with the words, "I'm in the mood for love." The singer then proceeds to explain why he or she is in the mood for love. This goes on until you hear the words "If it should rain, well let it; but for tonight forget it; I'm in the mood for love." This last sentence doesn't seem to make sense to a sensitive listener who is startled by "forget it" only to hear again "I'm in the mood for love." Louis Prima and Keely Smith avoid this paradox by substituting the phrase "if it should rain, well let it; but for tonight well let it; I'm in the mood for love."

It is interesting to speculate how a Turing machine would decide these "decision problems."

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