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High school graduates of the near future could be more sophisticated geometers than their professors. If the National Council of Teachers of Mathematics Standards [7] is adhered to, college-intending high-school students will learn more than basic Euclidean geometry. They will have worked with technological aides to make discoveries and then made deductive arguments to verify their conjectures ([7], 159) and spherical geometry ([7], 160). They will have developed both deductive and inductive reasoning in part by exploring geometry through short sequences of axioms. Students entering college will have an "... appreciation of Euclidean geometry as one of many axiomatic systems" ([7], 160).

That is, of course, if their high school teachers know the material and can teach it effectively using the pedagogical techniques suggested in the NCTM guidelines ([7], [8]). The NCTM expects high school teachers to introduce their students to spherical geometry, software programs, axioms, and deductive reasoning and proofs. In the classroom they are expected to incorporate tasks that require students to make and test conjectures and use manipulatives as well as to have students work in cooperative groups. Moreover, they will be expected to use a variety of methods to assess the student's progress in the course.

The geometry course described in this note is an attempt to help prepare prospective teachers to meet the goals described in the Standards. Some recent literature confirms that teachers teach material in the way they were taught and that, to make effective use of a pedagogical technique, it helps to have learned the material by the same technique ([1], [2], [6]). With the above in mind, we replaced the traditional "Foundations of Geometry" course with one that incorporated technology, discovery learning in a cooperative-group setting, and introduced students to non-Euclidean geometry early in the course.

The "Foundations of Geometry" course we replaced exhibited many features common to the majority of the geometry courses offered to prospective high school teachers in the United States [5]. It was taken primarily by prospective teachers, lecture-based with some group work, and made little use of manipulatives and software tools. In our course, like in about half such courses, the material was developed following Hilbert's or Birkhoff's axioms. First, absolute geometry was developed and then, towards the middle of the course, the parallel postulate was introduced. At that time the students were introduced to some spherical geometry and hyperbolic geometry. The course then returned to Euclidean geometry to develop similarity, area and properties of the circle.

For instance, that the angle sum of a triangle in Euclidean geometry is 180° appeared as follows. The first half of the course developed the incidence, betweenness, and congruence theorems and geometric inequalities. The elliptic, hyperbolic and parallel postulates were then introduced along with models of the different types of geometries. Following this, using Saccheri quadrilaterals, it was shown (as a theorem of absolute geometry) that the angle sum of a triangle is less than or equal to 180°. It was then shown (without recourse to the result from absolute geometry) that in Euclidean geometry the angle sum of a triangle is 180°. This struck the students as quite a bit of work to get to something they "already knew." A more serious difficulty with this approach was that there was little to challenge the students' high-school-based knowledge of Euclidean geometry before hyperbolic geometry was introduced. Hyperbolic geometry was usually introduced several weeks into the course and then (nearly) abandoned while similarity and other Euclidean topics were studied. In addition, as the proofs appeared to be merely confirming what the students felt they already knew, the proofs did little to promote a deeper understanding of the material.

We replaced our traditional course with one that forces students to confront what they 'already know' early in the course. With the help of tools that secondary teachers will eventually use in their own teaching, the course creates a need for axioms and proofs to describe and work with different geometries. The new course currently develops the topics using cooperative group projects in the following order: area, the angle sum of a triangle and the parallel postulates, congruence of triangles, similarity of triangles, properties of circles, and transformational geometry. During these projects the students use tools such as the Geometers' Sketchpad, Non-Euclid (a software program that models the Poincaré disc), the Lenart sphere (a clear plastic sphere with a spherical protractor and compass), a MIRA (a plastic device that acts as a mirror to do reflections), Geoboards and, of course, a compass and straight edge. Material that cannot be easily introduced in the projects is introduced in lectures and used for individual homework assignments. For instance, the betweenness axioms are introduced along with the exterior angle theorem in a lecture. About 70% of class time is spent with the students working in cooperative groups with the remainder of the time being used for lectures and exams.

Each project consists of two or three subprojects that each require a written 'progress report,' and conclude with the preparation of a final report that has the students synthesize the findings of the subprojects and correct any errors that appeared in the progress reports. The progress reports and final reports form the basis of a written dialog between the students and the instructor.

A short discussion of the first two projects shows how the assorted elements of the course fit together. The first project has the students develop a theory of area. The first subproject asks them to develop a procedure for finding the area of a polygonal region assuming they know how to a) find the area of a square and b) find the area of a triangle. Students then use these procedures to justify the standard formulas for the area of a rectangle, parallelogram and trapezoid. This work is followed by a lecture on axiom systems and models. The second subproject has each group develop a set of area axioms and then use the axioms to prove their formulas from the first subproject. These axioms are also discussed as a class. The third subproject has them use the Lenart sphere to test the validity of their axioms on the sphere and derive a formula for the area of a spherical triangle. The final report has them integrate the (corrected) results of the subprojects into a single document. As part of their work, the students are asked to identify any apparent gaps or holes in their arguments, for instance, any assumptions that they are making about length, the area of a boundary, et cetera.

The second project addresses the angle sum of a triangle. The students are first asked to develop a system of axioms that allow them to prove that the sum of the measures of the angles of a triangle is 180°; this usually requires an axiom stating that alternate interior angles are congruent. In the second subproject they explore the validity of their axioms and the exterior angle theorem on the sphere and in the Poincaré disc using the software program non-Euclid. This progress report also requires each group to make a conjecture about the area of a triangle in the Poincaré disc. In the final report the students show that the Euclidean parallel postulate in conjunction with the exterior angle theorem yields that the angle sum of a triangle is 180°. During this project there is a lecture on the history of the parallel postulate and the development of non-Euclidean geometry. (The angle sum result for absolute geometry is proven later in the course.)

At this stage the students are 6 to 7 weeks into the course. The students have had significant exposure to spherical geometry and the Poincaré disc. They have been surprised to discover that the area of a triangle is not always 'half the base times the height' and that the angle sum of a triangle is not always 180°. The students go on to explore the congruence of triangles, similarity and transformational geometry in Euclidean and non-Euclidean geometry. In the last two projects the students investigate geometries through the fixed points and lines of reflections and classify motions in the plane.

In the early part of the course the students develop their own axioms and lemmas for each project. As the course progresses, to maintain some uniformity in the axiomatic development, assorted key 'axioms' are suggested to them; for instance, in the angle sum project they are given the exterior angle theorem as an axiom, and later in the course they establish it as a theorem. Eventually, during the last two projects on

transformational geometry they are given the definitions and axioms they need for each subproject and are asked to prove a variety of theorems. These projects have them work extensively with the assorted software tools and manipulatives that were introduced earlier in the course. The definitions and axioms are given without intuitive motivation or explanation; it is up to the students to 'discover' the intuitive content of the definition through the models developed during the course. For instance, the students are given the definition of fixed points and fixed lines of a motion and then, to help develop their intuition, are asked to find the fixed points and lines of reflections on a sphere and in a model of Euclidean geometry.

> ***...we concluded that students improved in their ability to discuss mathematics, explain their mathematical thinking, and work with others toward a common goal.***

Most students benefited from this new course structure. From our observations, we concluded that students improved in their ability to discuss mathematics, explain their mathematical thinking, and work with others toward a common goal. On course evaluations students reported that they deepened their understanding of geometry, that the group work and computer software facilitated their understanding, and that they increased their self-confidence to do geometry.

There are some drawbacks to the course. One is that, as we implemented it, it requires quite a bit of classroom time; to accommodate this we added a weekly two hour lab to the course. As one would hope, we were able to investigate the standard topics and address additional concepts in the replacement course; in particular, transformational geometry was explored in much greater depth than in the traditional course. It is possible that the same amount of material could be investigated in a course with fewer contact hours by having the groups do some work outside of class. In our course most groups were able to do most of their group work during class.

Another drawback is that since the students are developing the axioms in a nonstandard order, at least two different axiom systems are introduced, and what is an axiom one week may become a theorem the next week, the students have some difficulty in seeing different sets of geometric axioms as coherent systems. The instructor needs to monitor the groups closely to prevent errors due to improperly blending axiom systems. For instance, the first two times the course was taught transformations were introduced through MIRAs and, as a result, the students tended to assume that reflections exhibited all the properties of reflections in a Euclidean plane. Even after working with reflections on a sphere, students still slipped into making assumptions based on their initial work in the Euclidean plane. Some of these problems can be avoided by making the assumptions in the projects very explicit. Even though there is occasionally some confusion during the course, in the end it is worth the extra vigilance to help the students develop the perspective needed to appreciate the role of axioms in mathematics.

A drawback of a more mundane nature is that the course can be very time consuming for the instructor. Each group submits ten to twelve written reports during a quarter. As the subprojects build on one another and are used to prepare the final report, they need to be graded promptly and carefully. Homework and exams also need to be graded. In addition, organizing the class into cooperative groups requires the instructor to do more administrative work than a lecture-based course.

A possible philosophical objection is that much of the grade is based on cooperative work. Since cooperative group work plays a large role in the course, group grades constitute a significant portion of the individual student's final grade. In our courses thirty-five to forty percent of the final grade was based on the cooperative projects. Consequently, it is important to be sure that the group grade reflects the sum of each individual's understanding of the material. One way to help accomplish this is to give individual quizzes at the conclusion of a project and make the combined group score on the quizzes part of the group grade. According to journal entries and (anonymous) student evaluations, during our courses the students' attitudes regarding group grades changed from some concern that some students would get undeserved credit to a

general belief that they were fair.

Even though the course was designed with the needs of prospective secondary teachers in mind, the course is also appropriate for a mathematics student. It is, at its core, a mathematics course. Except for a brief discussion of cooperative learning and group work (20 minutes), no class time is spent discussing pedagogy. Since the course gives the student the experience of mathematical discovery and actively learning mathematics, we believe it benefits the typical junior level mathematics major and is a viable replacement for a traditional "Foundations of Geometry" course.

REFERENCES

[1] Joyner, Virginia Green. Research into Practice: The use of a student teaching study to develop and improve mathematics method courses for preservice teachers. *School Science Mathematics*, Vol. 91(6) (1991), 236-237.

[2] McDevitt, Troyer, Ambrosio, Heikkinen, Warren. Evaluating Prospective Elementary Teachers' Understanding of Science and Mathematics in a Model Preservice Program. *Journal of Research in Science Teaching*, Vol. 32 No. 7 (1995), 749-775.

[3] Moise, E. E. *Elementary Geometry from an Advanced Viewpoint*, 3rd ed. Addison-Wesley Publishing, Reading, Massachusetts, 1990.

[4] Greenberg, M.J. *Euclidean and Non-Euclidean Geometries, Development and History*. W. H. Freeman Co., San Francisco, California, 1974.

[5] Grover, B. and Connor, J. The College Geometry Course: What is it?, preprint.

[6] Stoffelett, Rene and Stoddart, Trish. The Ability to Understand and Use Conceptual Change Pedagogy as a Function of Prior Content Learning Experience. *Journal of Research in Science Teaching*, Vol. 31 No. 1, (1994), 31-51.

[7] [NCTM89] *Curriculum and evaluation standards for school mathematics*. National Council of Teachers of Mathematics, Reston, Virginia, 1981.

[8] *Professional standards for teaching mathematics*. National Council of Teachers of Mathematics, Reston, Virginia, 1981.

[9] Wallace, E. C. and West, S. F. *Roads to Geometry*, 2nd ed. Prentice-Hall, Upper Saddle River, New Jersey, 1998.

Tryst of Twins: Antarctica, Amazon

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How crystal white the ice cap Neptune head
Views the ocean; streaming ice waters beneath
Are lubricant carrying his body
Gliding steadily to the sea.

His equatorial giant twin, the Amazon,
Suckling the breast of dark rain clouds
Transfuses oxygen, a bloody, muddy flow,
The umbilical for starving embryos, millions.

A biopsy of the ice cap reveals
Microbes fungal, bacterial species,
As do probes in Amazon's forest hair:
Nesting plant, bird, lichen fair.
Who might guess 'neath the Atlantic deep
Antarctic waters meet unseen in tryst?

In kisses hugging, bedside currents, embraces
The Amazon body in earth's one living womb?
While the surface conflicts, retards proliferation
Of stagnating antigen-antibiotic abcesses
Deep under spherical transfusing blood says
Planet love, is, such flowing expecting no return.

Haiku: The Heart

Arnold L. Trindade
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Big Bang disperses
Heart rub-a-dub calibrates
How in tune palpitates