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SUMMARY
A set of problems is presented and discussed for which there is a tendency for students to ignore part of the given information in the problem and to substitute some extraneous assumptions. Typical student reactions are also discussed.

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This article is about a very interesting and very specific class of non-routine mathematics problems. I discovered this particular type of problem in part from teaching a mathematics content course for prospective teachers that has a strong emphasis on problem solving, and from giving workshops to prepare prospective teachers for the mathematics questions on the New York State Teachers Certification Examination.

Of course, there have been many books and papers written about problem solving in mathematics (for example, [2], [5]). There have been studies of problems with too little information, problems with redundant information, problems with no possible solution (for example, [1]), and many other classifications. However, the class of problems to be discussed here does not seem to have been considered separately.

Here is a simple initial problem to illustrate the main idea. Krutetskii ([1], p. 142) gives the following problem:

One leg of a right triangle is equal to 7 cm. Determine the other two sides if they are expressed in integers.

The solution would be that the other two sides are 24 cm and 25 cm, but Krutetskii states that students faced with this problem would often claim that it could not be solved because a triangle cannot be determined if only one side is given. The student is forgetting, or perhaps ignoring, the condition that the other two sides must be integers. This raises the question of what causes the student to ignore this condition. Is it that (s)he does not know how to use this information? Or is it that (s)he does not think that the condition is relevant?

This is an example of the class of problem I wish to consider here, namely problems *in which the student tends to ignore some given condition in the problem*. I have found remarkably few problems which seem to fit this requirement exactly, and I will present them here. It is interesting to consider just what it is about these particular problems that causes the student to ignore a relevant portion of the given information in a problem. Frequently, the students are inclined to substitute some extraneous assumption of their own for the information which they are ignoring, as will be illustrated in the following examples.

The problem which originally suggested to me this idea of ignoring part of the given information is the following ([3]):

Four men, one of whom has committed a crime, made the following statements:
Arch says: Dave did it.
Dave says: Tony did it.
Gus says: I didn't do it.
Tony says: Dave lied when he said I did it.
If only one of these statements is true, who was the guilty man?

I have presented this problem at large workshops, and I have been somewhat surprised to find that invariably the students ignored the condition that only one of the statements is true. Typically they suggest reasons for their choice that are based on some sort of psychological argument, for example: "Gus must have done it, because he didn't accuse anyone else," or "Tony must have done it, since he accused Dave of lying." The students are substituting some sort of extraneous assumption for the given condition that only one statement is true, perhaps because they do not see how to apply this condition, or do not see how it is relevant. To solve this correctly, one could use this condition to test the possibility of each man's guilt. This would reveal that the correct answer is that Gus was guilty, for only in that case is exactly one statement true; if any of the others were guilty, then two or more of the statements would be true. Perhaps the idea of considering separate cases (i.e., what if Arch is guilty, what if Dave is guilty, etc.) does not occur to the students because they have not experienced many problems which are solved through the strategy of using cases.

Another example of this phenomenon appears in [4]:

Exactly one of the following statements is false:
a) Audrey is older than Beatrice.
b) Clement is younger than Beatrice.
c) The sum of the ages of Beatrice and Clement is twice the age of Audrey.
d) Clement is older than Audrey.
Who is the youngest—Audrey, Beatrice, or Clement?

Here too, students are inclined to ignore the condition that exactly one of the statements is false. For example, they may look at only two of the statements, say (a) and (b), and conclude from them that Clement is the youngest, ignoring the presence of the other statements. Similarly, they may look at only statements (a) and (d), and conclude quickly that Beatrice is the youngest. Occasionally they will see that statements (a), (b), and (d) contradict each other and will erroneously conclude that the problem cannot be solved. The students may tend to pay less attention to statement (c) because it is harder to understand, or because it involves consideration of the arithmetic operation of summing. They fail to realize that statement (c) implies that Audrey's age must be in between the other two ages (if all ages are different). Again, one could simply consider each of the four statements in turn, and ask, "If this statement were the false one, could the other three be all true simultaneously without contradiction?" In this way one could eventually determine that the only possibility is for (b) to be the false statement, which would make Beatrice the youngest.

A fourth problem, which the readers may have seen in one form or another, is the following:

Mrs. Adams and Mrs. Brown, two math teachers, are walking over to Mrs. Brown's house after school.
Mrs. A: How many children do you have?
Mrs. B: I have three children.
A: What are their ages?
B: The product of their ages is 36.
A: [Thinks for a moment] That's not enough information to figure out their ages.
[By now, the two of them are at Mrs. Brown's driveway, so that Mrs. Adams can see the number on Mrs. Brown's house.]
B: The sum of their ages is the number on the house.
A: [Thinks for a moment] That's still not enough information, I still can't figure out their ages.
B: The oldest child is visiting her grandmother.
A: [Instantly] Now I know their ages!
What are the ages of Mrs. Brown's children?

To solve this problem, one would first list all combinations of three whole numbers whose product is 36, as possible candidates for the ages of the three children. When Mrs. Adams is told that the sum of the ages of the children is the number on Mrs. Brown's house (which she knows), she states that she still cannot determine the children's ages. Among the triplets of whole numbers whose product is 36, only two such triplets have the same sum: 2+2+9=13 and 1+6+6=13; all other sums are distinct. Therefore if the house number was anything other than 13, Mrs. Adams would know the ages of the children as soon as she is told that the sum of the ages is the house number. The fact that she still cannot determine their ages at this point implies that the house number must have been 13. Then, when Mrs. Adams is told something about the "oldest child," she knows that the answer must be 2, 2, and 9, because in the combination of 1, 6, and 6, there is no "oldest" child.

In this problem almost invariably students will ignore the given fact that the sum of the children's ages is the number on the house. It seems apparent that this is because they do not see how this information can be used without the actual value of the house number. In fact, when the problem is posed, the students often ask, "What is the number on the house?" Sometimes they will state that the problem cannot be solved without the house number being given. In this problem I have seen all sorts of extraneous assumptions introduced to replace the clause which is ignored. For example, they may assume that the oldest child must be at least, say, 12 years old in order to be allowed to visit her grandmother on her own. Or, they may assume that the oldest child must be under 6 years of age (i.e. the answer must be 3, 3, and 4) because otherwise she would have to be at school that day (it is a school day, since the teachers were going home from school!) and could not be visiting her grandmother. In one highly unusual case, a student came to me and asked "Is the grandmother dead?" The student explained that she wanted to know this because if the answer was yes, then the oldest child was actually visiting her grandmother's grave site at a cemetery, and she thought that children under age 18 might not be permitted to visit a cemetery. It seems that students think that the grandmother is somehow relevant because they are accustomed to textbook problems in which only the necessary information is given, and they assume that if the grandmother were not relevant, then she would not be mentioned.

The four problems shown above have in common a tendency for students to ignore an actual given condition in the problem. Below is one more problem which is closely related in this regard, but with a slight difference:

What is the greatest amount of money (i.e., the maximum VALUE) in coins (up through half-dollars; no dollar coins) that you can have and still not be able to give someone change for any of the following: a nickel, a dime, a quarter, a half dollar, or a dollar?

In this problem students sometimes erroneously assume that you must have at least one of each type of coin (from a penny through a half-dollar, inclusive). Alternatively, students may incorrectly assume that the amount of money must be less than a dollar. (Actually the correct answer is $1.19; a half dollar, a quarter, four dimes and four pennies, but no nickels.) This is similar to some of the preceding problems in that students tend to make extraneous assumptions. However, it differs from the foregoing in that in this case the assumptions which cause the students to overlook *possible solutions* do not involve ignoring a given *condition* of the problem, as in the earlier examples. However, the tendency to overlook possible solutions is more common than altogether disregarding a part of the given information in a problem.

An interesting but perhaps difficult question for future study would be to examine why it is that students respond differently to these types of problems as compared with other non-routine problems. Is it possible to identify some commonality among these problems which provokes this unusual response, and how can we help our students to focus more on the meaning of the given information rather than introducing superfluous assumptions? Finally, I would be interested in hearing from readers any suggestions of other non-routine problems which would fall into the category discussed here.

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"Common sense is the collection of prejudices acquired by age eighteen."
--Albert Einstein