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As I polish these remarks, it is 11/5/99. Need we say more to realize that our calendar is a mathematical model of time? This model is based on our position in the universe. One orbit of the Sun is a year. We judge our age in orbits; we often think of both current events and history in terms of tens and hundreds of orbits—that is, in decades and centuries. This shows the influence of base 10 on our thinking. As we hit the juncture of the beginning of an orbit numbered 2000, we are reminded of this mathematical model.

This seems to be an appropriate time to review recent orbits. My goal is to do this in a way that will be interesting and informative. I have picked the last 50 orbits as my time frame because this interval covers the schooling of most of your students and their parents.

THE NEW MATH ERA

There is also a conceptual reason for beginning in 1950. In 1951, three faculty members in mathematics and education at the University of Illinois began the first of the new math projects, UICSM. Six years later, in 1957, the new math received its biggest push when the Soviets launched Sputnik, the first artificial satellite. Sputnik was neither a small nor an isolated feat. It built on the work of German rocket scientists that had started 15 years earlier in World War II and its 186 pound weight, followed the next month by the half-ton Sputnik II, showed that the Soviets had the capability to send a large missile anywhere in the world.

Within a year, the U.S. Congress passed the National Defense Education Act, which included sizeable funds for curriculum reform. These funds allowed the fledgling School Mathematics Study Group (SMSG), which had been initiated just a year before, to become the largest research and development project in mathematics education the U.S. has ever seen.

The work of SMSG was hailed widely by all connected with mathematics and education. The euphoria of the time is perhaps best represented by a 1963 report of The Cambridge Conference on School Mathematics entitled *Goals for School Mathematics*. In it, a group of 25 distinguished mathematicians from Harvard, MIT, Stanford, and other top universities joined mathematics educators and other professors of education in an attempt "to express their tentative views upon the shape and content of a pre-college mathematics curriculum that might be brought into being over the next few decades." [p. iii]

These mathematicians were strongly affected by the modernization of mathematics that was the trademark of UICSM, SMSG, and the other new math projects, and the successes that the projects seemed to be having. It led them to believe that students could learn much more if the mathematics were presented in an abstract, clear, and logical way. So they proposed a curriculum for grades K-6 that included conic sections, equations of lines, 3-dimensional Cartesian coordinates, polar coordinates, the vocabulary of elementary logic, graphs of relations and functions, the logarithm function and trigonometric functions. This was before the appearance of handheld calculators, but the use of desk calculators, slide rules, and tables was encouraged at these grades. In grades 7 and 8, students would study rational forms and functions, the derivative of a polynomial, the Euclidean algorithm, and a huge amount of statistics, including expectation and variance and the Poisson distribution. Two curricular organizations were proposed for grades 7-12 for the following reasons: First, as the authors wrote, "It was recognized that there are many different routes to follow in teaching geometry and that each has its advantages." [p. 47] Second, the authors believed that more than one approach to algebra and to calculus seemed reasonable, and they admitted not to know which was best. In both proposed curricula, probability and linear algebra were to be studied more than once.

Although their suggestions remain extraordinarily unrealistic, the Cambridge Conference mathematicians recognized the role they were playing: "These views are intended to serve as a basis for widespread further discussion and, above all, experimentation by mathematicians, teachers, and all others who share the responsibility for the processes and goals of American education. At this stage of their development they can not pretend to represent guidelines for school administrators or mathematics teachers, and they should not be read as such." [p. iii]

The Cambridge Conference mathematicians recognized that the difference between a mathematician and a mathematics educator is as great as that between a research biologist and a practicing physician. The physician sees patients and knows both symptoms and potential cures. The good physician realizes that not all patients are alike, and that you can prescribe things but patients don't always do what you prescribe. Mathematics teachers and those who train them and deal with curriculum day in and day out are the physicians of our profession. Teachers are the experts, and are particularly expert about their community.

In these years, mathematics educators loved the new math, and the general public liked it as well. In 1966, Francis Mueller, after studying articles about mathematics education in popular magazines from 1956 to 1965, identified those years as "happy years for 'new math"' and concluded, "As these years pass, less and less is said about mathematics being a highly disliked subject; more and more is said about the brightness of the future along these new mathematical tracts." But Mueller noted that at the beginning of 1965 there began to appear articles in *Time* and *Newsweek* questioning the ideas behind the new math. He wondered whether these articles might "mark a point of transition at which the public began to revise its perception of 'new math'." [Francis Mueller, "The Public Image of New Mathematics," *Mathematics Teacher 59* (November 1966): 621.] We know today that the public did revise its perception—completely. New math is now often treated as a debacle in mathematics education.

What is not so well-known is that the evidence for a debacle is not there. If the new math was so bad, how come the evidence is so hard to find? In fact, the evidence often leads the other way. By the early 1970s we were producing more students majoring in mathematics and majoring in science than ever before. Advanced placement programs existed in many schools where fifteen years before there was no mathematics beyond trigonometry. Enrollments were up in all mathematics courses even though many states had not changed their graduation requirements.

The public was misled by false signs of failure and a lack of sophistication about statistics that made it impossible to read these signs accurately. The first false sign of failure was a 21-point drop in SAT scores from 1963 to 1973. Although everyone should have realized immediately that something outside of mathematics was affecting performance when the verbal scores dropped 35 points in the same time period, not until 1976 did an official College Board report indicate that the drop in the 1960s was due to the much larger numbers of students taking the test.

The second false sign of failure was the appearance in 1972 of the first National Assessment of Educational Progress data on how well our 13-year-olds and 17-year-olds performed. As virtually always happens in the first administration of every large-scale test, performance was lower than people expected. But the NAEP designers were not so naïve. They also purposely tested adults who had gone to school before new math and found that the 17-year-olds outperformed those adults. This result, however, had no effect on the public view.

There was a true sign of failure. Although, overall, students seemed to be helped by new math, many students—particularly slower ones—were not well served by an abstract mathematics curriculum. These students were blown away by the new math, and their parents commiserated with them. Teachers and other adults who had been against the new math from the beginning used every instance of failure of new math students as a sign that the entire movement was a failure, and rallied public support against these curricula.

In the mid-1970s, as a response to the new math, a back-to-basics textbook series for grades K-12 appeared. It encouraged competence on skills without properties or applications, and the books contained little or no explanation. For a couple of years the elementary school texts of this series were the most purchased in the country. Though the back-to-basics high school texts were used in many places, well into the 1980s the textbooks of the Dolciani series, written in the 1960s and showing great influence of new math, remained the most used books in the country for algebra and geometry students. Honors algebra and geometry classes and the more advanced courses in the best schools continued to teach a curriculum very much like the new math curricula of the 1960s.

Concurrent with the back-to-basics movement came a movement for minimum competence, and these two movements together had the effect of encouraging teachers to teach algebraic skills without understanding and to lessen attention to proof in geometry. There was also a positive effect: Books were cleansed of the excesses of the new math. For instance, the ubiquitous first chapter on sets that had little relation to the rest of the book was taken out, as were overzealous formalisms and explanations that were at too high a level for student understanding.

Most parents of today's students took the courses in the 1970s that their children are being taught now. So the experience of the parents of current students is likely to have been at the time when new math was being branded a failure and back-to-basics curricula were being touted.

It is difficult to find any value in the back-to-basics backlash other than the cleansing of the excesses of new math. Within a few years following the backlash, scores on the SATs were the lowest they have ever been. That situation prompted quite a number of reports in the late 1970s and early 1980s encouraging improvement in mathematics education. Some of these reports promoted problem-solving rather than skill development as the key goal of school mathematics. Others promoted a rethinking of the high school curriculum with continued attention to algebra, geometry and functions, but stronger attention to applications, to probability and statistics, and to the widespread use of calculators and computers and the mathematics related to them. Mathematicians and mathematics educators worked together on these reports. They were, for the most part, not the same people who had led the new math movement of 25 years earlier. The mathematicians included applied mathematicians, computer scientists, and statisticians. The mathematics educators included big city and state supervisors. The most well-known of these reports was *A Nation at Risk*, which appeared in 1983.

THE CURRENT ERA

The situation since 1983 has been strikingly parallel to that of the new math era. The first of the reform projects was UCSMP. Six years later the major catalyst for more reform appeared in the form of the NCTM *Curriculum and Evaluation Standards for School Mathematics*. Within a couple of years, the government—specifically, the National Science Foundation—poured massive amounts of money into curriculum reform. These events followed almost exactly the schedule of the development of the new math 32 years earlier, as Table 1 indicates.

Again there was euphoria. State after state adopted its own version of the *Standards*. NSF felt so good about its projects that it assumed they would be successful and planned for dissemination well before any data were collected. And there are statistics to back up these good feelings about the current era. During

Table I
Parallel Developments in New Math Era and Current Era

	New Math Era 1951-1973	*Current Era* 1983-
First project: year n	UICSM	UCSMP
Catalyst for more projects: year n+6	Sputnik	NCTM *Standards*
Government help years n+7 on	NDEA	NSF curricula
Sign of euphoria year n+11	Cambridge Conference	States follow NCTM
False signs of failure years n+14, n+21	SAT decline, NAEP	TIMSS
True sign of failure	Poorer students lost	not known

the 1990s, more students have taken more mathematics in high school than ever before. Until a decline of a single point this year, in every year of the 1990s SAT scores have stayed the same or increased from the previous year. ACT scores have also either increased or stayed the same for each year in the decade. Mean scores on the long-term trend data of the National Assessment of Educational progress have increased. It has been a decade of phenomenal growth.

However, again there is a false sign of failure. This time it is the misinterpretation of the results from the Third Intenational Mathematics and Science Study (TIMSS). The TIMSS researchers did not compare performance of our students now with the performance of our students on FIMS (First International Mathematics Study, 1964) or SIMS (Second International Mathematics Study, 1981). If they had, the headlines would have been different, because U.S. students seem to have performed quite a bit better comparatively on TIMSS than on the previous studies.

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> ***The disparities are tragic...even though, ostensibly, we do not teach different mathematics in these different places.***

Specifically, the U.S. is being compared to Singapore, which scored even higher than Japan at the 4th and 8th grade levels. (Singapore did not participate at the 12th grade level.) But I will argue that the U.S. performs strikingly well, even compared to Singapore. My argument has to do with economics, sociology and geography.

First, the economics and the sociology. Throughout the world, both FIMS and SIMS showed that performance within a country was higher in those places within the country that were more affluent. The one exception to this was Japan, where performance was quite uniform throughout the country.

It is well-known that performance within the U.S. fits the international pattern. That is, throughout our country the best performing students in general are found in our affluent suburbs and the lowest-performing students are found in our poorest rural and urban areas. In our affluent suburbs, the students do score as well as the students from Singapore. Our evidence for this comes from the performance of students in the First in the World Consortium outside Chicago on TIMSS. I am reasonably certain that performance would be matched in similarly affluent places elsewhere in the country where the schools can select their own curriculum and are not subject to state constraints. Just this week, Gerald Bracey, a writer for *Phi Delta Kappan* on the interpretation of educational research, has reported that such a study has been done of the data from TIMSS and that it shows our suburban areas would be second in the world. If true, it would indicate that these students score as high despite many of the students not having a curriculum that is as advanced as that of Singapore. It would thus show that our suburban students learn better what they are taught than students from Singapore.

Now for the economics and the geographic part of the argument. Singapore is an independent country, but viewed from a larger geographic perspective it is the most affluent area of southeast Asia. Its per capita gross national product is five times higher than that of Malaysia which surrounds it and is only surpassed by the very small country of Brunei. Singapore’s per capita GNP is higher than that of Spain or Hong Kong or New Zealand. As with our suburbs, in recent generations people migrated to Singapore from neighboring areas, mostly China, because they wanted a better life. Today over three-fourths of the population of Singapore is Chinese even though Singapore does not lie close to China. The population of Singapore is special for the same reason that the population of our suburbs is special. And for these reasons the performance is similar.

There is no question we can do better than we have been doing. The disparities are tragic between performance in some states and others, and between performance in our suburbs and our cities, even though, ostensibly, we do not teach different mathematics in these different places. The performance in our more affluent areas demonstrates that we can improve what we are doing without major changes in curricula, but it also suggests that we might have to change economic opportunity in order to do so. We still have huge numbers of mathematics teachers who do not know enough about the subject to teach it well. These teachers have trouble handling a curriculum like UCSMP's which wants students to have more than one way of doing a problem and asks students to apply mathematics and make connections.

But is there a sign that we are doing worse than we have done in the recent past? I don't know of a single national study in which such a signal is found. Furthermore, states such as North Carolina and Texas and Michigan, whose National Assessment results have increased the most of any states in the country, are those who claim to have implemented the current kinds of reforms. Nevertheless, there are those who claim that the present reforms are a failure.

BELIEFS OF THE ANTI-REFORMERS

While a great number of mathematicians support the reforms in K-12 mathematics education, another group opposes these reforms. The anti-reform mathematicians are from the same types of outstanding universities as the mathematicians of the Cambridge Conference. For the most part, they are research mathematicians. Some are quite eminent. We cannot expect their knowledge of mathematics education and of students in schools to be any greater than that of the mathematicians in the Cambridge Conference. But, unlike the Cambridge Conference mathematicians, who took their role to be provokers and were careful to say that their ideas needed to be tested, these mathematicians desire to directly affect mathematics education.

In one state of the union they have taken over, and from this state we can obtain a picture of the solution these mathematicians offer. Their solution is found in the *Mathematics Framework for California Public Schools*.

The catalyst for the *Mathematics Framework for California Public Schools* was California's poor performance on the 1996 National Assessment of Educational Progress. California scored 3rd lowest of the 44 states that participated in this assessment test. Its mean scale score of 138 was 10 points, or approximately one full grade level, behind the national norm of 148. But this disguises the differences among the performance of various subgroups. White students in California were only 3 points below the national mean for white students. Asian students scored only 2 points below the national mean for Asian students. Black students scored 1 point above the national mean for Black Students. But Hispanic students, constituting 39% of the student population, scored 27 points behind the total national norm and 6 points behind the national mean for Hispanic students, and they caused the state's overall mean to be so low compared to the nation. [*Science and Engineering Indicators*, p. A-12]

With such diversity, it would seem reasonable to leave decisions to local school districts about what mathematics should be taught. But there is a history in California of strong control from the state's Department of Education in Sacramento. For grades K-8 the state approves books, and the approved books must follow the state framework. *Mathematics Framework for California Public Schools*, therefore, is not just a theoretical document; it has teeth.

Twenty-four individuals are listed as having contributed to the present *Mathematics Framework*. Not one of these individuals is a university mathematics educator, and all the sample problems were developed by university mathematics professors.

The tone of the document reflects the excesses rather than the lessons of the new math. Here is the introduction to one of those problems [p. 154]: "Starting with grade eight, students should be ready for the basic message that logical reasoning is the underpinning of all mathematics...Students should begin to learn to ***prove*** every statement they make. Every textbook or mathematics lesson should try to convey this message and to convey it well. Consider the problem of solving this equation:

$x - \frac{1}{4}(3x - 1)= 2x - 5$

Multiply both sides by 4 to get:
$4x - (3x - 1) = 8x - 20$
Then simplify the left side to get:
$x + 1 = 8x - 20$
Transposing x from left to right yields:
$1 = 7x - 20$
One more transposition gives the result $x = 3$.
So far this seems to be an entirely mechanical procedure. No proof is involved."

No proof is involved? It looks very much like a proof to me, except that I would emphasize doing the same things to both sides of the equation and avoid words like "transposing" that suggest to students that math-

Recommended Proof of $x - \frac{1}{4}(3x - 1) = 2x - 5$ by the writers of the Mathematics Framework

1.	$x - \frac{1}{4}(3x - 1) = 2x - 5$	1.	Hypothesis
2.	$4(x - \frac{1}{4}(3x - 1)) = 4(2x - 5)$	2.	$a = b$ implies $ca = cb$ for all numbers a, b, c.
3.	$4x - 4(\frac{1}{4}(3x - 1)) = 4(2x) - 20$	3.	Distributive law
4.	$4x - (4 \bullet \frac{1}{4})(3x - 1) = (4 \bullet 2)x - 20$	4.	Associative law for multiplication
5.	$4x - (3x - 1) = 8x - 20$	5.	$1 \bullet a = a$ for all numbers a
6.	$4x + (-3x + 1) = 8x - 20$	6.	$-(a - b) = (-a + b)$ for all numbers a, b.
7.	$(4x + (-3x)) + 1 = 8x - 20$	7.	Associative law for addition
8.	$x + 1 = 8x - 20$	8.	$4x + (-3x) = (4 + (-3))\,x$, by the distributive law
9.	$-x + (x + 10) = (-x + 8x) - 20$	9.	Equals added to equals are equal.
10.	$(-x + x) + 1 = (-x + 8x) - 20$	10.	Associative law for addition: $0 + 1 = 1$.
11.	$1 = 7x - 20$	11.	$-x + 8x - (-1 + 8)x$, by the distributive law
12.	$1 + 20 = (7x - 20)+ 20$	12.	Equals added to equals are equal.
13.	$21 = 7x + [(-20) + 20]$	13.	Associative law for addition
14.	$21 = 7x$	14.	$-a + a = 0$ for all a; $b + 0 = b$ for all b.
15.	$3 = x$	15.	Multiply (14) by $\frac{1}{7}$ and apply the associative law to $\frac{1}{7}$(7x).
16.	$x = 3$	16.	$a = b$ implies that b = a Q.E.D.

Mathematics Framework for California Public Schools, p. 155

ematics is a bag of tricks. If you showed that 3 is indeed a solution to the first equation, then I would argue that this is a proof that $x - \frac{1}{4}(3x - 1) = 2x - 5 <=> x = 3$. But this does not constitute a proof for the *Mathematics Framework* writers. Their proof can be found at the top of the next page. It is, in my opinion, cruel and unusual punishment to inflict this kind of pedantry onto young children.

Under the criterion for proof used by the Mathematics Framework writers, virtually every argument labeled a "proof" in any college textbook or any article on mathematics would be disqualified. So we now have a new criterion for a written proof. It must be rigorous. But even the presented proof is not rigorous. A couple of reasons are missing in step 15. And where are the logical principles such as modus ponens and the transitivity of implication?

The authors of this example have confused rigor and proof. They have confused logic and proof. And in the process they have repeated one of the major excesses of the new math era: the overemphasis on rigor.

There is a significant marginal comment on this page. "Without the realization that a mathematical proof is lurking behind the well-known formalism of solving linear equations, a teacher would most likely emphasize the wrong points in the presentation of beginning algebra." I agree with the point that students should learn that solving an equation proves a statement. But this is not the time to learn that. The authors have made a natural but fundamental error about teaching young students. Every teacher learns through experience that students learn in different ways and that a multitude of explanations are needed, ranging from the formal to the intuitive, from the symbolic to the pictorial.

Because the countries that scored highest on TIMSS tend not to use calculators in early grades, the authors of this framework conclude that calculators cause our students to perform poorly. Having asserted that correlation does not imply causation, they reason as if it does. The assessment program that goes along with this framework does not allow the use of calculators from kindergarten to grade 11. The authors ignore the fact that Singapore, Japan, and China, in their newer elementary curricula, are introducing calculators because they have come to realize the necessity of their students being technologically facile with mathematics. If you are interested in reading about these international developments, examine the proceedings from UCSMP's Fourth International Conference on Mathematics Education held in the summer of 1998 and now available in *Developments in School Mathematics Education Around the World, Volume 4* from NCTM.

There are a number of applications presented in the *California Framework*, particularly in the sample problems. Statistics has a strong presence. But modeling, as essential to applied mathematics as proof is to pure mathematics, is completely absent. The student will leave high school not realizing that mathematics is applicable outside of money matters, statistics, and the physical sciences.

With the exception of statistics, the *Mathematics Framework* ignores virtually all of the developments in the mathematical sciences in the past 50 years. The authors have created a curriculum that asserts what was good for students 30-40 years ago is still what's good for students today. And they have not taken into account that such a curriculum destroyed students at the bottom end.

CONCERNS AT THE COLLEGE LEVEL

Anti-reform mathematicians appear to be motivated by three major concerns. A first concern is that, at the top end, we are not creating enough students with high mathematical competence. This includes the concern that we are not creating enough well-trained mathematics teachers. A second concern is that too many students enter college needing to take remedial mathematics courses because they lack sufficient paper-and-pencil manipulative algebraic skills. The third concern is the decreasing emphasis on proof in secondary school mathematics courses.

The third concern, that proof is disappearing from high school mathematics, is one that we in UCSMP feel is a valid concern. We have tried to incorporate proof into four of our courses, with strong attention in both Geometry and PDM.

Most of my data about the first concern, students at the top end, come from the National Science Board report entitled *Science & Engineering Indicators* for 1998. This means that the data go no further than 1996. Let us begin with AP calculus.

ADVANCED PLACEMENT STUDENTS

In 1994 7% of all high school graduates took an AP calculus course, compared with 4% in 1990, 3% in 1987, and 1.5% in 1982. Almost half of these students are female. [*Science & Engineering Indicators*, 1998, p. A-16] This enormous increase is due in part to increasing numbers of students taking algebra in eighth grade, a trend in which UCSMP has had a hand. Even the birth of AP statistics has not lowered the numbers of students taking AP calculus.

PERCENT OF STUDENTS INTENDING TO MAJOR IN SCIENCE OR ENGINEERING

I could not find data for all students, so these data are limited to white students only. In 1996 32.2% of white freshmen intended to major in science and engineering. This is the highest percent in the last twenty years. Of these, 11.6% planned to major in the natural sciences. That is lower than the 12.0% of 1995 and the 11.9% of 1994, but higher than every other year since 1976.

The percent of white freshmen planning to major in mathematics or computer science peaked in 1982 at 5.9% but by 1985 had gone down to 2.5% and by 1992 was 2.2%. Since 1992 there has been a reasonably steady rise in this percent. In 1996 2.7% of freshmen planned to major in mathematics or computer science, the highest percent since 1985. [p. A-57]

Table 2
Bachelor's Degrees in Mathematics and Computer Science Awarded in the US Between 1975 & 1995

	Mathematics	*Computer Science*
1995:	13,851	24,769
1987:	16,515 (relative max.)	39,927
1986:	16,388	42,195 (max.)
1981:	11,901 (relative min.)	15,233
1975:	18,346	5,039

From *Science & Engineering Indicators 1998*, p. A-64.

The net result is that more students are now coming into college with high-end mathematics and with a broad desire to major in science or engineering, and a specific desire to major in mathematics or computer science, than in the 1980s before the current reforms. Whether this is due to economics or to curriculum, I do not know.

Now let us ask what happens to these students.

Number of Bachelor's Degrees

The number of bachelor's degrees in mathematics has seesawed. It was 18,346 in 1975, a year in which most students would have had their high school education in new math-oriented curricula. From 1979-1984, years in which students would have had their high school education affected by back-to-basics, it went down to around 12,500. Since 1985, its peak was in 1987 and there has been a steady decline to the 1995 level of 13,851, which is about 20% below the 1987 level. (See Table 2.) This is a serious problem, because we need more mathematics majors, but it will have been solved by this year if the degree-intending students of 1996 get degrees in mathematics proportional to their numbers in prior years.

Even more surprising is a much more serious decline in the number of bachelor's degrees in computer science than the decline in mathematics. There were over 42,000 computer science degrees in 1986 but under 25,000 in 1995. (See Table 2.)

Foreign Citizens

There has been a reasonably steady increase in the percent of bachelor's degrees in mathematics or computer science given to foreign citizens, from 5% in 1985 to 7% in 1995. [p. A-67] It is very difficult to see how this is related to changes in the U.S. curriculum. It seems far more related to the easing of world tensions early in this decade, to the increase in the study of English worldwide as a second language, and to the increased desire for a college education by people in countries where college attendance is not as accessible as in the U.S.

Graduate Enrollment

Graduate enrollment in mathematics and computer science is double what it was in 1975 and has fluctuated in a narrow range since 1987. The percent of foreign nationals has also fluctuated, between 26% and 33% since 1983, peaking in 1991. It is currently at 32%. [pp. A-70, A-72] Graduate enrollment has to lag quite a bit behind school curriculum changes, so these data are not influenced by the current reform movements.

Table 3
Freshmen Reporting Need for Remedial Work in Mathematics in 1995

By major and gender

	All	Male	Female
Physical science:	13.0%	11.3%	15.5%
Engineering:	14.3%	13.9%	15.5%
Social science:	27.5%	20.2%	32.2%
non-science or eng:	25.0%	20.8%	28.0%

By major and ethnicity

	White	Black	Hispanic
Physical science:	11.1%	39.9%	20.7%
Engineering:	10.3%	31.1%	34.1%
Social science:	24.2%	47.5%	41.2%
non-science or eng:	22.0%	47.1%	37.0%

Doctoral Degrees

The number of doctorates in mathematics in 1995 was 1,190, the highest it has been since 1975 and 72% higher than its low value in 1985. The number of doctorates in computer science in 1995 was the highest it has ever been. While the total number of doctorates in mathematics and computer science granted to temporary residents of the U.S. has more than doubled since 1977, the number of doctorates granted to U.S. citizens has also increased significantly. [p. A-82]

These data do not suggest a crisis, and certainly not anything attributable to recent curricula.

Now let us consider the second concern—that students are more poorly trained than they used to be. I do not have long-term data on this, but 13% of all physical science and 14% of all engineering freshman majors in 1995 reported a need for remedial work in mathematics. This figure varies markedly with ethnicity: 11% of whites but over 30% of Blacks. In other words, 1 out of 9 white students and 1 out of 3 Black students majoring in the physical sciences or engineering reported a need for remedial work in mathematics. (See Table 3.)

Significantly more freshman majoring in social science need remedial work in mathematics: 20% of males and 32% of females; 24% of whites and 48% of Blacks. These high percentages seem to reflect the ancient view that if you are going to major in the social sciences, you don't need to take as much mathematics in high school. These students, generally not as proficient, will not be helped by a more theoretical mathematics curriculum.

I assume that these percents are higher today than they used to be. And so we must ask: if scores of high school students are going up, why are the percents of students needing remediation not going down?

The reasons are many. First, despite revolutions in applied mathematics and in the ways in which computers change how mathematics can be done, the mathematics departments of many, if not most, major universities have not changed their basic required curriculum in a generation. And, because they haven't changed their curricula, they haven't changed their tests to represent what students are taught in high school. So the students do not score as well as they used to score (though, frankly, I have yet to see a published study on college placement tests over time). Mathematics departments need to wake up and recognize statistics, computer science, and applied mathematics as topics that are as important for mathematics majors as algebra and analysis, and test incoming students on their knowledge of basic ideas from these areas. Placement exams need to recognize that computers are here to stay, and allow students to use calculators on the tests because they will have such calculators with them their entire lives.

A second reason for more remediation is that college mathematics requirements have increased. Fields that used to require very little mathematics—psychology, business, the biological sciences, and the social sciences—now require statistics or calculus and sometimes linear algebra and finite mathematics. Many institutions now require some mathematics of all their students. In this regard, high school counselors are often behind the times. And, consequently, some students come to college without having taken the mathematics they should have taken to major in these fields. And, when they have taken the mathematics, they thought that it was just to fulfill an entrance requirement and did not realize that they would have to demonstrate competence.

Third, high schools are doing a better job of interesting their students in mathematics, and so some students who are not the very best students still like the subject, and they want to major in it even though they will not be research mathematicians. They may not be as good as most mathematics students were in the past, but they are as interested, and we need them because we have a chronic shortage of mathematics teachers.

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They may not be as good as most mathematics students were in the past, but they are as interested, and we need them because we have a chronic shortage of mathematics teachers.

Mathematics departments in many institutions operate as if computers and calculators do not exist, have requirements that suggest that applications of mathematics are for the not-so-serious student, still think that writing logically-correct mathematics legibly is sufficient to be called good teaching. No wonder that they are losing students to statistics, to economics, to operations research, to business and to many other disciplines which yearn for students who like and are proficient at mathematics. Who wants to major in an area that ignores even major changes within it?

Some people have argued that baseball players are not as good today as they were fifty years ago. Fifty years ago if you wished to be a professional athlete, you had little choice but to go into baseball. So baseball got the best athletes. There may be an analogy with mathematics. Fifty years ago if you liked pure mathematics, your choice was limited to mathematics. But today, you can go into many disciplines where your talents will be utilized.

A fourth reason for the poor performance of incoming college freshmen on placement tests is that placement tests are often given under conditions that do not allow students to show off what they know. A test given to students who have just come to campus a few days before, who are concerned about their new roommates, about the medical exam they just had, about their new ID card, not to mention being away from home, and who may have stayed up quite late talking to others in their dorms, is not being taken under optimal conditions. Also, students have had different courses in high school and need to be informed in advance exactly what topics are going to be on the test, and the kinds of language and notation that are going to be used.

REASONS FOR DISAGREEMENT

Why do people come to different conclusions about what is happening? Why does a U.S. Department of Education panel's choice of the best mathematics materials in the country for grades K-8 have nothing in common with the books selected earlier this year in California, except for UCSMP Transition Mathematics and Algebra?

We who are in the field are privy to much information about what is going on. We may be aware of the politics on both sides. But what are parents and the public to think? When there are conflicting views about an issue, and there seems to be no overwhelming authority, the tendency is to believe the loudest or the boldest. The press does not help; they revel in publicizing conflicts and tend to select individuals with extreme positions to make the point that there is a conflict.

We who are in mathematics must fight this tendency, regardless of how we feel about the issues. Truth in our field is based on careful reasoning. If we are in pure mathematics, we reason from assumptions using logical deduction. If we are in applied mathematics, we analyze data using statistical principles. In neither case should we allow untested opinion to sway us. In those cases where we do not have enough evidence to make a conclusion, we should be willing to say that a problem is unsolved. If we come to different conclusions, we ought to try to apply the tools of mathematics to determine why.

I don't think the critics of current reforms are operating with the same assumptions that we have, and I would like to finish by asserting some of the assumptions under which we operate at UCSMP. We in schools must educate everyone, and we cannot assume our students are motivated by the same things that motivate university-level mathematicians. As the NCTM Professional Teaching Standards emphasize, teaching is a complicated process, not subject to simple prescriptions. In some cases logical approaches work, but for many topics a good application or a game or an activity works better, and representations can be particularly powerful. Capable mathematics teachers who teach students every day contributed to the NCTM Standards and the newer curricula. They are not ignorant of mathematics. We want our students to have the same appreciation for its beauty, its logical structure, and its applications that we have. We try to instill in our students an appreciation also for careful reasoning, for not assuming a conclusion without weighing all, of the evidence. Statistics and mathematical modeling help our students to weigh data, to recognize the importance of comparable samples when comparing groups, to realize that there may be more than one answer to a real problem. We teach the students of today for what they need tomorrow, not for what they needed yesterday, and we realize that to avoid the use of technology is to doom our students to ignorance of much of the world of mathematics. We recognize that mathematics is important in consumer affairs, in matters of public policy, and in business as well as in its traditional venues of science and engineering and a subject to study for its own sake. It is because mathematics is more important than ever that we must work to see that all students are not only taught a significant amount of mathematics, but that they learn it.

"Try not to become a man of success but rather to become a man of value."

--Albert Einstein
