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Book Review: *Algebra in Context:
Introductory Algebra from Origins to Applications*
by Amy Shell-Gellasch and J. B. Thoo

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Synopsis

This is a review of *Algebra in Context: Introductory Algebra from Origins to Applications*, a textbook authored by Amy Shell-Gellasch and J.B. Thoo. The text presents traditional mathematics through the lens of history, allowing students to gain a rich understanding of how mathematics works and where it comes from. In addition to providing the reader with a summary of the book contents, the reviewer suggests why and how the text may be incorporated into college-level mathematics courses.

Algebra in Context: Introductory Algebra from Origins to Applications. By Amy Shell-Gellasch and J. B. Thoo, Johns Hopkins University Press, Baltimore, 2015. (Hardback US\$99.50, ISBN: 978-1-4214-1728-8. 536 pages.)

“Glaisher, I believe, once said that no subject loses more than mathematics by any attempt to dissociate it from its history.” This is the opening sentence of an article authored by British mathematician, cosmologist, and science historian G. J. Whitrow. The article, “The Importance of the History of Mathematics in Relation to the Study of Mathematical Technique,” was published in 1932 in *The Mathematical Gazette*. ‘Glaisher’ refers to the respected mathematician and astronomer, James Whitbread Lee Glaisher (1848-1928), whose comment remains as true today as it was a century ago.

Despite best efforts to reform mathematics education by re-writing curriculum, using innovative teaching pedagogies, and teaching with technology,

a large portion of society continues to view mathematics as mechanistic and unintelligible. Learning mathematics devoid of its history encourages the notion that mathematics is a fixed immutable body of knowledge rather than a growing field of study. By dissociating mathematics from its history, we ignore the intuitiveness and simplicity that was inherent in the development of mathematics and the role intuition can play in doing, and thus understanding, mathematics. We miss connections between the assortment of practices that different cultures have employed for addressing the same mathematical questions. Comparing and contrasting these processes can inform and deepen our understanding of modern day mathematical practices.

Algebra in Context: Introductory Algebra from Origins to Applications is a textbook that combines history and mathematics in ways that allow readers to learn mathematical concepts and procedures through the lens of how these ideas came about over the course of history. Readers gain an appreciation for the many cultures in which the mathematics originated. The text assumes prerequisite knowledge of basic algebra commonly covered in a traditional high school course. Thus, the text is accessible to a wide range of audiences. In this review, the book's organization and features, a brief overview of the content, and several course scenarios of how the text may be used are presented.

Organization and Features

The text is organized topically, rather than chronologically, into four parts entitled: Numeration Systems; Arithmetic Snapshots; Foundations; and Solving Equations. Each part contains several chapters in which history and mathematical ideas are seamlessly intertwined. Photos, maps, graphics, proofs, and examples support the well-crafted writing. The exposition of the material makes apparent that authors Shell-Gellasch and Thoo are, first and foremost, master teachers of mathematics who have carefully written the text with student learning in mind.

A feature of *Algebra in Context* not typically found in other history of mathematics textbooks is the inclusion of student exercises. An examination of history of mathematics books reveals editions that either contain no exercises or a collection of problems located at the end of chapters. In *Algebra in Context*, the reader encounters two types of exercises integrated at appropriate places within the sections. The "Now You Try" exercises give students

an opportunity to try out a mathematics procedure immediately after it is presented, allowing students to make sense of material and practice before moving to the next mathematical concept. The “Think About It” exercises are also distributed throughout the chapters. These “pondering” questions are meant to elicit an appreciation of mathematical techniques or historical contexts.

In addition to these “real-time” exercises, the four parts of the text contain a concluding chapter of additional exercises. Some exercises are routine, while others are nonstandard and provide added depth, variety, and challenge. Questions may require students to do research, for example:

Write a history of the decimal separatrix that separates the integer part of a numeral from the fraction part. (p. 84)

How is the date of Easter calculated? Why would that require intricate mathematics? (p. 138)

Write about GIMPS, the Great Internet Mersenne Prime Search. (p. 263)

Explain why it could be argued that $1 - 1 + 1 - 1 + 1 + \dots = 1/2$. (p. 511)

Using these questions allows instructors to naturally incorporate research, writing, and proof assignments into courses.

Content

Part I: Numeration Systems

Where did mathematics begin? What cultures created and used symbols for counting collections of objects? How do numbers “work” in different numeration systems? Part 1: Numeration Systems contains content that addresses these questions. The notions of number bases along with the role of zero provide the foundation for understanding numeration and what is meant by a positional or place-value system. Readers are encouraged to practice converting base-ten numbers to base-six and base-four, and vice-versa, in order to conceptualize the important ideas associated with place value.

Many civilizations developed numeration systems throughout history, some positional and some non-positional. We are fortunate to have artifacts that give insight to the usage of these systems, and the remaining chapters in this section explicate the Babylonian, Egyptian, Roman, Chinese, Mayan, and Indo-Arabian numeration systems.

Part II: Arithmetic Snapshots

After learning about numeration systems, a natural progression is to examine how computation was performed in various cultures. “Arithmetic Snapshots” is an apropos title of Part II and conjures up an image of the authors sitting at the dining room table with an overflowing shoebox full of mathematical gems placed between them, observing, “We really need to bring some order to these algorithms that have been developed over the course of history so others can appreciate them. Heaven knows if we don’t do it, it won’t get done.” And thus begins the process of pulling arithmetic snapshots out of the box, uttering expressions such as “Oh, look, here’s the Rhind papyrus fragment that shows that neat doubling method the ancient Egyptians used to multiply.”

The snapshots included in the text share how arithmetic was done in several cultures. Some cultures use an abacus and others a counting board. These tools were especially helpful with nonpositional numeration systems such as Roman numerals. A multiplication method, developed in the Indian culture, was carried out on a “dust board.” The algorithm required erasing digits while factors are translated one place value to the right and partial products are recorded in the spots vacated by erased digits. One may wonder how errors would be detected as steps are erased throughout the process of doing the computation. Another snapshot, presented in a subsequent chapter, shows how medieval Europeans as well as other cultures could detect errors in “dust board” computations where intermediate steps are not recorded. “Casting out nines” does not require the use of partial products. While the method detects errors, it does not confirm correctness of computations. Another snapshot shows how ancient Egyptians used a doubling and halving method for performing multiplication and division. Part II concludes with snapshots on finding square roots—constructing roots geometrically and approximating roots using algebraic formulas.

Part III: Foundations

“Foundations” presents three areas of mathematics—set theory, logic, and number theory. Set theory is a relatively new area of mathematics and was primarily developed by George Cantor (1845-1918). Sets of numbers are examined using the definitions and properties of set theory. Natural or counting numbers, used since prehistoric times, are joined with other types of numbers to form new sets of numbers, namely the sets of rational, irrational, and real numbers. Determining the cardinality of these sets and comparing their relative size provided many interesting questions for mathematicians and are presented in ways that will challenge students using this text.

While logic has existed since the time of Aristotle, modern or symbolic logic was developed because of Cantor’s work on determining cardinality of infinite sets. Mathematicians wanted to find a way to systematically verify “new” mathematics. One of those methods, propositional logic, begins with simple statements that may be classified as true or false. Logical relationships and properties are derived from combining and modifying the simple statements using operators and connectives. Using the rules of logic, complex statements are determined true or false depending on the truth values of the simpler statements.

The chapter entitled “The Higher Arithmetic” contains many of the topics found in an elementary number theory course sans the proofs. Readers are exposed to aspects of number theory attributed to Pythagoras, Euclid, Nicomachus, and Diophantus. Included are discussions of even and odd numbers, figurate numbers, and Pythagorean triples. The Euclidean algorithm for finding the greatest common divisor is just one of the topics presented as part of the mathematics related to divisors, factors, and multiples. Prime numbers—finding them using the sieve of Erastosthenes and the fundamental theorem of arithmetic—conclude this section.

Part IV: Solving Equations

The final part, composing roughly half the book, is dedicated to the many methods different cultures used to solve linear, quadratic, cubic, and other polynomial equations. This is followed by the history of the development and use of logarithms. Students may be surprised to learn that algebra first existed sans notation and was basically verbal explanations of how to solve problems.

Several cultures used the “method of false proposition” to find solutions to problems that today are written as linear equations. The method is intuitive, a form of guess-and-check that makes use of proportional reasoning. As stated in the text, the method instructs one to “Guess a solution, say g . If g leads to a result h that is incorrect, but $c \times h$ is the desired result, then the solution is $c \times g$.” (pg. 274) This method works for linear equations that pass through the origin. Double false position is also presented as a method of solving linear equations that do not pass through the origin.

Solving quadratic equations dates back to the ancient Babylonians and the text shares a number of methods used throughout history. Solving quadratics by completing the square, a method that introduces geometry in solutions, is explained. Some cultures present the solutions of quadratics that rely heavily on geometric properties, while other cultures present rhetorical sets of instructions that make no explicit connection to geometry.

Additional topics covered in this part of the text include the development of algebraic notation, the consideration of exponents, solving cubic and higher degree polynomial equations, complex numbers, the notion of zeros of a polynomial, and culminate in the fundamental theorem of algebra.

Proportional reasoning has throughout history played an important role in the study of algebra. The “rule of three”, a method for solving proportion problems appearing in early Chinese mathematics, is presented as well as examples from Indian mathematics. This precedes discussion of the historic references of direct and inverse variation.

Wrapping up this part of the text, is a chapter on logarithms—how we use them today and how they were used to complete calculations prior to the invention of computing machines.

Usage

General Education for non-STEM students

What is an appropriate “terminal” college-level mathematics course for students wishing to study in fields that do not rely heavily on knowledge of science, technology, engineering, or mathematics? Discussed ad nauseum by the mathematics community, there is agreement that college algebra is not that course. Arguments against college algebra is that the course is a repeat

of what students have been studying in high school and the course is primarily skills-based. And it can be easily demonstrated that taking college algebra does not provide students insight into the true nature of mathematics.

In lieu of college algebra, most colleges and universities offer alternative courses with titles such as Modern Mathematics, Quantitative Literacy, Mathematical Thinking, or Math for the Liberal Arts. The content and goals of these courses typically fall into one of two categories. One category has an aim of getting students to appreciate how mathematics is being used in the world. These courses include content such as graph theory, voting methods and social choice, and fair division. The other category consists of courses that attempt to personalize mathematics for students, exposing them to questions related to money—managing personal finances, mortgages, insurance, and investments. With the advent of the text by Shell-Gellasch and Thoo, a third option presents itself to the mathematical community, teaching mathematics via historical roots.

At my institution, we have been exploring ways to overhaul our general education program to make it more cohesive, relevant, engaging, and transformative for our students. We are particularly interested in designing courses that are inter-disciplinary, have opportunities for oral and written communication, and involve critical thinking. We also have a long-standing tradition of promoting global awareness and appreciating cultural differences. Lofty goals, since general education courses typically do not rely on pre-requisite courses. However, using *Algebra in Context: Introductory Algebra from Origins to Applications*, one could create a course that seems to check off most, if not all, of our criteria—interdisciplinarity, integrated communication, critical thinking, global awareness, cultural differences, no pre-requisite college courses. In addition, such a course provides an opportunity to allow students to revisit their study of mathematics, and to see it not as a collection of abstract and arduous procedures but something that can be understood and appreciated.

Mathematics Education

The effectiveness of mathematics teaching is most likely determined by the teacher's understanding of the mathematics he or she teaches. Teachers who develop depth and breadth of mathematical content knowledge are more likely to engage students in meaningful learning of mathematics.

I envision a number of courses in which the text *Algebra in Context: Introductory Algebra from Origins to Applications* could be used as the catalyst for elementary and secondary teachers to construct rich understandings of the mathematics they will teach.

Most elementary and middle school preservice teachers complete as many as three courses devoted to learning K-8 mathematics. Topics include numbers and operations, algebraic thinking, geometry, and probability and statistics. Part I: Numeration Systems and Part II: Arithmetic Snapshots of *Algebra in Context: Introductory Algebra from Origins to Applications* would be an excellent choice as a text for the number and operations course. Preservice teachers would gain understanding of how numbers evolved and develop conceptual understanding of place value.

Secondary preservice teachers do not typically have the benefit of taking courses that focus on the content they will teach. It is often erroneously assumed that these preservice teachers have understanding of algebra and geometry because they have successfully completed these courses as well as advanced mathematics courses. Hence, a text like Shell-Gellasch and Thoo's can provide future secondary teachers with the deep understandings of algebra that will allow them to be more attuned to different ways of thinking about, using, and doing algebra.

History of Mathematics

It is more rarity than rule that a university possesses a faculty member who holds a specialty in history of mathematics. An institution's History of Mathematics course is typically taught by a mathematics faculty member who took and enjoyed a history of mathematics course as an undergraduate, or the faculty member assigned to teach the course who developed an interest in the subject out of necessity and survival.

Challenges for novice instructors include designing and teaching a course that is very different in nature than other mathematics courses. History of mathematics courses require reading—lots of reading by instructors and students. Lectures must be prepared that tell a story replete with people, places, dates, and context. Instructors often devote time and effort locating artifacts online that enhance and support lectures. Many history of mathematics courses require students to conduct research and write papers. Instructors need to develop a list of topics appropriate for short and long papers,

have some knowledge of what sources are essential for each topic, and learn how to develop and use formative and summative measures to evaluate student writing. History of mathematics textbooks are typically tomes—filled with copious information and the instructor must pick and choose what to use. In short, the work can be overwhelming.

To these instructors and their students, *Algebra in Context: Introductory Algebra from Origins to Applications* is a godsend. The material is robust but not overwhelming, making the selection of the course content easier. The readability of the text and the insertion of problems within the sections make the book more akin to a mathematics textbook than traditional history of mathematics texts. The additional problems and research questions found in the final chapter of each part provide topics for short or long papers, presentations, and class discussion.

The prerequisite of college algebra permits undergraduate mathematics majors and others to take the course earlier in their programs. Perhaps some students will discover they have a penchant for history of mathematics, or history, or mathematics, and pursue more study through additional coursework or on their own.

Using *Algebra in Context: Introductory Algebra from Origins to Applications* as the primary textbook for a history of mathematics course means that some topics included in the traditional course would be absent—most notable are history of geometry and history of calculus. However, I would contend that with foundational knowledge and skills developed in a course built on the topics of *Algebra in Context: Introductory Algebra from Origins to Applications*, the histories of both geometry and calculus could be addressed quite appropriately within the content courses themselves.

Conclusion

Algebra in Context: Introductory Algebra from Origins to Applications is an atypical textbook. It is well-written, has instructive exercises embedded in the material, and includes additional questions that may be assigned for additional research and writing. The text lends itself as the perfect selection for use in general education, teacher education, and history of mathematics courses. *Algebra in Context: Introductory Algebra from Origins to Applications* paves the way for users to change their notions about where algebra comes from and how to think algebraically.