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Review: Transitivity and bundle shifts

Stephan Ramon Garcia
Pomona College

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Douglas, Ronald G. [Douglas, Ronald George] (1-TXAM);
Xu, Anjian (PRC-CQUT-SMS)
Transitivity and bundle shifts. (English summary)
Invariant subspaces of the shift operator, 287–297, *Contemp. Math.*, 638, *Amer. Math. Soc.*, *Providence*, *RI*, 2015.

Let $B(\mathcal{H})$ denote the set of bounded linear operators on a separable Hilbert space $\mathcal{H}$. A unital subalgebra of $B(\mathcal{H})$ is *transitive* if it has only trivial invariant subspaces; that is, only $\{0\}$ and $\mathcal{H}$. We say that A is *catalytic* if every transitive subalgebra of $B(\mathcal{H})$ that contains A is strongly dense. In 1967, W. B. Arveson proved that the unilateral shift of multiplicity one and non-scalar Hermitian operators of multiplicity one are catalytic [Duke Math. J. **34** (1967), 635–647; MR0221293]. S. Richter proved that the Dirichlet shift is catalytic in 1988 [J. Reine Angew. Math. **386** (1988), 205–220; MR0936999]. More recently, G. Cheng, K. Y. Guo and K. Wang showed that the coordinate multiplication operators on a functional Hilbert space with complete Nevanlinna-Pick kernel are catalytic [J. Funct. Anal. **258** (2010), no. 12, 4229–4250; MR2609544].

A uniform algebra on X is a *Dirichlet algebra* if $\operatorname{Re} A$ is uniformly dense in $C(X)$. One says that A is a *logmodular algebra* on X if $\log|A^{-1}|$ is uniformly dense in $\operatorname{Re} C(X)$; since $\operatorname{Re} A \subseteq \log|A^{-1}|$, every Dirichlet algebra is a logmodular algebra. One says that A is a *hypo-Dirichlet algebra* if there is a finite set of elements $f_1, f_2, \dots, f_s$ in A^{-1} so that the linear span of $\operatorname{Re} A$ and $\log|f_1|, \log|f_2|, \dots, \log|f_s|$ is uniformly dense in $\operatorname{Re} C(X)$; the number s is taken to be as small as possible.

For a hypo-Dirichlet or logmodular algebra, the authors show that $A = H^\infty(m)$, acting on a generalized Hardy space $H^2(m)$ in which the representing measure m provides $H^2(m)$ with the structure of a reproducing kernel Hilbert space, is catalytic. They also show that for finitely-connected domains bounded by nonintersecting smooth Jordan curves, the "holomorphic functions" of a bundle shift yield a catalytic algebra. This generalizes a result of H. Bercovici et al. [J. Funct. Anal. **258** (2010), no. 12, 4122–4153; MR2609540].

{For the collection containing this paper see MR3309345}

Stephan R. Garcia