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There are a thousand rules on how to teach mathematics well. Some teachers tend to absorb these rules naturally and other teachers can't understand them at all.

However, rules can be irrelevant. There are a few teachers, virtuosos of the classroom, who violate the most basic rules and teach brilliantly. More often, there are teachers who have what seems to be excellent technique but fail to teach even adequately. The reason for all of this is that teaching mathematics (and to some extent all teaching) is an interpersonal skill and not merely a matter of disseminating knowledge.

Teaching mathematics means to communicate mathematics from teacher to students. Teaching is communication, and it is largely a social skill.

This point is not trite but rather profound, as so many would-be teachers do not understand it. It is in fact quite common — an everyday occurrence — for a class to be taught or lecture to be given without one individual in the audience knowing at all what the teacher is talking about. This is why I am not addressing the difficult question of how do we tell a good teacher from a bad one. There are many teachers that are not borderline. These are teachers of whom no student can say anything good. Undergraduates, graduates, D-students, A-students; they all condemn the teacher. And amazingly, nearly every university department in the country (of any size) has one or more teachers of this quality. Teachers who communicate with no one at all are largely a phenomena of the mathematical sciences, and are a primary reason that mathematics and mathematical reasoning is so poorly integrated into our society at large. Teachers like these make the mediocre teachers look good.

The Personal Side of Teaching

The most important aspect of teaching as communication is that communication with students is a two-way process. To teach effectively you need to gauge accurately what is being digested and what isn't. A crude way to do this, when you are unsure, is by giving an unannounced quiz (which doesn't have to count). The best tool, when lecturing, for judging the students is body language: faces and eyes. This is the window to the student, and if you do not know how to use it, you teach at a disadvantage. When you ask the students if they understand a certain point, their faces will give more information than their voices. At first their faces will not necessarily reveal much. The first week, students tend to regard certain questions as rhetorical. However, when they realize that you are genuinely concerned with their understanding (just as many teachers are not) they loosen up and it is then that their faces become revealing. It is this technique of watching faces that enables the teacher to set the proper pace.

It is incumbent in all of this that you are teaching the majority of your students. Again, teaching is communication and to maximize communication, you teach to as many students as you can. In many classes there are students who go into the course knowing much of the material. Many teachers invariably identify these students as their *good* students. However, with these students there is less communication, since it has taken place for them already. They should be taking another class! Similarly, there are students whose background is so deficient that they also should be in another class. However, when we say that a student's background is deficient, we mean that it is deficient with respect to the class at large. As a rule, virtually none of the students will have mastered the prerequisites to the degree that you feel that they should have. This is a basic law of the universe, that falls right behind Newton's laws of motion: *The students never know what they are supposed to know.* The exceptions are generally those students we have just encountered who already know the material of the course as well as the prerequisites. They should be taking another class and learning new material. With respect to the class at large, how much of the prerequisite material you review is a matter of judgment. It is nearly always wise to reiterate some of that material, but is inappropriate to spend much time on it.

Teaching Style

Pedagogical studies and treatises on teaching tend to concentrate on technique and style. However, these things should be entirely subordinate to communication. Style will not turn a bad teacher into a good one or vice versa. In fact, it is almost irrelevant to good teaching.

A unique style of teaching mathematics is due to R. L. Moore and is known as the *Texas school of teaching*. Although this style is worth study, it is not in itself a subject for this essay. Moore used his style with great success and it has been used by his "descendants" and their "descendants" (Moore's *descendants* are the people who received Ph.D.'s under him). The Texas school of teaching has been the subject of much attention including documentary films. It is clear that he was an inspirational teacher. Many teachers slavishly follow the precepts of Moore's teaching method to the letter. Yet remarkably often they achieve the precise opposite result. R. L. Moore would have been a great teacher had he used any style at all. Conversely, and unfortunately, no style will turn a bad teacher into a good one. To be a good teacher, it is necessary to be sensitive to other people as human beings. This is particularly true with the Texas school of teaching.

That style will not turn a bad teacher into a good one, has a corollary with respect to the incorporation of computers into mathematics instruction. Yes, computers can enhance math education at all levels, from preschool through graduate school. However, the successful integration of computers into education will never obviate the need for competent instructors unless the need for instructors is eliminated altogether.

Developmental Psychology

There is an order by which students learn material. I do not mean the tautological order; for example, one must study what a derivative is before studying differential equations. That much is trivial. I am referring to a couple of psychological truths. The first psychological truth is that students need to learn concepts in an ordered manner. Unfortunately, there are individual differences here, but some generalities hold. Most people learn from the specific to the general. One studies the real number line before one studies fields. At first, the student understands fields as generalizations of the reals. Eventually, the perspective changes. The scholar thinks in terms of fields, and sees the real number line as a specific case. And it is precisely at this point that many teachers make a critical mistake. Because this latter point of view has become clearer for them, many mathematics teachers teach abstractions before they teach specifics. The mistake is this: people learn mathematics in a different order than which they (later) come to understand it!

The second psychological truth is that students need to learn material in a paced manner. That is, some material needs to be digested before new material is learned. This is true at several levels. At the macro level, we don't go straight from the definition of the derivative to differential equations. At the micro level, I have seen a teacher severely undermine his effectiveness, simply by not pausing enough during his lectures. Lectures can be like (well-told) jokes: you need to pause for a beat or two. Here again the best tool for judging pace is body language. Many students seem to light up when an idea sinks in. If the idea isn't sinking in everywhere, you need to reiterate. If the idea is sinking in nowhere, you probably need to back up. It is a tragedy that so many teachers will continue when no one at all is absorbing anything.

Motivation

The foremost tool for teaching mathematics is motivation. Generally, the less advanced the students are, the more the need for motivation. Conversely, a characteristic of strong students and professionals is that they find their own motivation. Frequently, a proof does *not* motivate a theorem. The student wants to know where the theorem came from. For example, Lagrange Multipliers are usually *explained* with a proof that is largely algebraic. But the insight for the technique comes from a simple picture (and undoubtedly it was this picture that inspired Lagrange). Pictures tend to motivate better than does algebra, but not always. Since textbooks too frequently fail to provide motivation, it is the job of the lecturer to provide this motivation. It is generally supplied in the form of pictures, examples, or simply an indication of the where the topic is heading. Many teachers more or less write a textbook on the board. However, students nearly always prefer a textbook in hand; the teacher exists to supplement the book.

Homework

At all levels, to learn mathematics, it is necessary to do mathematics. Most students understand this, but elementary students in particular do not have the discipline to do mathematics without "encouragement." I find that most of them appreciate a little coercion. Here I am going to deviate from the usual pattern of this essay, and I will tell you what I personally have found to work in assigning homework.

When I have a grader, I assign homework at the end of each lecture on the basis of what I cover that day (and how well the students handle it). I collect homework at the beginning of each class and go over the problems on the board. I ask my grader to give me total scores at the end of the semester, standardized to some preset maximum, say 50 points. This should work out to be about 10% to 15% of the total grade. It should count little enough that a missed assignment will make little difference (and little hassle). On the other hand, homework should count enough that a student that consistently does not do the homework loses a letter grade. Both of these points are impressed on the students so that they feel compelled to do the homework, but it is not a matter for hysteria. Also they are more or less free to copy homework on the principal that what they gain on the homework, they more than lose on the tests. I tell the students that the purpose of the homework is to prepare them for the tests. I often use old test problems for homework. This mitigates against any advantage some students attain by having old tests in their possession. Also, students are very much interested in old test problems. In fact, if you run off copies of old test pages to hand out as homework, you cannot keep the students from doing the problems. I favor certain homework problems as being particularly pedagogical and tend to give similar homework problems from one semester to another. However as a matter of basic ethics, I have never given any test twice.

If I do not have a grader, I am inclined to give a quiz each week. Frequent quizzes will motivate students to do their homework, since that is how they study for the quiz. I find quizzes much quicker to grade than homework.

Textbooks

As a rule students prefer courses that are textbook oriented, and they are keenly interested in having a text that they can read and understand. Given the abundance of texts in most areas the instructor's job is to wade through a sea of mediocre texts and retrieve the exceptional text. However, not all texts are at least mediocre. There are not only poorly written texts published, but even major publishers will publish incompetent texts. These texts occur at all levels and in late editions. That such texts get selected is shameful. However, many teachers will favor a poorly written text. Such texts are often like computer manuals: you can only understand them if you already know the material. Clearly, such a text will be readable to the teacher but will be frustrating and nearly useless to the student. That teachers will insist upon using bad textbooks (when good ones are available) is an indictment of their sensitivity to the students' needs. There are many rationales for this behavior, but not one of them is any good.

The state of calculus education is an indicator of all mathematics education. High school calculus is most often a wasted year; it almost never prepares the student for the second semester of college calculus. We will only concern ourselves here with college calculus.

Current calculus texts are incredibly alike. This is demonstrated in a wonderful review — essay actually — by Underwood Dudley [*American Mathematical Monthly*, Vol 95, November 1988]. Anyone interested in calculus texts should read this essay. (See also Professor Dudley's review of three calculus texts, to appear in *The UMAP Journal* probably in 1990, Vol 11, no. 2). Calculus texts tend to be far too long and far too formal. Remember, we are talking about college freshmen. Most of them do not understand proofs, and they are not yet ready for long proofs. *Which brings me to an important digression. Proofs are not something you either understand or not. It takes practice to learn to understand proofs, and as with learning most things, one should start out with the simple and work up to the more complex. This is an ongoing process requiring the entire undergraduate four years. Delta epsilon proofs are difficult for freshmen. Yes, the students need to be exposed to delta epsilon arguments, but it is a mistake to over-emphasize it (for example with choose-the-epsilon-as-a-function-of-delta problems). Time is important; with exposure now, the same students will find delta epsilon proofs easy when they are juniors. It is a mistake to emphasize formalism in freshman calculus. Doing so actually gets in the way of teaching the many difficult concepts of basic calculus.* Current calculus texts do a bad job of teaching calculus. For an example of what a calculus texts should be, see the text by Gilbert Strang [to be published in 1990 by Wellesley-Cambridge Press]. Professor Strang's text features some material which is current, but many of the other sections, such as the material on trigonometry, could have been written two hundred years ago, but likely has never been written as well. I think Strang's text is the best calculus text since Courant's first calculus text and possibly since Euler's (eighteenth century) text.

Preparation

The key to a good lecture is organization. The teacher should go into the classroom intending to expound on one or two ideas. He (or she) should have a predetermined sequence of points. These points are illustrated by examples. The teacher should also be aware of the examples in the text. Sometimes it is sufficient to tell the students that a point is illustrated by a particular example

in the text. Yet the teacher should be prepared to be flexible. A well-prepared lecture is like a flow chart. The actual words are supplied during the lecture and are not prepared beforehand. A lecture should appear somewhat spontaneous.

Lectures

Lectures are a framework for the course. This is where you put your organizing into action. You motivate what is in the text, and you order the material. For example, given ten formulas, probably two are more important than others. Typically, I remember only a few formulas out of any group and I remember how to derive the others from what I have memorized. It is *very* appropriate for the lecturer to share with the students his (or her) mnemonic devices (at this point the students need these devices more than the teacher does). The teacher in effect structures the course through the lecturing process. Again, it is not the lecturer's job to provide another book on the board. Not only is that inappropriate, but it is a waste of the lecturing environment. The primary purpose of the board should be for emphasis and enumeration. The lecturer simultaneously augments the text and incorporates it into the lecture. (Don't get me wrong, I've been forced many times to teach without a text, or to make minimal use of my text, but except for advanced graduate courses, this is usually not desirable.)

The War of the Students and the Teachers

Bad students abound. By bad students I do not mean slow learners, but the lazy and the rude (a good teacher needs to know how to deal with rude students without letting them have an adverse affect on his style — however that is beyond this essay). But given a class of thirty students you should have a fair sample of most basic human characteristics. Unless a class requirement is that the students have been convicted of violent felonies, every class should contain some good students. This is true even at the weakest schools. You just have to open your eyes and look.

A student has a class on Monday and Thursday evenings. One day the instructor says that he can't make the following Thursday, and class will be held on Friday instead. Two students object that they have conflicts. "Tough" he says, if you want to come to class, you will attend on Friday. Fortunately, the administration cancels the class.

So what is unusual about the preceding story? Absolutely nothing. It is merely the most recent story of its type that I have heard. Too many teachers regard the students as the enemy. Students do not have real illnesses or deaths in the family. Or as in the story, they do not deserve the common decency ordinarily accorded to human beings.

Teachers control the teaching environment. A good teacher should be able to handle the bad apples, but students are completely vulnerable to bad teachers. This is true at all levels, but becomes absolutely critical at the graduate level and especially the Ph.D. level. More then one Ph.D. student has blown four years or more down the drain because he has offended a professor.

As we are going into the 1990's, students are paying more and more in tuition and other education expenses and getting less and less in return. Do you remember ever seeing the following notice?: *XYZ University recognizes that Professor Glck's class was not properly taught. All academic records related to that course are being deleted. Please find enclosed a refund of your tuition along with interest.*

Teaching well is not simple. There are many difficult issues to address such as philosophy of testing, philosophy of grading, and so on. However, these issues only become relevant given some competence to begin with. If the teacher does not understand the basics of human communication, then chances are that he is not going to appreciate the fine points of lecturing.